linear Algebra 2

Dr.Jawad Kadhim Khalaf Al-Delfi

Lecture7

Standard form

The objective function lines are drawn for $Z = 15, 30$ and 36. The optimal value for the linear program is 36, and the corresponding objective function line $6x_1 + 10x_2 = 36$ pass through the point C of the feasible region. Thus the feasible solution $x_1 = 1$ and $x_2 = 3$ is the optimal solution.

Definition: A linear programming problem with m constraints and n variables which can be represented as follows:

Maximize (or Minimize): $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

القيو Subject to:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$
$$x_1 \geq 0,\ x_2 \geq 0,\ \ldots,\ x_n \geq 0$$
$$b_1 \geq 0,\ b_2 \geq 0,\ \ldots,\ b_m \geq 0$$

is called a linear program in standard form.

i.e. The linear program is in standard form if it is satisfy the following:

i) The objective function is of the maximization or minimization type.

ii) All the constraints are expressed as equations. القيود جميعها متغيرات

iii) All the variables are non-negative متغيرات غير سالبة

iv) The constant which is on the right hand side of each constraint equation is non-negative. جميع النواتج مال المعادلات
الارقام الموجودة على جهة اليمين من كل معادلة غير سالبة ≥0

Converting a linear programming problem to a problem in standard form:

Since not all the linear programming problems come in the standard form and usually the constraints in the linear program expressed as inequalities rather than equations and some of its decision variables to be not non-negative. Hence to convert the linear program to the standard form we need to go through the following steps:

1) Handling inequality constraints:

Since the standard form requires all the constraints to be equations, we need to convert the inequalities to equations. We can do this by introducing new variables to represent the slack between the left hand side and the right hand side of each inequality. The new variables is called slack variables.

As example if we have the inequality

$$x_1 \leq 24 \qquad x_1 + x_3 = 24$$

we convert it to an equation by introducing a slack variable say x_3 as follows

$x_1 + x_3 = 24$, where $x_3 \geq 0$.

And for the inequality

$$5x_1 + 3x_2 \geq 135$$

we can convert it to an equation by introducing a slack variable say x_4 as follows $\quad 5x_1 + 3x_2 - (5x_1 + 3x_2 - 135) = 135$

$5x_1 + 3x_2 - x_4 = 135$, where $x_4 \geq 0$.

2) Handling variables unrestricted in sign:

In some of the linear programming problems

we got some of their decision variables can take both negative and non-negative values. And since the standard form requires all the variables to be non-negative, then we can write the unrestricted variable as the difference of two non-negative variables.

i.e. if x_1 is an unrestricted variable then we can write it as follows

$x_1 = x_9 - x_{10}$, where $x_9 \geq 0$ and $x_{10} \geq 0$.

Example :

The linear program

Maximize : $Z = 2x_1 - x_2 + x_3$

Subject to : $x_1 + 2x_2 + x_3 \leq 14$

$x_1 - x_2 - 2x_3 \geq 1$

$x_1 + x_2 = -2$

$x_1 \geq 0$, $x_3 \geq 0$ and x_2 is unrestricted,

can be converted to a standard form as follows :

i) Replace x_2 by $x_4 - x_5$ where $x_4 \geq 0$ and $x_5 \geq 0$.

the L.P. becomes

Maximize $Z = 2x_1 - x_4 + x_5 + 3 \rightarrow x_1 + 2x_4 - 2x_5 + x_3 \leq 14$

ii) Multiply both sides of the constraint $x_1 + x_2 = -2$ by -1.

iii) Introduce slack variables x_6 and x_7 to the constraints $x_1 + 2x_2 + x_3 \leq 14$ and $x_1 - x_2 - 2x_3 \geq 1$ respectively.

Thus the above program will becomes

Maximize : $Z = 2x_1 - x_4 + x_5 + x_3$

Subject to : $x_1 + 2x_4 - 2x_5 + x_3 + x_6 = 14$

$$x_1 - x_4 + x_5 - 2x_3 - x_7 = 1$$

$$-x_1 - x_4 + x_5 = 2$$

$x_1 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0,\ x_7 \geq 0.$

Definition: The collection of all possible solutions to the system of constraints is called the solution set.

Definition: Two systems of equations are said to be equivalent if both systems have the same solution set.

Definition: A variable x_i is said to be

a basic variable in a given equation if it appears with a unit coefficient in that equation and zeros in all other equations.

Definition: A variable which is not basic is called non-basic variable.

Definition: A system of m equations which got in each equation at least one basic variable is called a canonical system.

$x_1 + 3x_2 + x_3 + x_5 \leq 12$

$4x_2 - x_3 + x_5 \leq 7$

هذا نظام قانوني

نظام قانوني

Definition: A pivot operation is a sequence of elementary operations which reduces a given system to an equivalent system in which a specified variable has a unit coefficient in one equation and zeros elsewhere.

عملية محورية

Definition: The solution of a canonical system which obtained by setting the non-basic variables to zero and solving for the basic variables is called a basic solution.

21.

ع

Definition: A basic solution in which the values of the basic variables are non-negative is called a basic feasible solution.

Definition: An Adjacent basic feasible solution differs from the present basic feasible solution in exactly one basic variable.

Definition: The net change in the value of Z by increasing the value of the non-basic variable x_i one unit in the basic feasible solution is called the relative profit of the non-basic variable x_i. i.e. the relative profit of x_i = The new value of Z (by changing the value of x_i from 0 to 1) − The old value of Z.

22.

The Simplex Method

$x_1 + 2x_2 + 3x_3$

The basic steps of the simplex method for a maximization problem to find the optimal solution and the optimal value are as follows:

1) Start with an initial basic feasible solution in canonical form.

2) Check whether the current basic feasible solution is optimal. It is optimal if the relative profits of all non-basic variables are negative or zero. If we got a non-basic variable with positive relative profit then the current basic feasible solution is not optimal and we should go to step 3.

3) Select the non-basic variable which has the largest relative profit to be the new basic variable.

4) Find the basic variable which will be replaced by the non-basic va[illegible]ble. This can be done by finding the constr[illegible]t with the lowest limit, and the basic variable i[illegible] that constraint will be replaced.

[illegible]