linear Algebra 2

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Lecture9

Simplex method

Tableau 3

C_B	basis \ c_j	4	3	1	-1	constants	
		x_1	x_2	x_3	x_4		
4	x_1	1	0	$\frac{3}{5}$	$-\frac{1}{5}$	3	$\frac{3}{5}R_1$
3	x_2	0	1	$-\frac{1}{5}$	$\frac{2}{5}$	3	$R_2 - \frac{1}{5}R_1$
$\bar{c}$-row		0	0	$-\frac{4}{5}$	$-\frac{7}{5}$	$Z_j = 21$	$R_2 - \frac{1}{5}\left(\frac{3}{5}R_1\right)$

[illegible]

Since we got all the values of the c_j's in the $\bar{c}$-row are non-positive. Therefore the solution is optimal and hence we got:

The optimal solution is $x_1 = 3$, $x_2 = 3$, $x_3 = 0$, $x_4 = 0$

and the optimal value is $Z = 21$.

Example:

Find the optimal value and the optimal solution of the following linear program

Maximize : $Z = 4x_1 + 3x_2$

Subject to :

$$-x_1 + 2x_2 \le 5$$
$$3x_1 + 4x_2 \le 19$$
$$2x_1 - x_2 \le 2$$
$$x_1 \ge 0 \;,\; x_2 \ge 0$$

[illegible]

Solution : Converting the problem to standard

form by the addition of slack variables, we obtain

Maximize : $Z = 4x_1 + 3x_2$

Subject to :

$$-x_1 + 2x_2 + x_3 = 5$$
$$3x_1 + 4x_2 + x_4 = 19$$
$$2x_1 - x_2 + x_5 = 2$$
$$x_1 \ge 0 \;,\; x_2 \ge 0 \;,\; x_3 \ge 0 \;,\; x_4 \ge 0 \;,\; x_5 \ge 0$$

we have a basic feasible solution in canonical form with x_3, x_4, x_5 as basic variables

Tableau 1

C_B	basis \ c_j	4	3	0	0	0	Constants
		x_1	x_2	x_3	x_4	x_5	
0	x_3	-1	2	1	0	0	5
0	x_4	3	4	0	1	0	19
0	x_5	2	-1	0	0	1	2
$\bar{c}$-row		4	3	0	0	0	$Z = 0$

x_1 will be the new basic variable and the minimum ratio of ∞, $\frac{19}{3}$, $\frac{2}{2}$ is $\frac{2}{2} = 1$ thus the variable x_5 will be non-basic (The circle element in the tableau is called the pivot element.

[illegible]

Tableau 2

C_B	basis \ c_j	4 x_1	3 x_2	0 x_3	0 x_4	0 x_5	Constants	
0	x_3	0	$\frac{3}{2}$	1	0	$\frac{1}{2}$	6	$R_1 + \frac{1}{2} R_3$
0	x_4	0	$\frac{11}{2}$	0	1	$-\frac{3}{2}$	16	$R_2 - \frac{3}{2} R_3$
4	x_1	1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	1	$\frac{1}{2} R_3$
$\bar{c}$-row		0	5	0	0	-2	$Z = 4$	

x_2 will be the new basic variable and the minimum ratio of $\frac{6}{\frac{3}{2}}$, $\frac{16}{\frac{11}{2}}$, ∞ is $\frac{16}{\frac{11}{2}} = \frac{32}{11}$.

Thus the variable x_4 will be non-basic.

Tableau 3

C_B	basis \ c_j	4 x_1	3 x_2	0 x_3	0 x_4	0 x_5	Constants	
0	x_3	0	0	1	$-\frac{3}{11}$	$\frac{10}{11}$	$\frac{18}{11}$	$R_1 - \frac{3}{11} R_2$
3	x_2	0	1	0	$\frac{2}{11}$	$-\frac{3}{11}$	$\frac{32}{11}$	$\frac{2}{11} R_2$
4	x_1	1	0	0	$\frac{1}{11}$	$\frac{4}{11}$	$\frac{27}{11}$	$R_3 + \frac{1}{11} R_2$
$\bar{c}$-row		0	0	0	$-\frac{10}{11}$	$-\frac{7}{11}$	$Z = 18\frac{6}{11}$	

Thus the optimal solution is

$x_1 = \frac{27}{11}$, $x_2 = \frac{32}{11}$, $x_3 = \frac{18}{11}$, $x_4 = 0$, $x_5 = 0$

and the optimal value is $Z = 18\frac{6}{11}$.

Example:

Find the optimal value and the optimal solution of the following linear program

Minimize: $Z = 10x_1 + 7x_2$

Subject to: $x_1 \le 9$

$x_2 \le 11$

$4x_1 + 3x_2 \ge 32$

$x_1 \ge 0$, $x_2 \ge 0$

Solution: This linear program is equivalent to the following linear program problem

Maximize: $Z' = -10x_1 - 7x_2$

Subject to: $x_1 \le 9$

$x_2 \le 11$

$4x_1 + 3x_2 \ge 32$

$x_1 \ge 0$, $x_2 \ge 0$

and by converting the problem to standard form by adding slack variables, we obtain

Maximize: $Z' = -10x_1 - 7x_2$

Subject to: $x_1 + x_3 = 9$

$x_2 + x_4 = 11$

$4x_1 + 3x_2 - x_5 = 32$

$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0, x_5 \ge 0$

31. We convert it to a canonical system in standard form as follows:

Maximize : $Z' = -10x_1 - 7x_2$

Subject to:

$$-\frac{3}{4}x_2 + x_3 + \frac{1}{4}x_5 = 1 \qquad R_1 - \frac{1}{4}R_3$$

$$x_2 + x_4 = 11 \qquad R_2$$

$$x_1 + \frac{3}{4}x_2 - \frac{1}{4}x_5 = 8 \qquad \frac{1}{4}R_3$$

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0$$

We have now a basic feasible solution in canonical form with x_3, x_4, x_1 as basic variables.

Tableau 1

C_B	basis \ C_j	-10 x_1	-7 x_2	0 x_3	0 x_4	0 x_5	constants
0	x_3	0	$-\frac{3}{4}$	1	0	$\frac{1}{4}$	1
0	x_4	0	1	0	1	0	11
-10	x_1	1	$\frac{3}{4}$	0	0	$-\frac{1}{4}$	8
$\bar{C}$-row		0	$\frac{1}{2}$	0	0	$-\frac{10}{4}$	$Z' = -80$

x_2 will be the new basic variable and the minimum ratio of $\infty, 11, \frac{8}{\frac{3}{4}}$ is $\frac{32}{3}$ thus the variable x_1 will be non-basic

32. Tableau 2

C_B	basis \ C_j	-10 x_1	-7 x_2	0 x_3	0 x_4	0 x_5	constants	
0	x_3	1	0	1	0	0	9	$R_1 + R_3$
0	x_4	$-\frac{4}{3}$	0	0	1	$\frac{1}{3}$	$\frac{1}{3}$	$R_2 - \frac{4}{3}R_3$
-7	x_2	$\frac{4}{3}$	1	0	0	$-\frac{1}{3}$	$\frac{32}{3}$	$\frac{4}{3}R_3$
$\bar{C}$-row		$-\frac{2}{3}$	0	0	0	$-\frac{7}{3}$	$Z' = -\frac{224}{3}$	

Thus the optimal solution is $x_1 = 0,\ x_2 = \frac{32}{3},\ x_3 = 9,\ x_4 = \frac{1}{3},\ x_5 = 0$ and the optimal value is $Z = \frac{224}{3} = 74\frac{2}{3}$

Solving Minimization Problems by Using The Simplex Method in Tableau Form:

The basic steps for solving the minimization problem by using the simplex method in tableau forms are:

1.) Express the problem in standard form, and start with an initial basic feasible solution in canonical form, and then set up the initial tableau.

2) If all the $\bar{c}_j$ coefficients in the $\bar{C}$-row.

33. are non-negative, the current basic feasible solution is optimal. Otherwise, select the non basic variable with the lowest value in the $\bar{C}$-row to enter the basis. نستخدم اكل تطيف حاصل

3) Apply the minimum ratio rule to determine the basic variable to leave the basis

4) Find the new tableau of the new canonical system by using the pivot operation, and then return to step 2.

Ties in The Selection of The Non Basic Variable:

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In maximization problems (minimization problems) If there exists more than one variable with the same largest positive value (the same lowest negative value) in the $\bar{C}$-row. Then we select any one of them.

Definition: If one or more than one basic variable are equal to zero in the basic feasible solution. then the basic feasible is called

34. a degenerate basic feasible solution.

مثال

Ties in The Minimum Ratio Rule: تعادل

When we got two constraints having the same least ratio value This result in a tie for selecting which basic variable will leave the basis, and this may introduce some complications in solving the problem

[redacted]

Maximize: $Z = 4X_1 + 3X_3$

Subject to: $X_1 - X_2 \leq 2$

$$2X_1 + X_3 \leq 4$$

$$X_1 + X_2 + X_3 \leq 3$$

$$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0$$

Converting the problem to standard form by adding slack variables, we get