linear Algebra 2

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Lecture10

Simplex method

33. are non-negative, the current basic feasible solution is optimal. Otherwise, select the non basic variable with the lowest value in the $\bar{C}$-row to enter the basis. نستخدم اكل تقلية حاصل

3) Apply the minimum ratio rule to determine the basic variable to leave the basis

4) Find the new tableau of the new canonical system by using the pivot operation, and then return to step 2.

Ties in The Selection of The Non Basic Variable:

إذا كانت 2 قيمة نفس الرقم والإشارة أي واحد نأخذه عادي (بالجدول)

In maximization problems (minimization problems) If there exists more than one variable with the same largest positive value (the same lowest negative value) in the $\bar{C}$-row. Then we select any one of them.

Definition: If one or more than one basic variable are equal to zero in the basic feasible solution. then the basic feasible is called

34. a degenerate basic feasible solution.

تعادل

Ties in The Minimum Ratio Rule: تعادل

When we got two constraints having the same least ratio value. This result in a tie for selecting which basic variable will leave the basis, and this may introduce some complications in solving the problem

[redacted]

Maximize: $Z = 4X_1 + 3X_3$

Subject to: $X_1 - X_2 \leq 2$

$$2X_1 + X_3 \leq 4$$

$$X_1 + X_2 + X_3 \leq 3$$

$$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0$$

Converting the problem to standard form by adding slack variables, we get

35. Maximize : $Z = 4x_1 + 3x_3$

Subject to :

$$x_1 - x_2 + x_4 = 2$$
$$2x_1 + x_3 + x_5 = 4$$
$$x_1 + x_2 + x_3 + x_6 = 3$$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0.$

We have a basic feasible solution in canonical form with x_4, x_5, x_6 as basic variables

Tableau 1

C_B	basis \ c_j	4	0	3	0	0	0	Constants
		x_1	x_2	x_3	x_4	x_5	x_6	
0	x_4	1	-1	0	1	0	0	2
0	x_5	2	0	1	0	1	0	4
0	x_6	1	1	1	0	0	1	3
$\bar{c}$-row		4	0	3	0	0	0	Z=0

الجدول الأساسي

هذا اعلى قيمة

x_1 will be the new basic variable, but we have a tie in the minimum ratio of $\frac{2}{1}, \frac{4}{2}, \frac{3}{1}$ which is 2 thus we have either x_4 will be non basic or x_5 will be non basic. Let us first select x_4 to be non-basic

صف اول

نفس الناتج

Tableau 2a

C_B	basis \ c_j	4	0	3	0	0	0	Constants	
		x_1	x_2	x_3	x_4	x_5	x_6		
4	x_1	1	-1	0	1	0	0	2	R_1
0	x_5	0	2	1	-2	1	0	0	$R_2 - 2R_1$
0	x_6	0	2	1	-1	0	1	1	$R_3 - R_1$
$\bar{c}$-row		0	4	3	-4	0	0	Z = 8	

الصف الرئيسي

x_2 will be the new basic variable and the minimum ratio of $\infty, \frac{0}{2}, \frac{3}{2}$ is 0 and thus x_5 will be non-basic.

Tableau 3a

C_B	basis \ c_j	4	0	3	0	0	0	Constants	
		x_1	x_2	x_3	x_4	x_5	x_6		
4	x_1	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	2	$R_1 + \frac{1}{2}R_2$
0	x_2	0	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	$\frac{1}{2}R_2$
0	x_6	0	0	0	1	-1	1	1	$R_3 - R_2$
$\bar{c}$-row		0	0	1	0	-2	0	Z = 8	

Notice that there is no improvement in the value of Z happen in tableau 3a.

x_3 will be the new basic variable and the minimum ratio of $\frac{2}{\frac{1}{2}}, \frac{0}{\frac{1}{2}}, \infty$ is 0, and x_2 will be non-basic

Tableau 4a

C_B	basis \ c_j	4 (x_1)	0 (x_2)	3 (x_3)	0 (x_4)	0 (x_5)	0 (x_6)	Constants	
4	x_1	1	-1	0	1	0	0	2	$R_1 - R_2$
3	x_3	0	2	1	-2	1	0	0	$2R_2$
0	x_6	0	0	0	1	-1	1	1	R_3
$\bar{c}$-row		0	-2	0	2	-3	0	$Z=8$	

x_4 will be the new basic variable, and the minimum ratio of $\frac{2}{1}, \infty, \frac{1}{1}$ is 1 thus x_6 will be non-basic

Tableau 5a

C_B	basis \ c_j	4 (x_1)	0 (x_2)	3 (x_3)	0 (x_4)	0 (x_5)	0 (x_6)	Constants	
4	x_1	1	-1	0	0	1	-1	1	$R_1 - R_3$
3	x_3	0	2	1	0	-1	2	2	$R_2 + 2R_3$
0	x_4	0	0	0	1	-1	1	1	R_3
$\bar{c}$-row		0	-2	0	0	-1	-2	$Z=10$	

Thus the optimal solution is $x_1 = 1$, $x_2 = 0$, $x_3 = 2$, $x_4 = 1$, $x_5 = 0$, $x_6 = 0$ and the optimal value is $Z = 10$.

We go back now to tableau 1 and we select x_5 to be non-basic

Tableau 2b

C_B	basis \ c_j	4 (x_1)	0 (x_2)	3 (x_3)	0 (x_4)	0 (x_5)	0 (x_6)	Constants	
0	x_4	0	-1	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	$R_1 - \frac{1}{2}R_2$
4	x_1	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	2	$\frac{1}{2}R_2$
0	x_6	0	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	1	$R_3 - \frac{1}{2}R_2$
$\bar{c}$-row		0	0	1	0	-2	0	$Z=8$	

العنصر الرئيسي

x_3 will be the new basic variable and the minimum ratio of $\infty, \frac{2}{\frac{1}{2}}, \frac{1}{\frac{1}{2}}$ is 2 and thus x_6 will be non-basic

Tableau 3b

C_B	basis \ c_j	4 (x_1)	0 (x_2)	3 (x_3)	0 (x_4)	0 (x_5)	0 (x_6)	Constants	
0	x_4	0	0	0	1	-1	1	1	$R_1 + R_3$
4	x_1	1	-1	0	0	1	-1	1	$R_2 - R_3$
3	x_3	0	2	1	0	-1	2	2	$2R_3$
$\bar{c}$-row		0	-2	0	0	-1	-2	$Z=10$	

Thus the optimal solution is $x_1 = 1$, $x_2 = 0$, $x_3 = 2$, $x_4 = 1$, $x_5 = 0$, $x_6 = 0$ and the optimal value is $Z = 10$

Un Bounded Solution :

One of the complication of the minimum ratio rule when we are not able to determine which basic variable should leave the basis. This is happen when we have all the coefficients of the selected variable to enter the basis are non-positive. In this case we have unbounded solution.

Example: Find the optimal solution of the following linear program

Maximize : $Z = 3x_1 + 4x_2$

Subject to : $2x_1 - 3x_2 + x_3 = 2$

$-3x_1 + 2x_2 + x_4 = 4$

$x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$.

We have a basic feasible solution in canonical form with x_3 , x_4 as basic variables

Tableau 1

C_B	basis \ c_j	3	4	0	0	Constants
		x_1	x_2	x_3	x_4	
0	x_3	2	-3	1	0	2
0	x_4	-3	2	0	1	4
$\bar{C}$-row		3	4	0	0	$Z=0$

x_2 will be the new basic variable and the minimum ratio of ∞ and $\frac{4}{2}$ is 2 , thus x_4 will be non-basic.

Tableau 2

C_B	basis \ c_j	3	4	0	0	Constants	
		x_1	x_2	x_3	x_4		
0	x_3	$-\frac{5}{2}$	0	1	$\frac{3}{2}$	8	$R_1 + \frac{3}{2}R_2$
4	x_2	$-\frac{3}{2}$	1	0	$\frac{1}{2}$	2	$\frac{1}{2}R_2$
$\bar{C}$-row		9	0	0	-2	$Z=8$	

x_1 will be the new basic variable but both ratios are infinite. Thus when x_1 increase then x_2 and x_3 will also increase and hence the linear program has unbounded solution.

Finding a basic feasible solution:

After we convert the problem to the standard form, we may find that some of the constraints have no basic variable. In this case we can add a new variable to act as the basic variable in each of those constraints, and then we will have the system in canonical form.