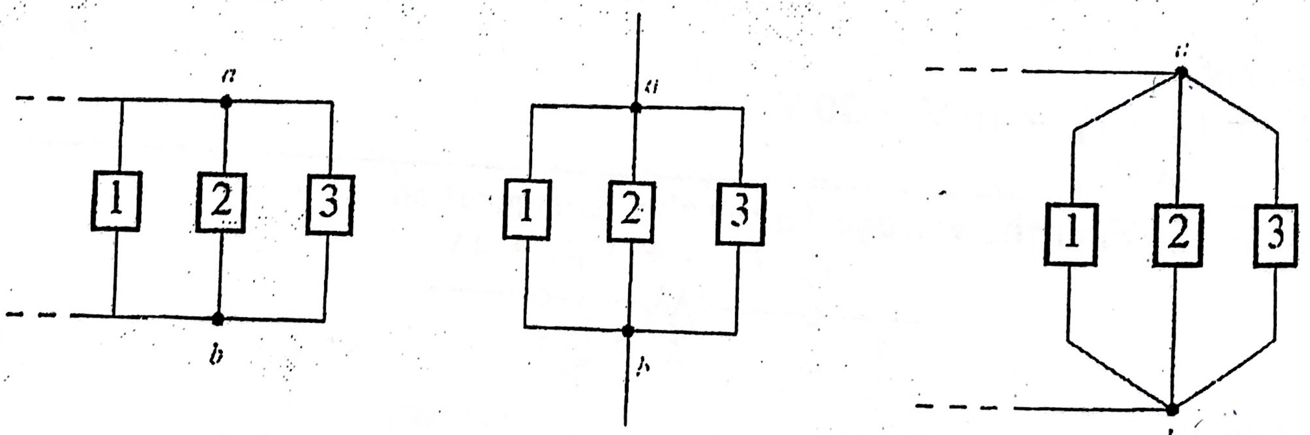
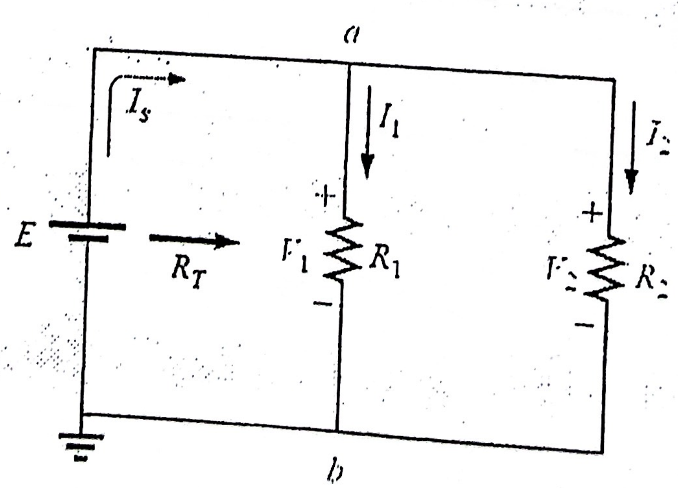
Parallel Circuits:

Two elements, branches, or networks are in parallel if they have two points in common.



(a) (b) (c)

- The voltage across parallel elements is the same, but the current is different.

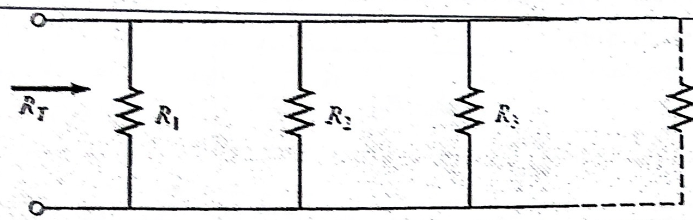


$$E = V1 = V2$$

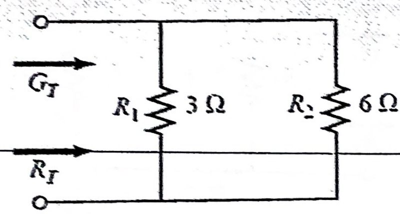
- For parallel elements, the total conductance is the sum of the individual conductances.
- The total resistance of parallel resistors is always less than the value of the smallest resistor.

$$\boxed{G_T = G_1 + G_2 + G_3 + \cdots + G_N}$$

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$



EXAMPLE: Determine the total conductance and resistance for the parallel network.

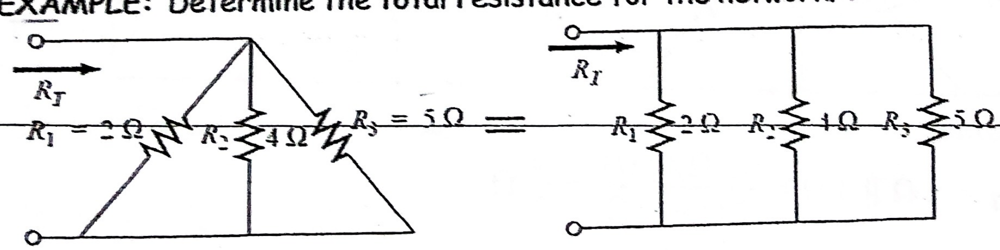


Solution:

$$G_T = G_1 + G_2 = \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega} = 0.333\text{ S} + 0.167\text{ S} = 0.5\text{ S}$$

and
$$R_T = \frac{1}{G_T} = \frac{1}{0.5\text{ S}} = 2\,\Omega$$

EXAMPLE: Determine the total resistance for the network.



Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{5\,\Omega} = 0.5\text{ S} + 0.25\text{ S} + 0.2\text{ S}$$

$$= 0.95\text{ S}$$

and $$R_T = \frac{1}{0.95\text{ S}} = 1.053\,\Omega$$

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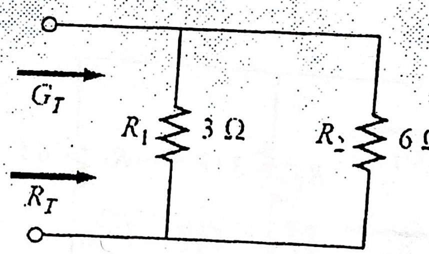
* The total resistance of two parallel resistors is the product of the two divided by their sum.

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

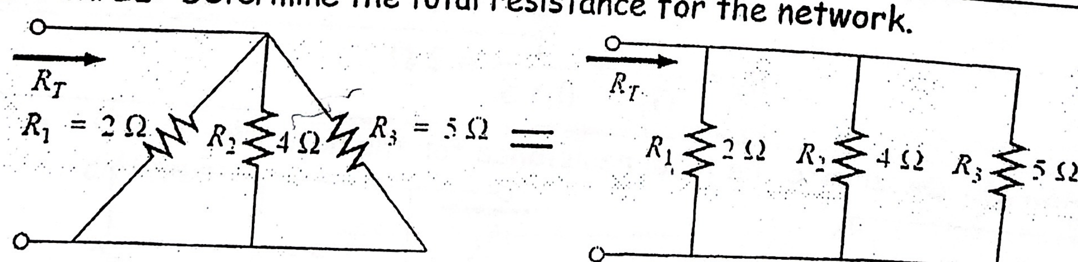
EXAMPLE: Determine the total conductance and resistance for the parallel network.



Solution:

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(3\,\Omega)(6\,\Omega)}{3\,\Omega + 6\,\Omega} = \frac{18\,\Omega}{9} = 2\,\Omega$$

EXAMPLE: Determine the total resistance for the network.

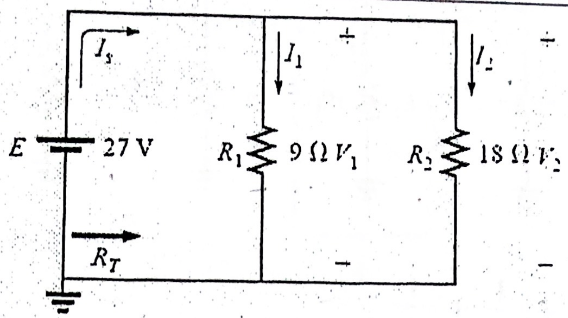


Solution:

$$R'_T = 2\,\Omega \parallel 4\,\Omega = \frac{(2\,\Omega)(4\,\Omega)}{2\,\Omega + 4\,\Omega} = \frac{4}{3}\,\Omega$$

$$R_T = R'_T \parallel 5\,\Omega = \frac{\left(\frac{4}{3}\,\Omega\right)(5\,\Omega)}{\frac{4}{3}\,\Omega + 5\,\Omega} = 1.053\,\Omega$$

EXAMPLE: For the parallel network.



a. Calculate R_T.
b. Determine I_s.
c. Calculate I_1 and I_2, and demonstrate that $I_s = I_1 + I_2$.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

Solution:

a. $R_T = \dfrac{R_1R_2}{R_1 + R_2} = \dfrac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \dfrac{162\ \Omega}{27} = 6\ \Omega$

b. $I_s = \dfrac{E}{R_T} = \dfrac{27\ \text{V}}{6\ \Omega} = 4.5\ \text{A}$

c. $I_1 = \dfrac{V_1}{R_1} = \dfrac{E}{R_1} = \dfrac{27\ \text{V}}{9\ \Omega} = 3\ \text{A}$

$I_2 = \dfrac{V_2}{R_2} = \dfrac{E}{R_2} = \dfrac{27\ \text{V}}{18\ \Omega} = 1.5\ \text{A}$

$I_s = I_1 + I_2$

$4.5\ \text{A} = 3\ \text{A} + 1.5\ \text{A}$

$4.5\ \text{A} = 4.5\ \text{A}$ (checks)

d. $P_1 = V_1I_1 = EI_1 = (27\ \text{V})(3\ \text{A}) = 81\ \text{W}$

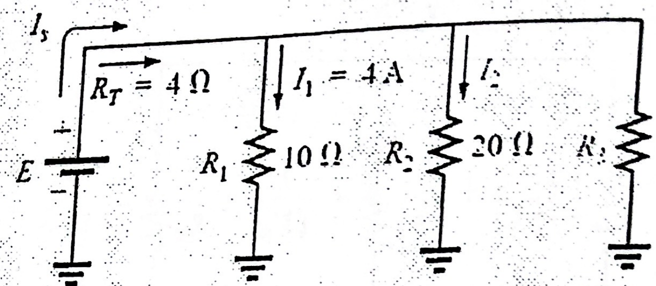
$P_2 = V_2I_2 = EI_2 = (27\ \text{V})(1.5\ \text{A}) = 40.5\ \text{W}$

e. $P_s = EI_s = (27\ \text{V})(4.5\ \text{A}) = 121.5\ \text{W}$

$= P_1 + P_2 = 81\ \text{W} + 40.5\ \text{W} = 121.5\ \text{W}$

EXAMPLE: Given the information provided.

a. Determine $R3$.
b. Calculate E.
c. Find Is.
d. Find $I2$.
e. Determine $P2$.



Solution:

a. $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4\ \Omega} = \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{R_3}$$

$$0.25\ \text{S} = 0.1\ \text{S} + 0.05\ \text{S} + \frac{1}{R_3}$$

$$0.25\ \text{S} = 0.15\ \text{S} + \frac{1}{R_3}$$

$$\frac{1}{R_3} = 0.1\ \text{S}$$

$$R_3 = \frac{1}{0.1\ \text{S}} = 10\ \Omega$$

b. $E = V_1 = I_1 R_1 = (4\ \text{A})(10\ \Omega) = 40\ \text{V}$

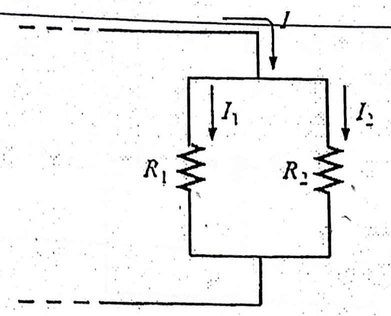
c. $I_s = \dfrac{E}{R_T} = \dfrac{40\ \text{V}}{4\ \Omega} = 10\ \text{A}$

d. $I_2 = \dfrac{V_2}{R_2} = \dfrac{E}{R_2} = \dfrac{40\ \text{V}}{20\ \Omega} = 2\ \text{A}$

e. $P_2 = I_2^2 R_2 = (2\ \text{A})^2(20\ \Omega) = 80\ \text{W}$

CURRENT DIVIDER RULE:

- For two parallel elements of equal value, the current will divide equally.
- For parallel elements with different values, the smaller the resistance, the greater the share of input current.
- For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

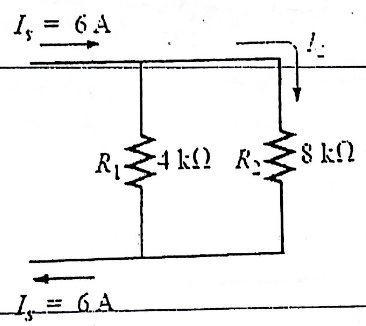


Note difference in subscripts.

$$\boxed{I_1 = \frac{R_2 I}{R_1 + R_2}}$$

$$\boxed{I_2 = \frac{R_1 I}{R_1 + R_2}}$$

EXAMPLE: Determine the current I_2 for the network using the current divider rule.

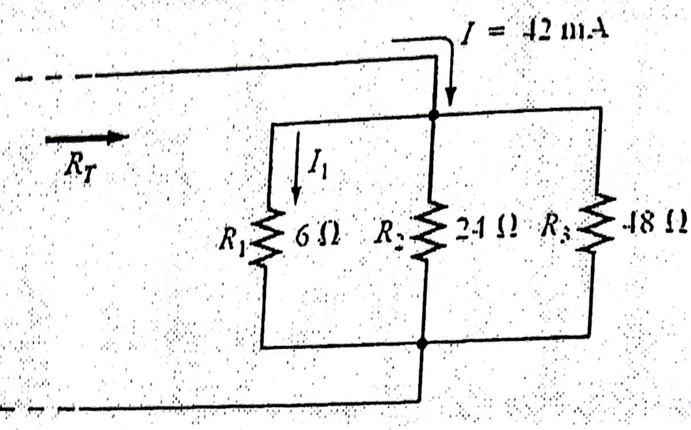


Solution:

$$I_2 = \frac{R_1 I_s}{R_1 + R_2} = \frac{(4\ k\Omega)(6\ A)}{4\ k\Omega + 8\ k\Omega} = \frac{4}{12}(6\ A) = \frac{1}{3}(6\ A)$$

$$= 2\ A$$

EXAMPLE: Find the current I_1 for the network.

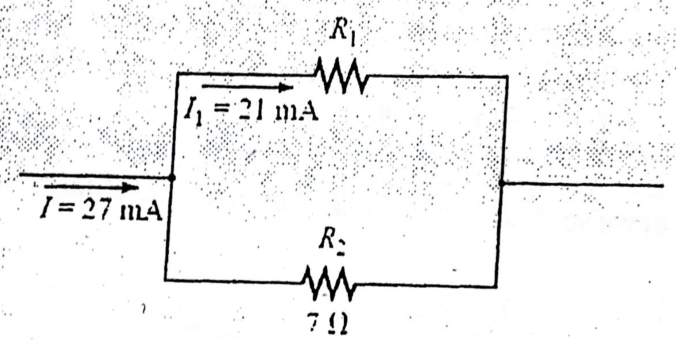


Solution:

$$24\ \Omega \parallel 48\ \Omega = \frac{(24\ \Omega)(48\ \Omega)}{24\ \Omega + 48\ \Omega} = 16\ \Omega$$

$$I_1 = \frac{16\ \Omega(42\ \text{mA})}{16\ \Omega + 6\ \Omega} = 30.54\ \text{mA}$$

EXAMPLE: Determine the resistance *R1* to effect the division of current.



Solution:

$$I_1 = \frac{R_2 I}{R_1 + R_2}$$

$$(R_1 + R_2)I_1 = R_2 I$$
$$R_1 I_1 + R_2 I_1 = R_2 I$$
$$R_1 I_1 = R_2 I - R_2 I_1$$
$$R_1 = \frac{R_2(I - I_1)}{I_1}$$

$$R_1 = \frac{7\ \Omega(27\ \text{mA} - 21\ \text{mA})}{21\ \text{mA}}$$

$$= 7\ \Omega\left(\frac{6}{21}\right) = \frac{42\ \Omega}{21} = 2\ \Omega$$