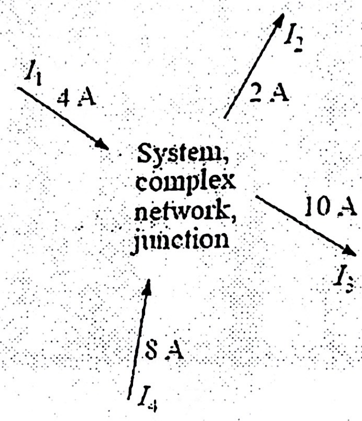
Kirchoff's Laws

* Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero.

Or,

* The sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction.

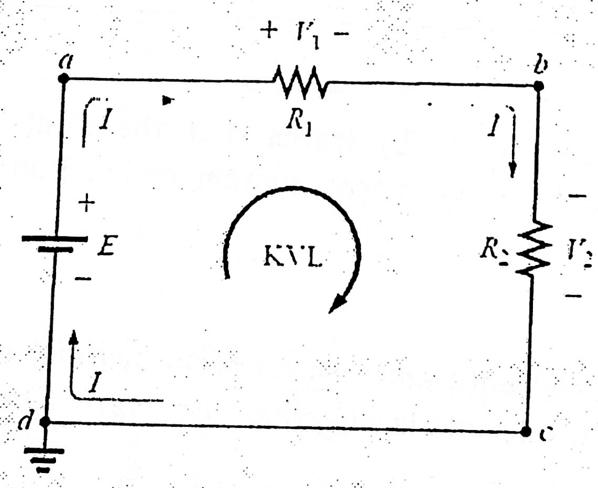
$$\boxed{\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}}$$



$$I_1 + I_4 = I_2 + I_3$$
$$4\,A + 8\,A = 2\,A + 10\,A$$
$$12\,A = 12\,A$$

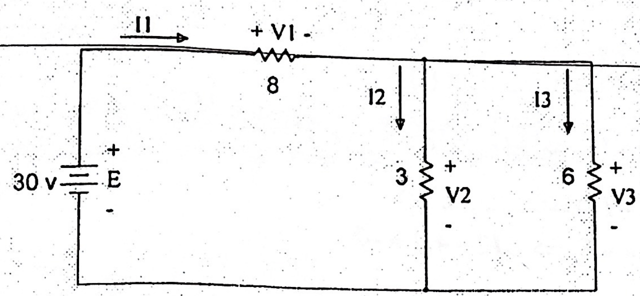
* Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

$$\boxed{\Sigma_{\circlearrowright} V = 0}$$



$$+E - V_1 - V_2 = 0$$
$$E = V_1 + V_2$$

EXAMPLE: using Kirchhoff's lows. Find the currents and voltages in the circuit shown.



Solution:

$$E - V1 - V2 = 0 \quad \Rightarrow \quad 30 - V1 - V2 = 0$$

$$30 - 8I1 - 3I2 = 0 \quad \Rightarrow \quad I1 = \frac{30 - 3I2}{8}$$

$$V2 - V3 = 0 \quad \Rightarrow \quad V2 = V3$$

$$6I3 = 3I2 \quad \Rightarrow \quad I3 = \frac{I2}{3}$$

$$I1 = I2 + I3 \quad \Rightarrow I1 - I2 - I3 = 0$$

By substitute I1 and I3

$$\frac{30 - 3I2}{8} - I2 - \frac{I2}{2} = 0 \quad \Rightarrow I2 = 2A$$

$$I1 = \frac{30 - 3I2}{8} = \frac{30 - 3(2)}{8} = 3A$$

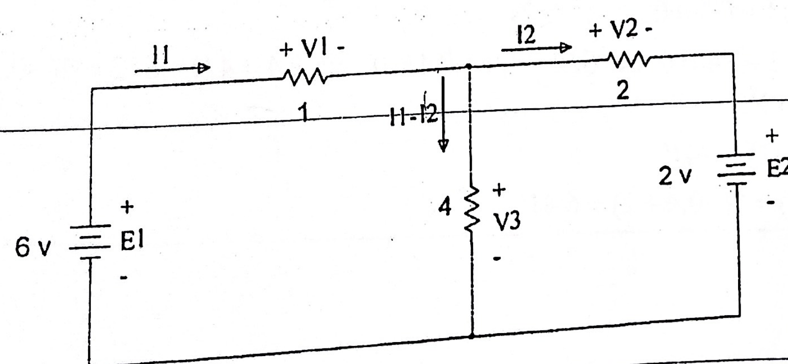
$$I3 = \frac{I2}{2} = \frac{2}{2} = 1A$$

$$V1 = 8I1 \quad \Rightarrow V1 = 8(3) = 24V$$

$$V2 = 3I2 \Rightarrow V2 = 3(2) = 6V$$

$$V3 = 6I3 \Rightarrow V3 = 6(1) = 6V$$

EXAMPLE: using Kirchhoff's lows. Find the currents and voltages in the circuit shown.



Solution:

$I1+4(I1-I2)-E1=0$ ....(1)

$I1+4I1-4I2=E1 \Rightarrow 5I1-4I2=6 \Rightarrow I1=\dfrac{6+4I2}{5}$

$2I2+E2-4(I1-I2)=0$ ....(2)

$2I2-4I1+4I2=-E \Rightarrow 6I2-4I1=-2$

By substitute I1

$6I2-4(\dfrac{6+4I2}{5})=-2 \Rightarrow 6I2-\dfrac{24}{5}-\dfrac{16I2}{5}=-2 \Rightarrow 30I2-24-16I2=-10$

$14I2=14 \Rightarrow I2=\dfrac{14}{14}=1A$

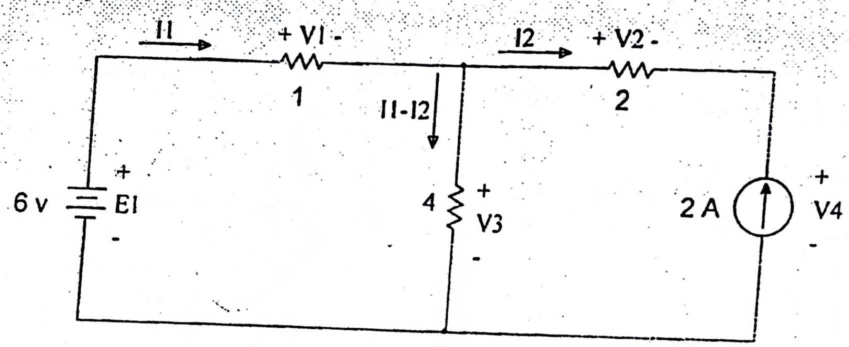
$I1=\dfrac{6+4(1)}{5}=2A$

$V1=1I1=2V$

$V2=2I2=2(1)=2V$

$V3=4(I1-I2)=4(2-1)=4V$

EXAMPLE: using Kirchhoff's lows. Find the currents and voltages in the circuit shown.



Solution:

From the circuit I2=-2 A

$I1+4(I1-I2)-E1=0$ ....(1)

$I1+4I1-4I2=E1 \Rightarrow 5I1-4I2=6 \Rightarrow I1=\dfrac{6+4I2}{5}=\dfrac{6+4(-2)}{5}=-0.4A$

$2I2+V4-4(I1-I2)=0$ ....(2)

$2I2-4I1+4I2+V4=0 \Rightarrow 6I2-4I1+V4=0 \Rightarrow V4=4I1-6I2=4(-0.4)-6(-2)=10.4$

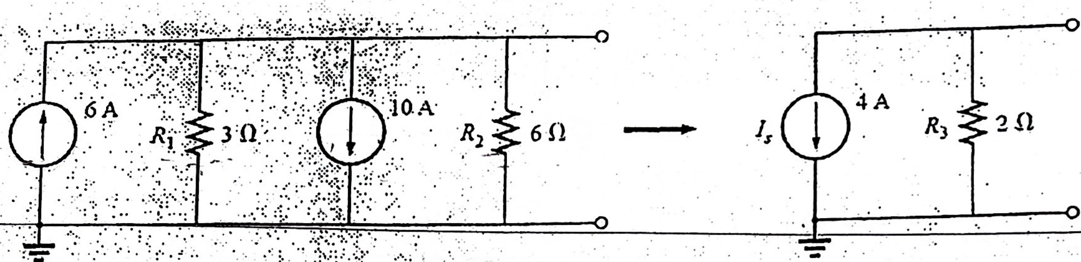
$V1=1I1=-0.4\,V$

$V2=2I2=2(-2)=-4V$

$V3=4(I1-I2)=4(-0.4+2)=6.4V$

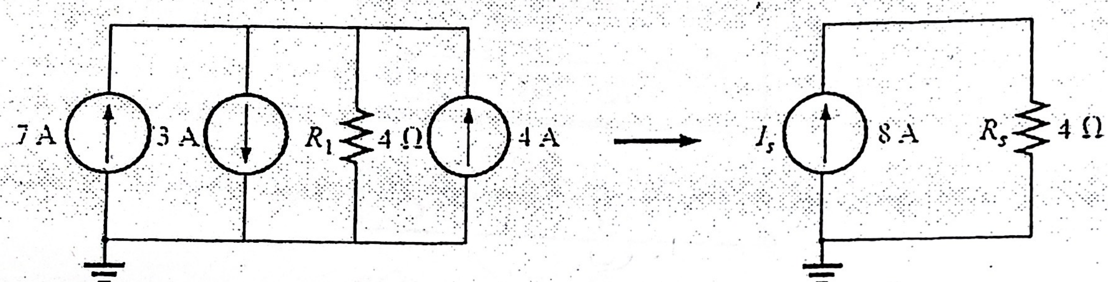
Current Sources in Parallel

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant



$I_s = 10\text{ A} - 6\text{ A} = 4\text{ A}$
$R_s = 3\ \Omega \parallel 6\ \Omega = 2\ \Omega$ $\Rightarrow R_s = \dfrac{R_1 * R_2}{R_1 + R_2}$

$R_s = \dfrac{3*6}{3+6} = \dfrac{18}{9} = 2$



$I_s = 7\text{ A} + 4\text{ A} - 3\text{ A} = 8\text{ A}$
$R_s = R_1 = 4\ \Omega$