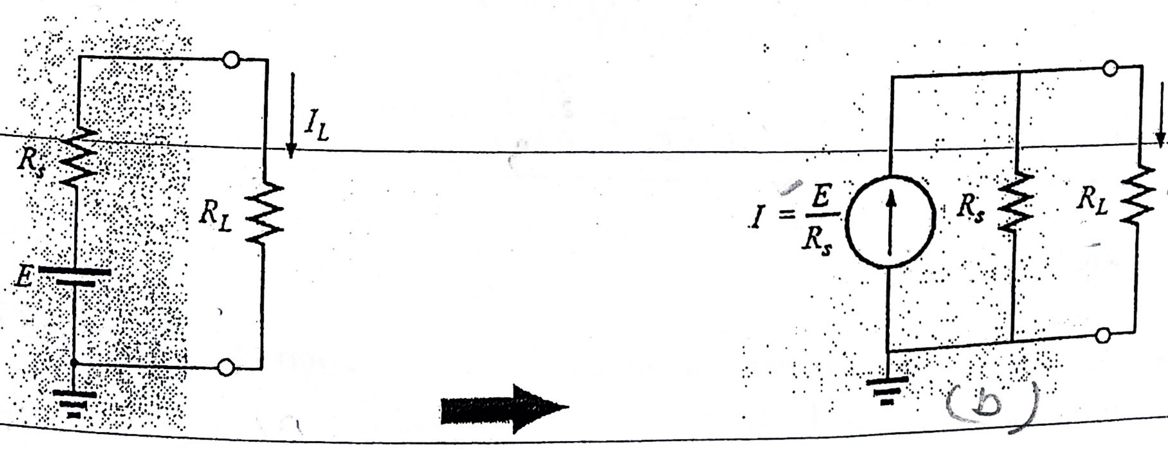
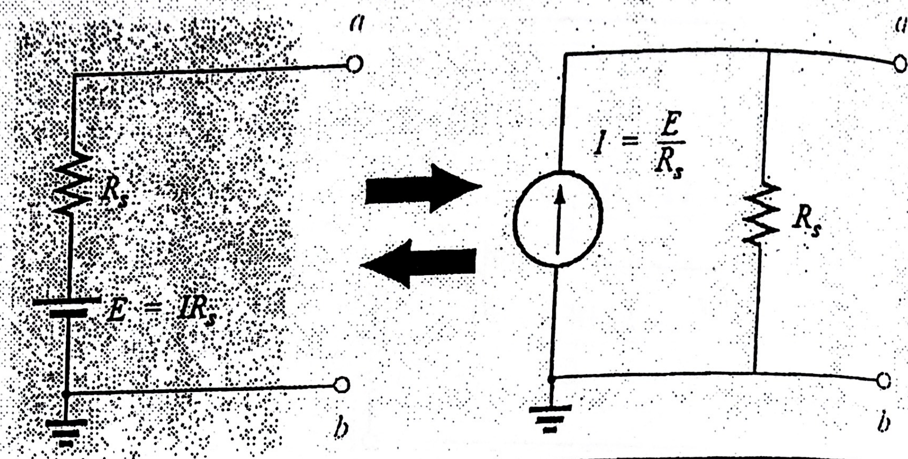
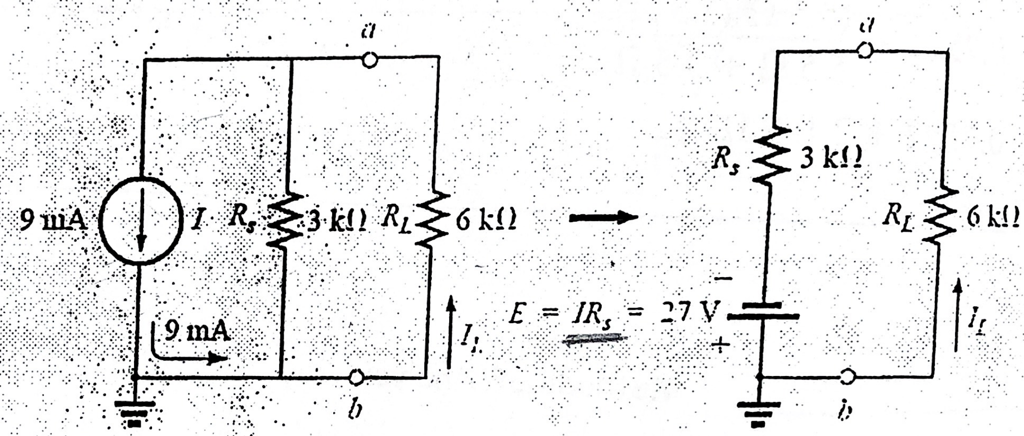
Source Conversions

To perform a conversion from one type of source to another, a voltage source must have a resistor in series with it, and a current source must have a resistor in parallel.





EXAMPLE: Convert the current source of the circuit below to a voltage source, and find the load current for each source.



(a) (b)

Solution:

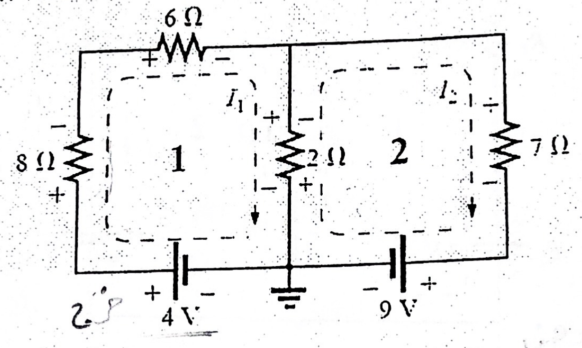
(a): $I_L = \dfrac{R_s I}{R_s + R_L} = \dfrac{(3\ \text{k}\Omega)(9\ \text{mA})}{3\ \text{k}\Omega + 6\ \text{k}\Omega} = 3\ \text{mA}$

(b): $I_L = \dfrac{E}{R_s + R_L} = \dfrac{27\ \text{V}}{3\ \text{k}\Omega + 6\ \text{k}\Omega} = \dfrac{27\ \text{V}}{9\ \text{k}\Omega} = 3\ \text{mA}$

Mesh Analysis (Loop)

- Assign a distinct current in the clockwise direction to each independent, closed loop of the network.
- Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop.
- Write the linear equations for each loop.
- Solve the resulting linear equations for the assumed loop currents.

EXAMPLE: Write the mesh equations for the network, and find the current through the 7-Ω resistor.



Solution:

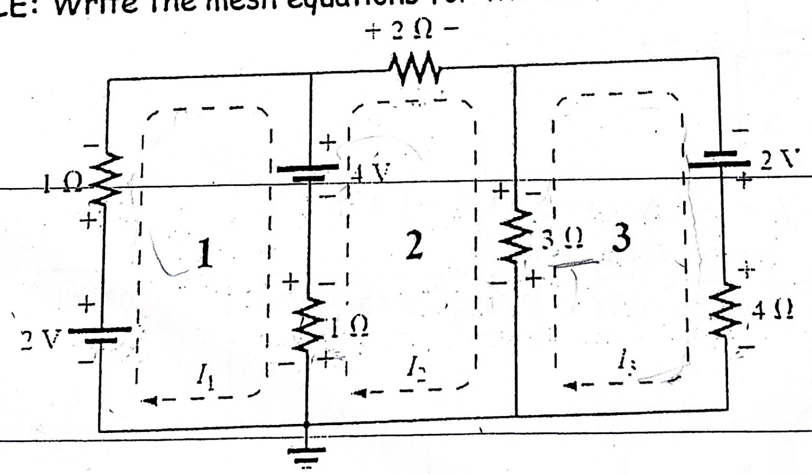
I_1: $(8\ \Omega + 6\ \Omega + 2\ \Omega)I_1 - (2\ \Omega)I_2 = 4\text{ V}$
I_2: $(7\ \Omega + 2\ \Omega)I_2 - (2\ \Omega)I_1 = -9\text{ V}$

$$16I_1 - 2I_2 = 4$$
$$9I_2 - 2I_1 = -9$$

$$16I_1 - 2I_2 = 4$$
$$-2I_1 + 9I_2 = -9$$

$$I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144 + 8}{144 - 4} = \frac{-136}{140}$$
$$= \mathbf{-0.971\ A}$$

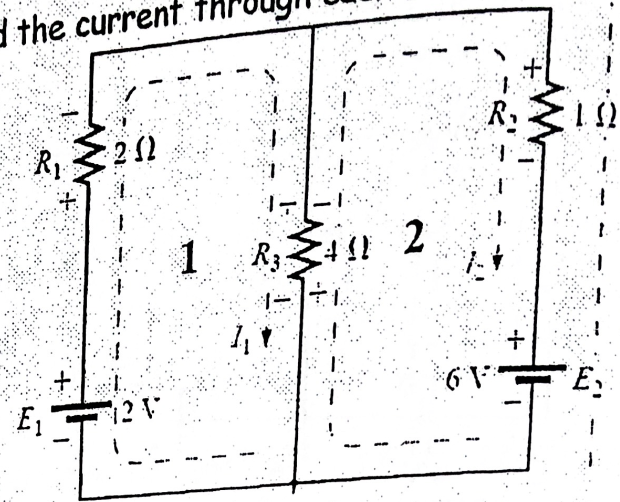
EXAMPLE: Write the mesh equations for the network



Solution:

$$2I_1 - I_2 + 0 = -2$$
$$6I_2 - I_1 - 3I_3 = 4$$
$$7I_3 - 3I_2 + 0 = 2$$

EXAMPLE: Find the current through each branch of the network



Solution:

$6I1-4I2=2$ ....(1)

$5I2-4I1=-6$ ....(2)

$6I1\ \ -4I2=2$

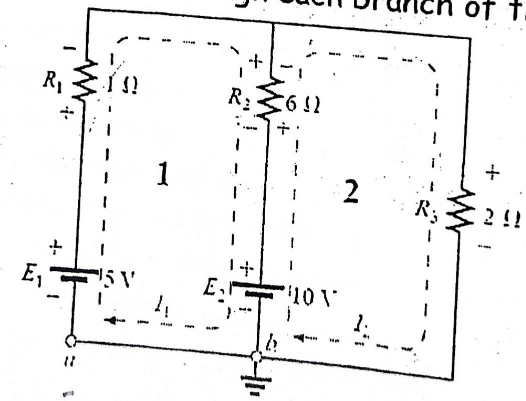
$-4I1+5I2=-6$

$$I1=\frac{\begin{bmatrix}2 & -4\\ -6 & 5\end{bmatrix}}{\begin{bmatrix}6 & -4\\ -4 & 5\end{bmatrix}}=\frac{(2\times5)-(-4\times-6)}{(6\times5)-(-4\times-4)}=\frac{10-24}{30-16}=\frac{-14}{14}=-1\,A$$

$$I2=\frac{\begin{bmatrix}6 & 2\\ -4 & -6\end{bmatrix}}{\begin{bmatrix}6 & -4\\ -4 & 5\end{bmatrix}}=-2\,A$$

$$I_{4\Omega}=I_1-I_2=-1\text{ A}-(-2\text{ A})=-1\text{ A}+2\text{ A}$$
$$=1\text{ A}\quad(\text{in the direction of } I_1)$$

EXAMPLE: Find the current through each branch of the network



Solution:

$7I1-6I2=5-10$ ....(1)

$8I2-6I1=10$ ....(2)

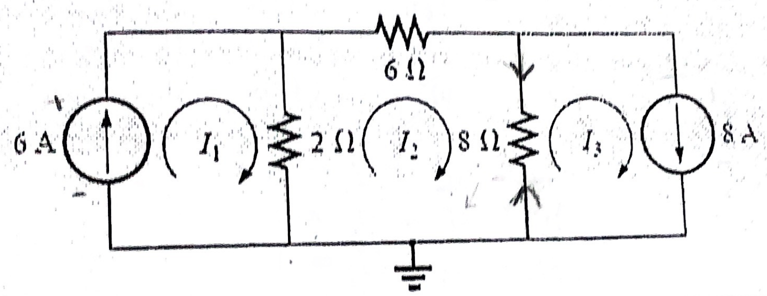
$-7I_1+6I_2=5$
$+6I_1-8I_2=-10$

$$I_1=\frac{\begin{vmatrix}5 & 6\\ -10 & -8\end{vmatrix}}{\begin{vmatrix}-7 & 6\\ 6 & -8\end{vmatrix}}=\frac{-40+60}{56-36}=\frac{20}{20}=1\text{ A}$$

$$I_2=\frac{\begin{vmatrix}-7 & 5\\ 6 & -10\end{vmatrix}}{20}=\frac{70-30}{20}=\frac{40}{20}=2\text{ A}$$

$I_{R_2}=I_2-I_1=2\text{ A}-1\text{ A}=1\text{ A}$ in the direction of I_2

EXAMPLE: Using mesh analysis, determine the currents of the network



Solution:

$I_1=6\text{ A}$

$I_3=8\text{ A}$

$2I_1-16I_2+8I_3=0$

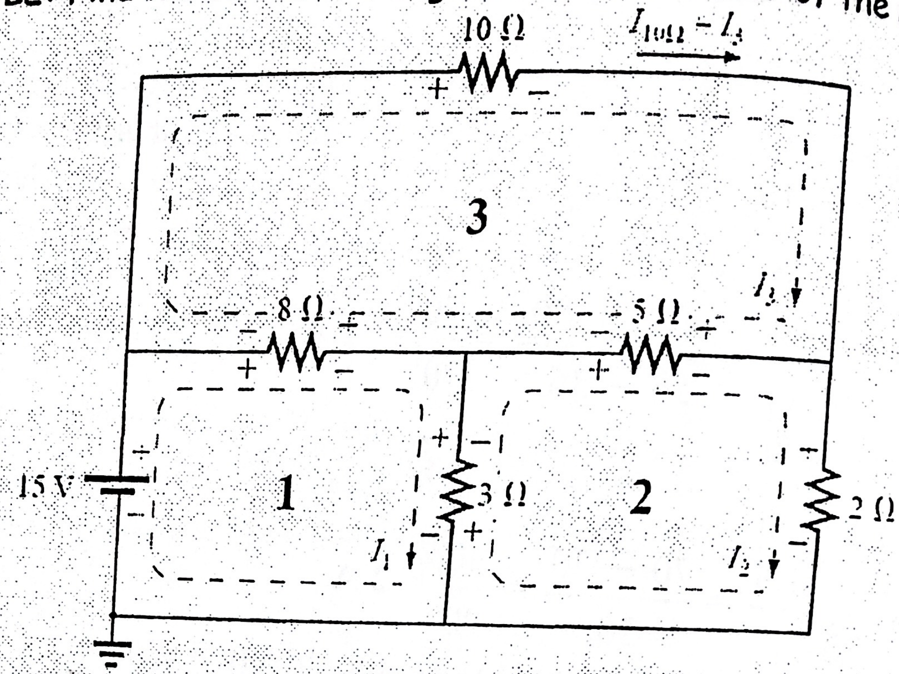
$2(6\text{ A})-16I_2+8(8\text{ A})=0$

$$I_2=\frac{76\text{ A}}{16}=4.75\text{ A}$$

$I_{2\Omega}\downarrow=I_1-I_2=6\text{ A}-4.75\text{ A}=1.25\text{ A}$

$I_{8\Omega}\uparrow=I_3-I_2=8\text{ A}-4.75\text{ A}=3.25\text{ A}$

EXAMPLE: Find the current through the 10-Ω resistor of the network



Solution:

I_1: $(8\ \Omega + 3\ \Omega)I_1 - (8\ \Omega)I_3 - (3\ \Omega)I_2 = 15\text{ V}$

I_2: $(3\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (3\ \Omega)I_1 - (5\ \Omega)I_3 = 0$

I_3: $(8\ \Omega + 10\ \Omega + 5\ \Omega)I_3 - (8\ \Omega)I_1 - (5\ \Omega)I_2 = 0$

$$11I_1 - 8I_3 - 3I_2 = 15$$
$$10I_2 - 3I_1 - 5I_3 = 0$$
$$23I_3 - 8I_1 - 5I_2 = 0$$

$$\begin{aligned} 11I_1 - 3I_2 - 8I_3 &= 15 \\ -3I_1 + 10I_2 - 5I_3 &= 0 \\ -8I_1 - 5I_2 + 23I_3 &= 0 \end{aligned}$$

$$I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \mathbf{1.220\ A}$$