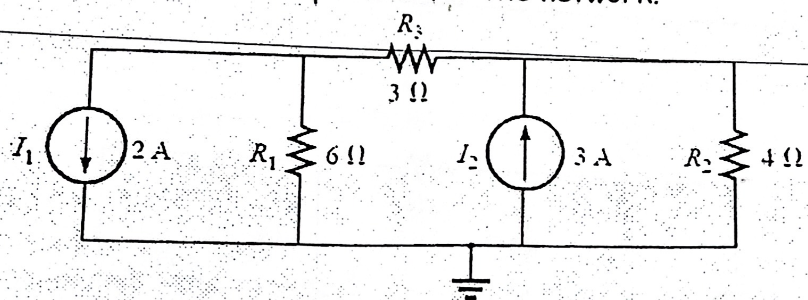
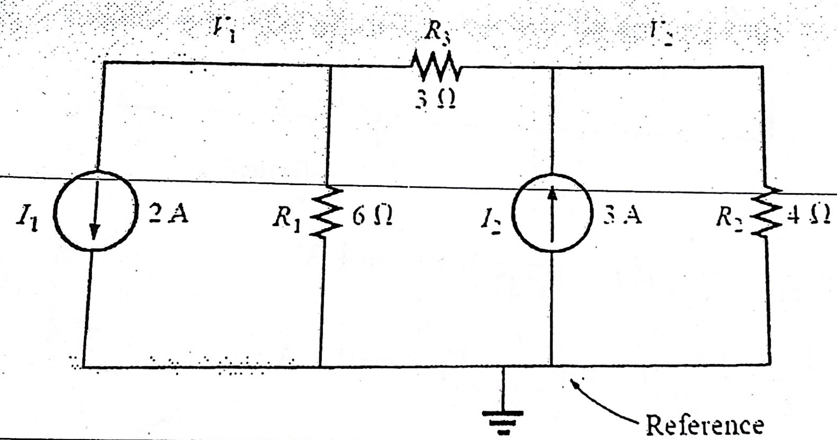
Nodal Analysis

- Choose a reference node and assign a subscripted voltage label to the (N _ 1) remaining nodes of the network.
- Writ the equations of node so that the column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.
- Solve the resulting equations for the desired voltages.

EXAMPLE: Write the nodal equations for the network.



Solution:



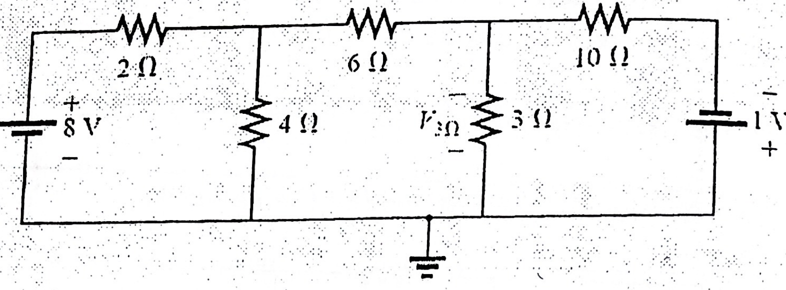
$$V_1:\ \underbrace{\left(\frac{1}{6\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\text{Sum of conductances connected to node 1}} V_1-\underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\text{Mutual conductance}} V_2=\overset{\substack{\text{Drawing current}\\\text{from node 1}\\\downarrow}}{-2\text{ A}}$$

$$V_2:\ \underbrace{\left(\frac{1}{4\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\text{Sum of conductances connected to node 2}} V_2-\underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\text{Mutual conductance}} V_1=\overset{\substack{\text{Supplying current}\\\text{to node 2}\\\downarrow}}{+3\text{ A}}$$

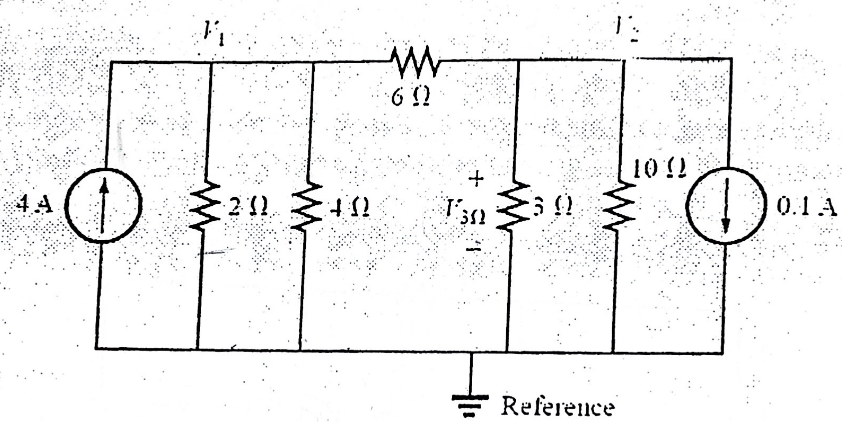
$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

EXAMPLE: Find the voltage across the 3-Ω resistor by nodal analysis.



Solution:



$$\left(\frac{1}{2\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{6\,\Omega}\right)V_1 - \left(\frac{1}{6\,\Omega}\right)V_2 = +4\text{ A}$$

$$\left(\frac{1}{10\,\Omega} + \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega}\right)V_2 - \left(\frac{1}{6\,\Omega}\right)V_1 = -0.1\text{ A}$$

$$\frac{11}{12}V_1 - \frac{1}{6}V_2 = 4$$

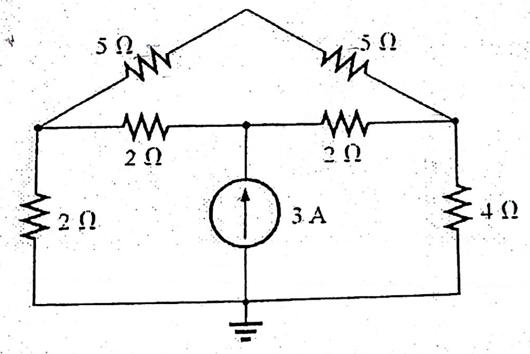
$$-\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1$$

$$11V_1 - 2V_2 = +48$$

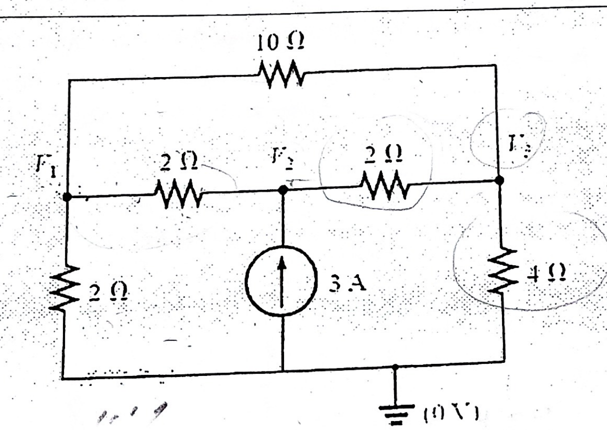
$$-5V_1 + 18V_2 = -3$$

$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = \frac{207}{188} = \mathbf{1.101\ V}$$

EXAMPLE: Using nodal analysis, determine the potential across the 4-Ω resistor and it's current.



Solution:



$$V_1:\quad \left(\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{10\,\Omega}\right)V_1-\left(\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{10\,\Omega}\right)V_3=0$$

$$V_2:\quad \left(\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{2\,\Omega}\right)V_1-\left(\frac{1}{2\,\Omega}\right)V_3=3\text{ A}$$

$$V_3:\quad \left(\frac{1}{10\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}\right)V_3-\left(\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{10\,\Omega}\right)V_1=0$$

$$1.1V_1-0.5V_2-0.1V_3=0$$
$$V_2-0.5V_1-0.5V_3=3$$
$$0.85V_3-0.5V_2-0.1V_1=0$$

$$V_3' = V_{4\Omega} = \frac{\begin{vmatrix} 1.1 & -0.5 & 0 \\ -0.5 & +1 & 3 \\ -0.1 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & +1 & -0.5 \\ -0.1 & -0.5 & +0.85 \end{vmatrix}} = 4.645\ \text{V}$$

$$I_{4\Omega} = \frac{V3}{4} = \frac{4.645}{4} = 1.16\,A$$