كلية العلوم

قسم الفيزياء

المغناطيسية

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2025

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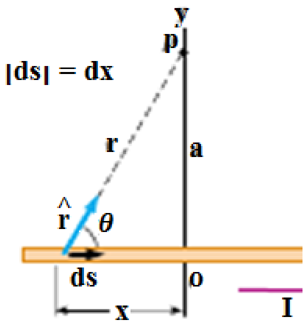
Sources of the Magnetic Field

2.1. Magnetic Field Surrounding a Thin Straight Conductor

Example 1: Consider a thin, straight wire carrying a constant current I and placed along the x axis as shown in Figure 2.2. Determine the magnitude and direction of the magnetic field at point **P** due to this current.

Solution: From the Biot–Savart law, we expect that the magnitude of the field is proportional to the current in the wire and decreases as the distance from the wire to point P increases. We start by considering a length element **ds** Located a distance **r** from **P**.



The direction of the magnetic field at point **P** due to the current in this element is out of the page because $\boldsymbol{ds} \times \hat{\boldsymbol{r}}$ Is out of the page. In fact, because all of the current elements **I ds** lie in the plane of the page, they all produce a magnetic field directed out of the page at point **P**. Thus, we have the direction of the magnetic field at point **P**, and we need only fiend the magnitude. Taking the origin at **O** and letting point **P** be along the positive y axis, with $\hat{\boldsymbol{k}}$ being a unit vector pointing out of the page, we see that

من قانون بايو سفرت نستطيع ان نستنتج ان قيمة المجال المغناطيسي B تتناسب مع قيمة التيار المار في الموصل وان B تقل كلما ابتعدنا النقطه p من الواير كما في الشكل 2.2 لنفرض ان المقطع الازاحه الصغير ds يبعد مسافه مقدارها r عن النقطه p اذا سيكون اتجاه المجال المغناطيسي B في النقطه p خارج عن مستوي الورقه

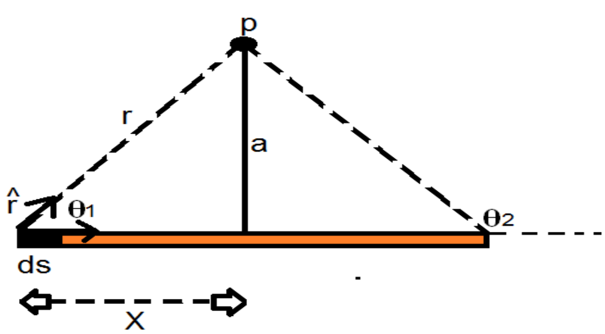


Figure 2.3

$$\boldsymbol{ds} \times \hat{\boldsymbol{r}} = |\boldsymbol{ds} \times \hat{\boldsymbol{r}}|\hat{\boldsymbol{k}} = (\boldsymbol{dx\,sin\theta})\hat{\boldsymbol{k}}$$

Where represents the magnitude of $\boldsymbol{ds} \times \hat{\boldsymbol{r}}$. Because $\hat{\boldsymbol{r}}$ is a unit vector, the magnitude of the cross product is simply the magnitude of **ds**, which is the length **dx**.

$$\boldsymbol{dB} = (\boldsymbol{dB})\hat{\boldsymbol{k}} = \frac{\boldsymbol{\mu_o I}}{\boldsymbol{4\pi}}\frac{\boldsymbol{dx\,sin\theta}}{\boldsymbol{r^2}}\hat{\boldsymbol{k}}$$

$$\boldsymbol{dB} = \frac{\boldsymbol{\mu_o I}}{\boldsymbol{4\pi}}\frac{\boldsymbol{dx\,sin\theta}}{\boldsymbol{r^2}} \qquad \dots\dots\dots\dots(\mathbf{1})$$

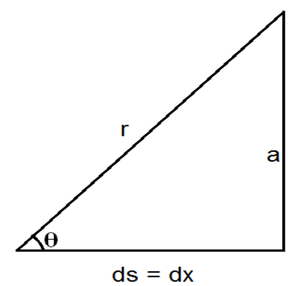
الان نحاول ان نضع كل من ds و r بصيغة θ من الشكل 2.3

$$\mathbf{sin\theta} = \frac{\boldsymbol{a}}{\boldsymbol{r}} \Rightarrow \boldsymbol{r} = \frac{\boldsymbol{a}}{\boldsymbol{sin\theta}} \Rightarrow \boldsymbol{r} = \boldsymbol{a\,csc\theta} \dots\dots\dots\dots\mathbf{2}$$

Because $\boldsymbol{tan\theta} = \frac{\boldsymbol{a}}{\boldsymbol{-x}}$ From the right triangle

In Figure (the negative sign is necessary

Because **ds** are located at a negative value of **x**),



We have $\boldsymbol{x = -a\,cot\theta}$

$$\mathbf{tan\theta} = \frac{\boldsymbol{a}}{\boldsymbol{-x}} \Rightarrow \boldsymbol{x} = \frac{\boldsymbol{-a}}{\boldsymbol{tan\theta}} \Rightarrow \boldsymbol{x = -a\,cot\theta} \dots\dots\dots\dots(\mathbf{3})$$

∴ dx $= \boldsymbol{a\,csc^2\theta d\theta}$ **………(4)** **(the cotθ derivative is -csc²θdθ)**

الان نضع معادله ٢ و ٣ و ٤ في معادله ١

Substitution of Equations 1 we get:

$$\boldsymbol{dB} = \frac{\boldsymbol{\mu_o I}}{\boldsymbol{4\pi}}\frac{\boldsymbol{a\,csc^2\theta\,d\theta\,sin\theta}}{(\boldsymbol{a\,csc\quad\theta})^2} = \frac{\boldsymbol{\mu_o I}}{\boldsymbol{4\pi a}}\boldsymbol{sin\theta d\theta} \qquad \dots\dots\dots(\mathbf{5})$$

الان اصبحت المعادله فقط مع المتغير θ نكامل من $\boldsymbol{\theta_1}$ الى $\boldsymbol{\theta_2}$

An expression in which the only variable is **θ**. We now obtain the magnitude of the magnetic field at point **P** by integrating Equation 2 over all elements, where the subtending angles range from θ_1 to θ_2 as defined in Figure **2.3**

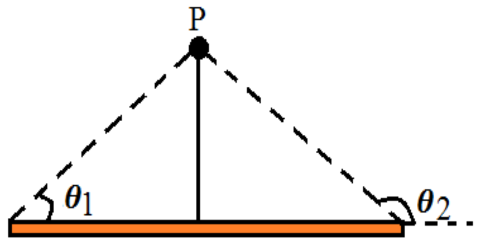


Fig. 2.3

$$dB = \frac{\mu_o I}{4\pi a}\int_{\theta 1}^{\theta 2} sin\theta d\theta$$

$$= \frac{\mu_o I}{4\pi a}(cos\theta 1 - \cos\theta 2) \ldots\ldots 6$$

في حالة كون السلك الموصل طويل جدا

$\theta_1 = 0 \qquad \theta_2 = \pi$

We can use this result to field the magnetic field of any straight current-carrying wire if we know the geometry and hence the angles θ_1 and θ_2. Consider the special case of an infinitely long, straight wire. If we let the wire in Figure 2 become infinitely long, we see that $\theta_1=0$ and $\theta_2= \pi$ for length elements ranging between positions $x = -\infty$ and $x = +\infty$.

Because **($\cos\theta_1 - \cos\theta_2$)**

$= (\cos 0 - \cos\pi) =$

$= (1-(-1) = 2$

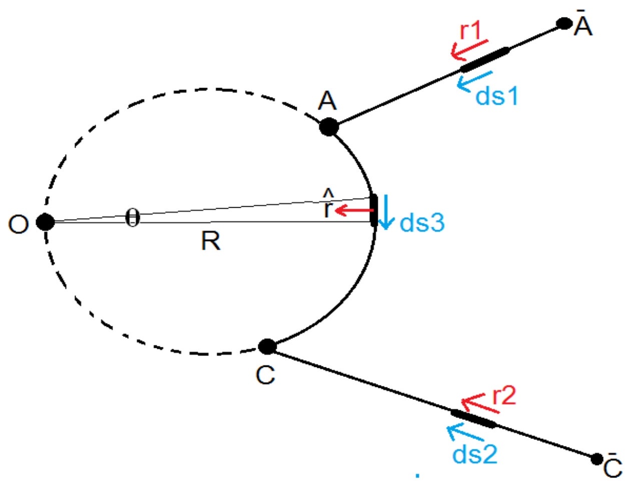
Equation 6 becomes

$$B = \frac{\mu_o I}{2\pi a} \quad \ldots\ldots.7$$

2.2. Magnetic Field Due to a Curved Wire Segment

Example 2: Calculate the magnetic field at point **O** for the current-carrying wire segment shown in Figure 2.4. The wire consists of two straight portions and a circular arc of radius **R**, which subtends an angle **θ**. The arrowheads on the wire indicate the direction of the current.

Solution:



نقسم الشكل الى ثلاث مقاطع

1- $A\acute{A}$ **With a unit area** ds_1
2- $C\acute{C}$ **With a unit area** ds_2
3- *AC* **With a unit area** ds_3

The magnetic field at O due to the current in the straight segments $A\acute{A}$ And $C\acute{C}$ Is zero because ds is parallel to r̂ along these paths; this means

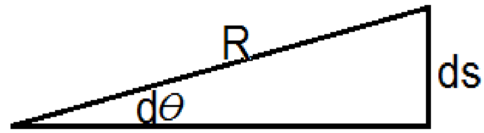
$$\left.\begin{array}{l} ds_1 \,//\, \hat{r}_1 \Longrightarrow ds_1 \times \hat{r} = 0 \\ ds_2 \,//\, \hat{r}_2 \Longrightarrow ds_2 \times \hat{r} = 0 \end{array}\right] \quad \sin\theta = 0$$

And $ds_3 \perp \hat{r}_3 \Longrightarrow ds_3 \times \hat{r} = ds \qquad \sin 90 = 1$

That $\boldsymbol{ds} \times \hat{\boldsymbol{r}} = \boldsymbol{0}$. Each length element ds along path **AC** is at the same distance **R** from **O**, and the current in each contributes a field element dB directed into the page at O.

Furthermore, at every point on **AC**, ds is perpendicular to $\hat{r}$ Hence, hence, $|\boldsymbol{ds} \times \hat{\boldsymbol{r}}| = \boldsymbol{ds}$ using this information and Equation1, we can find the magnitude of the field at **O** due to the current in an element of length ds:

ds= Rdθ



Now from Bio- Savart law

$$dB = \frac{\mu_o I}{4\pi}\frac{ds \times \hat{r}}{r^2}$$

$$dB = \frac{\mu_o I}{4\pi}\frac{Rd\theta}{R^2}$$

Because **I** and **R** are constants in this situation, we can easily integrate this expression over the curved path **AC**:

$$B = \int dB = \frac{\mu_o I}{4\pi R}\int_0^\theta d\theta$$

$$B = \frac{\mu_o I}{4\pi R}\,\theta$$

Or

$$B = \frac{\mu_o I}{4\pi R^2}\,S$$ remember that ds= Rdθ is equal to S=Rθ

Where we have used the fact that $s = R\theta$ with measured in radians. The direction of **B** is into the page at **O** because $ds \times \hat{r} = 0$ is into the page for every length element.

2.3. Magnetic Field on the Axis of a Circular Current Loop

Example 3: Consider a circular wire loop of radius **R** located in the **yz** plane and carrying a steady current ***I***, as in Figure 2.5. Calculate the magnetic field at an axial point **P** a distance **x** from the center of the loop.

Solution: In this situation, every length element ds is perpendicular to the vector $\hat{\boldsymbol{r}}$ At the location of the element.

في هذه الحاله كل قطعة طول ds هي عموديه على وحده المتجه $\hat{\boldsymbol{r}}$ كما في الشكل 2.5

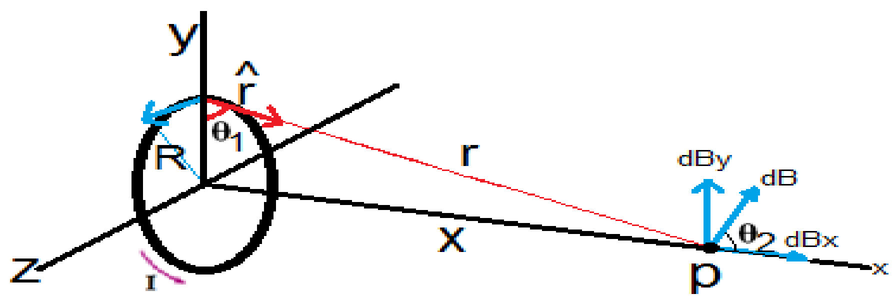


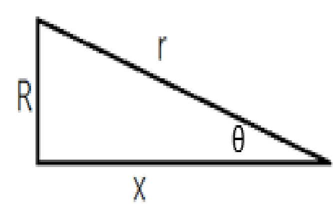
Fig. 2.5

$$|\boldsymbol{ds} \times \hat{r}| = \boldsymbol{ds}\ \boldsymbol{sin}90° = \mathbf{ds}$$

Furthermore, all length elements around the loop are at the $\boldsymbol{B} = \boldsymbol{B_x}\hat{\boldsymbol{\imath}}$ Where

اتجاه المجال B لكل وحدة طول هو باتجاه المحور X

$\boldsymbol{B_x} = \boldsymbol{dB}\ \boldsymbol{sin}\ \boldsymbol{\theta}$
$\boldsymbol{B_y} = \boldsymbol{dB}\ \boldsymbol{cos}\ \boldsymbol{\theta}$



$\boldsymbol{B_y} = \mathbf{0}$

المركبه السالبه تلغي المركبه الموجبه

$$\boldsymbol{B} = \frac{\boldsymbol{\mu_o I}}{\boldsymbol{4\pi}} \frac{\boldsymbol{ds}\ \boldsymbol{sin}\ \boldsymbol{\theta}}{\boldsymbol{r^2}}$$ **...... Biot- saver...... (1)**

$$\boldsymbol{B_x} = \frac{\boldsymbol{\mu_o I}}{\boldsymbol{4\pi}} \oint \frac{\boldsymbol{ds}\ \boldsymbol{sin\theta}}{\boldsymbol{r^2}}$$ **......................(2)**

But $r^2 = x^2 + R^2$ **………. (3)**

Sinθ $= \frac{R}{(x^2+R^2)^{1/2}}$ **………. (4)**

Put equation (3) and (4) in (2)

$$B_x = \frac{\mu_o I}{4\pi} \oint \frac{ds\, R}{(x^2+R^2).\,(x^2+R^2)^{1/2}} \text{………(5)}$$

And we must take the integral over the entire loop. Because **θ, x** and **R** are constants for all elements of the loop and because $\sin\theta = \frac{R}{\sqrt{(x^2+R^2)}}$, we obtain

$$B_x = \frac{\mu_o IR}{4\pi(x^2+R^2)^{3/2}} \oint ds$$

Where we have used the fact that $\oint ds = 2\pi R$ (the circumference of the loop).

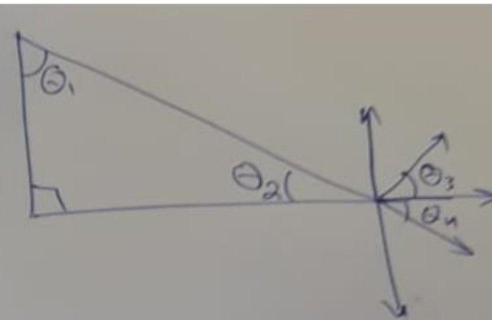
$$B_x = \frac{\mu_o IR}{4(x^2+R^2)^{3/2}} \quad 2\pi R \ldots (6)$$ المجال المغناطيسي لسلك مستقيم طويل

$$B_x = \frac{\mu_o I\pi R^2}{2(x^2+R^2)^{3/2}} \ldots (7)$$

<u>To find the magnetic field at the center of the loop</u>, we set **x = 0** in Equation 1 at this special point, therefore,

$$B_x = \frac{\mu_o I}{2R} \text{ ……(7)}$$

H.W. Prove that $\theta_{1=}\ \theta_2$ $\quad use\ figure\ 2.5$



$$\theta_1 + \theta_2 + 90 = 180^\circ$$

$$\therefore \ \theta_1 + \theta_2 = 90^\circ \text{ ——— } (1)$$

$$\theta_2 = \theta_4 \text{ - - - } (2)$$ تقابل بالرأس

$$\therefore \ \theta_1 + \cancel{\theta_4} = 90^\circ \text{ ——— } (3)$$

$$\theta_3 + \cancel{\theta_4} = 90^\circ$$

$$\therefore \ \boxed{\theta_1 = \theta_3} \text{ ——— } \boxed{4}$$

2.4. The Magnetic Force between Two Parallel Conductors

Example 4:

Consider two long, straight, parallelwires separated by a distance and carrying currents I_1 and I_2 in the same direction, as in Figure 2.6.

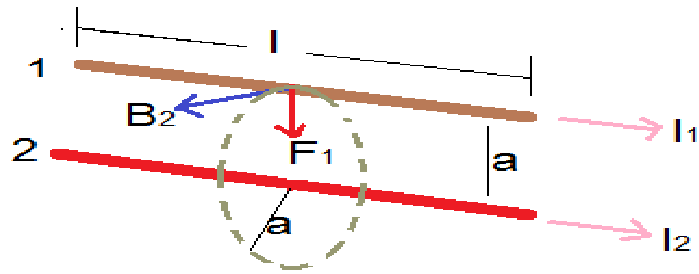


Figure 2.6

في حالة اخذنا تاثير الواير W2 على الواير W1 ...الواير W2 يمتلك مجال مغناطيسي $\vec{B}$ عمودي على الواير W1 ويعتبر في هذه الحاله هو المصدر للمجال المغناطيسي .اذا سيكون ال W1هونقطة الدراسه و بالاستناد الى المعادله

F= I L×B ………..(*) H.W prove

We can determine the force exerted on one wire due to the magnetic field set up by the other wire. Wire 2, which carry a current I_2 and is identified arbitrarily as the source wire, creates a magnetic field $\mathbf{B_2}$ at the location of wire 1, the test wire. The direction of $\mathbf{B_2}$ is perpendicular to wire 1, as shown in Figure 2.6.

According to Equation $\boldsymbol{F_B = IL \times B}$, the magnetic force on a length of wire 1 is:

$\boldsymbol{F_1 = I_1 \ell \times B_2}$ ………….. القوه المغناطيسيه التي يسلطها الواير ٢ على الواير ١

Because $\boldsymbol{\ell}$ is perpendicular to $\mathbf{B_2}$ in this situation,

the magnitude of $\mathbf{B_2}$ is given by Equation $= \dfrac{\mu_o I}{2\pi a}$,

the magnetic field for long wire we see that

$$F_1 = I_1\ell \times B_2 = I_1\ell\left(\frac{\mu_o I_2}{2\pi a}\right)$$

$$F_1 = \left(\frac{\mu_o I_1 I_2}{2\pi a}\right)\ell \text{...........(1)}$$

The direction of F_1 is toward wire 2 because ℓB_2 is in that direction. If the field set up at wire 2 by wire 1 is calculated, the force F_2 acting on wire 2 is found to be equal in magnitude and opposite in direction to F_1. This is what we expect because Newton's third law must be obeyed.

وبنفس الطريقه نجد القوه المغناطيسيه التي يوثر بها الواير ١ على الواير ٢

$$F_2 = \left(\frac{\mu_o I_1 I_2}{2\pi a}\right)\ell \text{.........(2)}$$

If $F_1 = F_2$ and is equal to F_B

Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F_B. We can rewrite this magnitude in terms of the force per unit length:

عندما يمر نفس التيار في السلكين ولنفس المجال المغناطيسي (مجال مغناطيسي منتضم) تكون القوه المغناطيسيه الاولى تساوي الثانيه وتساوي F_B

$$\frac{F_B}{\ell} = \left(\frac{\mu_o I_1 I_2}{2\pi a}\right)$$

When the currents are in opposite directions (that is, when one of the currents is reversed in Fig. 2.6), the forces are reversed and the wires repel each other.

١ – القوه المغناطيسيه ستكون تنافر اذا كان التيارين متعاكسين

Parallel conductors carrying currents in the same direction attract each other,

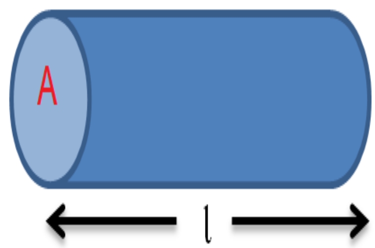
٢ – القوه المغناطيسيه ستكون تجاذب اذا كان التيارين بنفس الاتجاه

The force between two parallel wires is used to define the **ampere** as follows: When the magnitude of the force per unit length between two long parallel wires that carry identical currents and are separated by **1m** is **2x10^{7} N/m**, the current in each wire is defined to be **1A**.

يمكن الاستفاده من هذه الفكره في تعريف الامبير : اذا كان لدينا سلكين متوازيين طويلين والمسافه الفاصله بينهما واحد متر (1m) و النسبه بين القوه المغناطيسيه (N) على وحدة الطول (m) تساوي 2x10^{7} فان التيار المار بالسلك يساوي 1A.

Prove that F= I L×B……… (*) this relation used to find the current carrying conductor (C. C. C.) Lies in the magnetic field.

Solution:



Magnetic force is $F = q\, v \times B$ ……..(1)

Volume (V) is equal to V = AL ……(2)

Let n = number of charges (N), per unit volume (V)

$n = \frac{N}{V}$ …… N= nV ………(3)

put eq. (2) in (3)

N= n AL …….(4)

And let the, Total charge q= Ne………(5)

Put eq. (4) in (5)

q= NAL. e ……..(6)

$I = \frac{q}{t}$ … … (7) التيار يساوي كمية الشحنه الماره في سلك لوحده الزمن

Put eq. (6) in (7)

$I = \frac{NAL\,.\,e}{t}$ recall that $velocity\ ie\ equal\ to: v = \frac{l}{t}$

I = n A v e

$v = \frac{I}{nAe}$

$\because$ F= q v B $\Rightarrow$ F = (nAL. e) . $(\frac{I}{nAe})\ B$

F= LIB

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