كلية العلوم   قسم الفيزياء

المغناطيسية

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Induced Electro motive Force القوه الدافعه الكهربائيه المحثه

3.1. Faraday's Law of Induction قانون فراداي للحث

The **emf** induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit. This statement, known as Faraday's law of induction, can be written

$$\varepsilon = -\frac{d\phi_B}{dt}$$

تتولد emf محتثه عندما يتغير الفيض المغناطيسي مع الزمن في الدائره الكهربائيه وهذا ما يعرف بقانون **فراداي للحث**. الاشاره السالبه تشير الى معاكسه المجال المحتث للمجال الاصلي

Where $\phi_B = \int BdA$ Is the magnetic flux through the circuit? If the circuit is a coil consisting of **N** loops all of the same area and if ϕ_B Is the magnetic flux through one loop, an **emf** is induced in every loop. The loops are in the series, so their **emf** adds; thus, the total induced **emf** in the coil is given by the expression:

حيث ϕ_B الفيض المغناطيسي خلال الدائره الكهربائيه فاذا كانت الدائره عباره عن ملف (coil) مؤلف من عدد N من اللفات واذا كان ϕ_B هو الفيض خلال كل حلقه من الحلقات . اذا سستولد القوه الدافعه الكهربائيه المحتثه emf في كل حلقه وتضاف (تجمع) ال emf لكل حلقه في الملف . اذا ويمكن التعبير عن مجموع القوه الدافعه الكهربائيه emf بـ

$$\varepsilon = -N\frac{d\phi_B}{dt}$$

Suppose that a loop enclosing an area **A** lies in a uniform magnetic field **B**, as in Figure (1). The magnetic flux through the loop is equal to **BA cosθ**; hence, the induced **emf** can be expressed as:

لنفرض ان حلقة الملف لها مساحه A ضمن الحلقه المغلقه وهي ضمن مجال مغناطيسي B (مجال مغناطيسي خارجي وليس المحتث) كما في الشكل (1). اذا سيكون الفيض المغناطيسي خلال الحلقه **BA cosθ** اذا يمكن التعبير عن القوه الدافعه الكهربائيه المحتثه emf بـ

$$\boldsymbol{\varepsilon = -N\frac{d}{dt}(BAcos\theta)} \qquad (1)$$

From this expression, we see that an **emf** can be induced in the circuit in several ways:

1. The magnitude of B can change with time.
2. The area enclosed by the loop can change with time.
3. The angle **θ** between **B** and the normal to the loop can change with time.
4. Any combination of the above can occur.

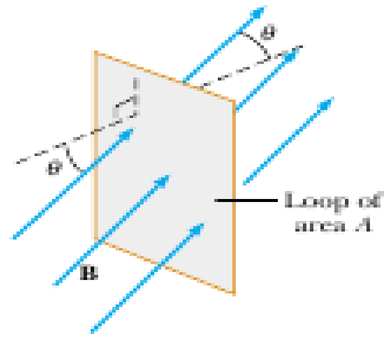


Figure 1

من خلال هذه العلاقه (1) يمكن ان نلاحظ تولد القوه الدافعه الكهربائيه المحتثه emf بعدة طرق

1. تغير الفيض مع الزمن
2. تغير المساحه المغلقه بالحلقه مع الزمن
3. تغير الزاويه بين خطوط الفيض و العمود المقام على سطح المساحه
4. او تغير اي من النقاط اعلاه مجتمعة

Example: A coil of **15 turns** and radius **10cm** surrounds a long solenoid of radius 2**cm** and 1**x10^3 turns/meter** (show in the Figure) The current in the solenoid changes as **I = (5.00 A) sin(120t)**. Find the **<u>induced emf</u>** in the **15-turn** coil as a function of time.

Solution:

From Gauss law $\Phi_B = BA\cos\theta$**........(1) A: A is the area**

And for solenoid $B = \mu_O\, n\, I$ **..........(2)**

Put 2 in 1 .... $\Phi_B = (\mu_o nI)\, A_{soenoid}$ **........... (3)**

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\mu_o n(\pi r^2)\frac{dI}{dt}$$

$$\varepsilon = -15\left(4\pi\times10^{-7}T.\frac{m}{A}\right)(1\times10^3 m^{-1})\pi(0.02m)^2\left(\frac{500A}{s}\right)cos120t$$

$$\varepsilon = -14.2cos(120t)mV$$

3.2. Induced emf and Electric Fields القوه الدافعه الكهربائيه المحتثه و المجال الكهربائي

We have seen that a changing magnetic induces an emf and a current in a conducting loop. In our study of electricity, we related a current to an electric field that applies electric forces on charged particles. In the same way, we can relate an induced current in a conducting loop to an electric field by claiming that an electric field created in the conductor because of the changing magnetic flux this induced electric field is **non-conservative**, unlike the electrostatic field produced by stationary charges.

سبق وان درسنا في الموضوع السابق ان التغير في الفيض المغناطيسي مع الزمن يولد قوة كهربائيه دافعه محتثه induced emf (فراداي) ويتولد عنها تيار محتث ومجال كهربائي محتث induced electric field ولكن هذا المجال غير محافظ non conservative على العكس من المجال الكهربائي المتولد من الشحنات الاساسيه stationary charges

To prove that the induced electric field is non conservative?

We can illustrate this point by considering a conducting loop of radius r situated in a uniform magnetic field that is perpendicular to the plane of the loop, as in Figure1. If the magnetic field changes with time, then, according to Faraday's law, an **emf** $\mathcal{E} = -\boldsymbol{d\phi_B/dt}$ Is induced in the loop The induction of a current in the loop implies the presence of an induced electric field **E**, which must be tangent to the loop because this is the direction in which the charges in the wire move in response to the electric force

يمكن اثبات هذه الحاله من خلال فرضنا ان لدينا موصل على شكل حلقه في مجال مغناطيسي حيث r تمثل نصف قطر الحلقه فاذا تغير الفيض المغناطيسي مع الزمن وحسب قانون فراداي تتولد قوه دافعه كهربائيه محتثه induced emf وذلك يؤدي الى توليد تيار محتث induced current وبالتالي يتولد مجال كهربائي محتث induce electric field يكون اتجاهه مماسا متاثرا بحركة الشحنات في الموصل (الحلقه) ....الشغل = القوة × الازاحة

القوة الكهربائية = المجال الكهربائي × الشحنة

اذا كانت القوة الكهربائية = الشغل

The work done by the electric field in moving a test charge **q** once around the loop is equal to **q$\mathcal{E}$**. Because the electric force acting on the charge is **qE**, the work done by the electric field in moving the charge once around the loop is **qE(2πr),** where **2πr** is the circumference of the loop. These two expressions for the work done must be equal; therefore, we see that

اذا سيكون الشغل الازم لتحريك الشحنه حول الحلقه **qE(2πr)** يساوي القوة الكهربائية **qƐ .** اذا سيكون الشغل المنجز من قبل المجال الكهربائي لتحريك الشحنه حول الحلقه هو

القوه الكهربائيه = الشغل اللازم لتحريك الشحنه

$\text{Electric force } (q\mathcal{E}) = \text{work requerd to moves } qE(2\pi r)$

Where the Circumference of the loop is $2\pi r$

$q\mathcal{E} = qE\,.2\pi r$

$\mathcal{E} = E.\,2\pi r$

$\because \mathcal{E} = -\frac{d\phi_B}{dt}$ (Faraday law of induction)

And $\phi = BA = B\,\pi r^2$ (Gauss)

$\therefore\ E\,.\,2\pi r = -\frac{d}{dt}\,(\pi r^2 B)$

$E = -\frac{r}{2}\frac{dB}{dt} \neq 0$ is no equal to zero

The **emf** for any closed path can be expressed as the line integral of **E. ds** over that path: $\mathcal{E} = \oint \boldsymbol{E}\,.\,\boldsymbol{ds}$ (حث كهربائي). In more general cases, **E** may not be constant, and the path may not be a circle. Hence, Faraday's law of induction,

$\boldsymbol{\varepsilon} = -\frac{\boldsymbol{d\phi_B}}{\boldsymbol{dt}}$ (حث مغناطيسي), can be written in the general form

القوه الدافعه الكهربائيه لاي مسار مغلق يمكن التعبير عنها بتكامل خطي لـ **E. ds** لكل المسار $\mathcal{E} = \oint \boldsymbol{E}\,.\,\boldsymbol{ds}$ وبشكل عام قد يكون المجال الكهربائي غير ثابت و المسار ليس دائريا وعليه يمكن كتابة قانون فراداي للحث بصيغته العامه

$$\oint \boldsymbol{E}.\boldsymbol{ds} = -\frac{\boldsymbol{d\phi_B}}{\boldsymbol{dt}} \qquad (\mathbf{2})$$

The induced electric field **E** in Equation **1** is a **non-conservative field** that is generated by a changing magnetic field. The field **E** that satisfies Equation**1** cannot possibly be an electrostatic field because if the field were electrostatic, and hence conservative, the line integral of ***E.ds*** over a closed loop would be zero.

من المعادله (2) نلاحظ ان المجال الكهربائي المحتث ليس محافظا. ذلك بسبب تغير المجال المغناطيسي الاصلي .ولو فرضنا العكس ان المجال الكهربائي في المعادله (2) محافظ سيكون التكامل الخطي للـ $\boldsymbol{E}.\boldsymbol{ds}$ لكل مسار الحلقه المغلق يساوي صفرا .

$$\oint \boldsymbol{E}.\boldsymbol{ds} = \begin{cases} = 0 & \textbf{conservative} \\ \neq 0 & \textbf{non- conservative} \end{cases}$$

التكامل الخطي لمسار مغلق للمجال الكهربائي هو مجال محافظ اذا = صفر (حقيقة لاتقبل النقاش)

Example: A long solenoid with 1000 **turns** per meter and radius **2cm** carries an oscillating current given by **I= (5A) sin(100πt)**. What is the <u>electric field induced</u> at a radius **r =1cm** from the axis of the solenoid? What is the direction of this electric field when the current is increasing counterclockwise in the coil?

Solution:

$$E.\,2\pi r = \frac{d\Phi_B}{dt}$$

$\overrightarrow{B}_{solenoid} = \mu_o \,.\, n\, I$ Biot-Savart Law

$\phi_B = AB$ Magnetic flux

$\therefore \quad \phi_B = A\mu_o.n\,.I$

When r = 0.02 m from the center of the solenoid

$$E = n\mu_o \mathrm{A} \frac{d}{dt}(5A)\sin(100\pi t)$$

$$\mathrm{E} = \frac{(1000)(4\pi \times 1010^{-7}\mathrm{T})(2\pi r^2)(5\mathrm{A})(100\pi)\cos(100\pi t)}{2\pi r}$$

$$\mathrm{E} = 2\pi^2 \ (\mathrm{mv})$$

3.3. Lenz's Law قانون لنز

The induced current produces magnetic fields, which tend to oppose the change in magnetic flux that induces such currents.

يولد التيار المحتث مجالا مغناطيسيا معاكسا لاتجاه المجال الاصلي

تغير الفيض ⟸ تيار محتث ⟸ مجال مغناطيسي محتث ⟸ يعاكس الاتجاه الاصلي للمجال

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector $\vec{A}$.

2. Assuming that $\vec{B}$ is uniform, take the dot product of $\vec{B}$ and $\vec{A}$. This allows for the determination of the sign of the magnetic flux Φ_B. .

3. Obtain the rate of flux change $\frac{d\Phi_B}{dt}$ by differentiation. There are three possibilities:

$$\frac{d\Phi_B}{dt} = \begin{cases} > 0 \Rightarrow \text{Induced emf} \quad \varepsilon < 0 \\ < 0 \Rightarrow \text{Induced emf} \quad \varepsilon > 0 \\ = 0 \Rightarrow \text{Induced emf} \quad \varepsilon = 0 \end{cases}$$

4. Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of $\vec{A}$, curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if **ε >0**, and the opposite direction if **ε < 0**, as shown in Figure.

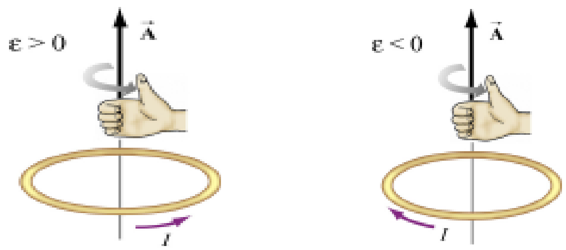
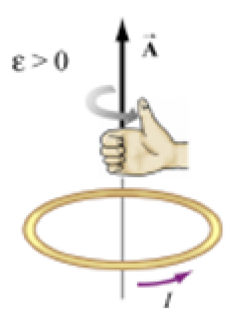
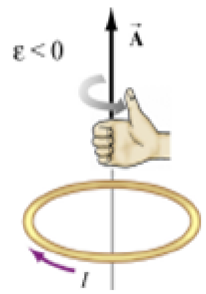


Figure 2. Determination of the direction of induced current by the right-hand rule

***** clarify the Lenz aw and illustrate the induced emf (ε) both experimentally and theoretically?

لتحديد كيف يعمل قانون لنز عمليا ونضريا . لنفرض ان لدينا موصلا على شكل حلقه موضوعه في مجال مغناطيسي منتضم

عمليا **Experimentally**	نضريا **Theoretically**
1 – نجعل الابهام باتجاه متجه المساحه **A** فاذا كان اتجاه دوران الاصابع بنفس اتجاه حركة التيار المحتث **induced current** عندها يكون **induced emf** موجب **ε > 0**  2 - فاذا كان اتجاه دوران الاصابع عكس اتجاه حركة التيار المحتث **induced current**	1 – نحدد قيمة الفيض المغناطيسي $\Phi_B = BA\cos\theta$ 2 – نجد قيمة مشتقة الفيض $\frac{d\Phi_B}{dt} \begin{cases} > 0 \rightarrow \varepsilon < 0 \\ = 0 \rightarrow \varepsilon = 0 \\ < 0 \rightarrow \varepsilon > 0 \end{cases}$ و هذا يعني ان تغير في الفيض عندما يزداد هذا يؤدي الى تقليل **emf** حتى بالنتيجة تقل **I induced** ويقل معها **B induced**

<table>
<tr>
<td>عندها يكون induced emf سالب $\varepsilon < 0$

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ملاحظه : ان حاصل ضرب المجال B.A يساوي كميه ثابته contestant فاذا زاد تغير الفيض الاصلي يجب ان يكون اتجاه الفيض المحتث بالتجاه معاكس ليقلل من قيمة التغير لتبقى القيمه ثابته .

<u>3. 4. Motional EMF</u> القوه الدافعه الكهربائيه المحتثه المتحركه

هي فرق الجهد (V) بين طرفي قطعه معدنيه موصله متحركه في مجال مغناطيسي منتظم

$$\boldsymbol{\varepsilon} = \oint(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}).\boldsymbol{d}\vec{\boldsymbol{s}}$$

Consider a conducting bar of length l moving through a uniform magnetic field which points into the page, as shown in Figure 3. Particles with charge q > 0 inside experience a magnetic force $\vec{\boldsymbol{F}} = \boldsymbol{q}\vec{\mathbf{v}} \times \vec{\boldsymbol{B}}$ which tends to push them upward, leaving negative charges on the lower end.

لنفرض ان لدينا موصل على شكل قضيب معدني بطول l يتحرك خلال مجال مغناطيسي منتظم الذي يتجه داخل الصفحه (×) كما مبين في الشكل (3) الشحنات الموجبه في داخل القضيب سوف تتاثر بقوه مغناطيسيه تعمل على دفعها الى اعلى القضيب فيما تندفع الشحنات السالبه الى الاسفل بفعل نفس القوه

The separation of charge gives rise to an electric field $\vec{E}$ inside the bar, which produces a downward Electric force $\vec{F} = q\vec{E}$. At equilibrium where the two forces cancel, we have

$$qvB = qE, \text{ or } E = vB$$

Between exists a potential difference given by

$$V_{ab} = V_a - V_b$$

$$\varepsilon = E\ell$$

$$= vB\ell$$

فصل الشحنات بهذه الطريقه يولد مجلا كهربائيا في داخل القضيب المعدني والتي تولد قوه كهربائيه متجه الى الاسفل وبالتالي سوف تلغي القوتين بعضهما لانهما في اتجاهين متعاكسين وسفوف تظهر على الموصل فرق جهد في نهايتي الموصل

Since ε arises from the motion of the conductor, this potential difference is called the motional emf. In general, motional emf around a closed conducting loop can be written as:

$$\varepsilon = \oint (\vec{v} \times \vec{B}) . d\vec{s}$$

Where $d\vec{s}$ is a differential length element?

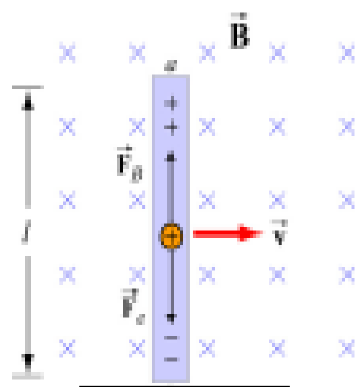


Figure 3

بما ان ε = induced emf سوف تنشاء من حركه الموصل . سوف نسمي فرق الجهد هذا القوه الدافعه الكهربائيه المتحركه motional emf حيث $d\vec{s}$ تمثل متجه المسافه .

Now suppose the conducting bar moves through a region of uniform magnetic field $\vec{B} = B\hat{k}$ (pointing into the page) by sliding along two frictionless conducting rails that are at a distance ℓ apart and connected together by a resistor with resistance **R**, as shown in Figure 4.

لنفرض ان لدينا الموصل يتحرك بواسطه سكتين متوازيتين عديمتي الاحتكاك في مجال مغناطيسي منتضم متجه داخل الصفحه $\vec{B} = B\hat{k}$ وان المسافه بين السكتين هي ℓ ومربوطه الى مقاومه **R** كما مبين في الشكل 4 .

Let an external force be applied so that the conductor moves to the right with a constant velocity

$\vec{v} = v\hat{i}$. The magnetic flux through the closed loop formed by the bar and the rails is given by:

$$\Phi_B = BA = B\ell x \qquad \textbf{(4)}$$

Thus, according to Faraday's law, the induced **emf** is:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -BI\frac{dx}{dt} = -B\ell v$$

Where $dx/dt = v$ is simply the speed of the bar. The corresponding induced current is:

$$I = \frac{|\varepsilon|}{R} = \frac{B\ell v}{R}$$

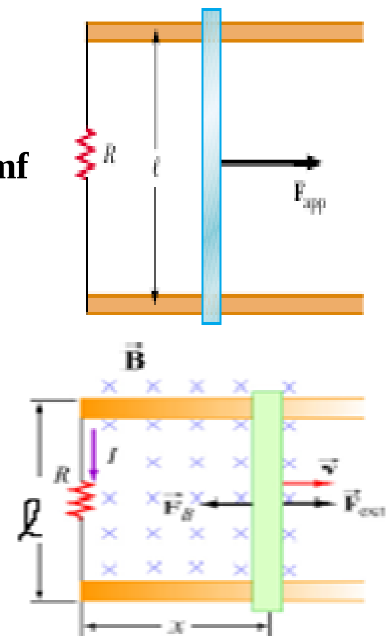


Figure 4

ولنفرض اننا سلطنا قوه خارجيه (ميكانيكيه) .اذا سوف يتحرك الموصل الى اليمين من الصفحه بسرعه $\vec{v} = v\hat{i}$ اذا سيكون الفيض المغناطيسي خلال الحلقه المغلقه والناتج من حركه الموصل و السكتين يعطى من خلال العلاقه (4) .

Example: Figure shows a top view of a bar that can slide without friction. The resistor is **6Ω** and a **2.5T** magnetic field is directed perpendicularly downward, into the paper. Let **ℓ=1.2m**. (a) Calculate the applied force required to move the bar to the right at a constant speed of **2m/s**. (b) At what rate is energy delivered to the resistor?

Solution:

a. $|F_B| = I|\ell \times B|$

$When\ I = \frac{\varepsilon}{R} \quad and\ \varepsilon = B\ell v$

$We\ get\ F_B = \frac{B\ell v}{R}(\ell v) = \frac{B^2\ell^2 v}{R}$

$F_B = \frac{(2.5)^2(1.2)^2(2)}{6} = 3N \quad to\ the\ right$

b. $P = I^2R = \frac{B^2\ell^2v^2}{R} = 6W$

Problems of Chapter three

1-A circular loop of wire is held in a uniform magnetic field, with the plane of the loop perpendicular to the field lines. Which of the following will not cause a current to be induced in the loop? (a) Decrease the loop; (b) rotating the loop about an axis perpendicular to the field lines; (c) keeping the orientation of the loop fixed and moving it along the field lines; (d) pulling the loop out of the field. *Why (c) In all cases except this one, there is a change in the magnetic flux through the loop.)*

2-A circular loop of wire is held in a uniform magnetic field, with the plane of the loop perpendicular to the field lines. Which of the following will not cause a current to be induced in the loop? (a) Decrease the loop; (b) rotating the loop about an axis perpendicular to the field lines; (c) keeping the orientation of the loop fixed and moving it along the field lines; (d) pulling the loop out of the field? *Why (b) The magnetic field lines around the transmission cable will be circular, centered on the cable. If you place* *your loop around the cable, there are no field lines passing through the loop, so no emf is induced. The loop must be placed next to the cable, with the plane of the loop parallel to the cable to maximize the flux through its area.*

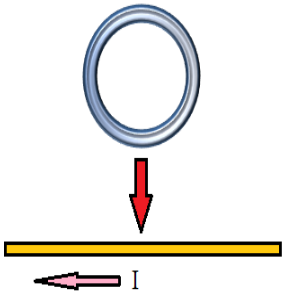


Fig.6

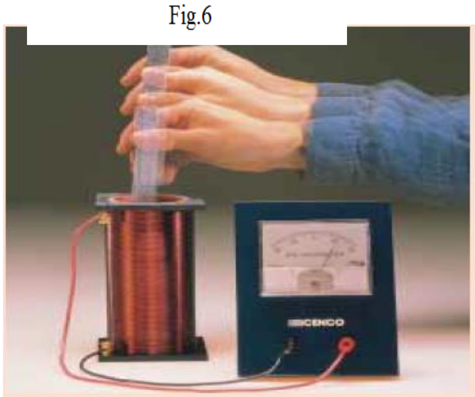


Figure 31.15 (Quick Quiz 31.7)

Figure 5

3- Figure 5 shows a magnet being moved in the vicinity of a solenoid connected to a sensitive ammeter. The south pole of the magnet is the pole nearest the solenoid, and the ammeter indicates a clockwise (viewed from above) current in the solenoid. Is the person (a) inserting the magnet or (b) pulling it out? *Why (a) Because the current induced in the solenoid is clockwise when viewed from above, the magnetic field lines produced by this current point downward in Figure 5. Thus, the upper end of the solenoid acts as a south pole. For this situation to be consistent with Lenz's law, the south pole of the bar magnet must be approaching the solenoid.*

4- Figure 6 shows a circular loop of wire being Dropped toward a wire carrying a current to the left. The direction of the induced current in the loop of wire is (a) clockwise (b) counterclockwise (c) zero (d) impossible to determine.

Why (b) ?? At the position of the loop, the magnetic field lines due to the wire point into the page. The loop is entering a region of stronger *magnetic field as it drops toward the wire, so the flux is increasing. The induced current must set up a magnetic field that opposes this increase. To do this, it creates a magnetic field directed out of the page. By the right-hand rule for current loops, this requires a counterclockwise current in the loop.*

5-In a region of space, the magnetic field increases at a constant rate. This changing magnetic field induces an electric field that (a) increases in time (b) is conservative (c) is in the direction of the magnetic field (d) has a constant magnitude. *Why (d) The constant rate of change of B will result in a constant rate of change of the magnetic flux. According to Equation 31.9, if d!B/dt is constant, E is constant in magnitude.*

6. The south end of the magnet is toward the loop of wire. The magnet is dropped toward the loop. Find the direction of the current through the resistor (a) while the magnet is falling toward the loop and (b) after the magnet has passed through the loop and moves away from it.

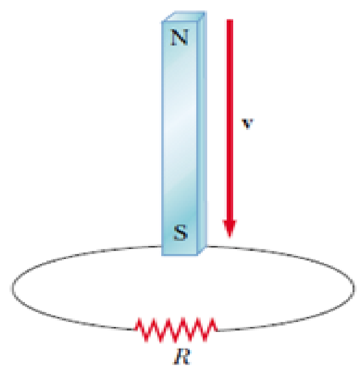


Figure 7

Answer

(a) The south pole of the magnet produces an upward magnetic field that increases as the magnet approaches.

The loop opposes change by making its own downward magnetic field; it carries current clockwise, which goes to the left through the resistor.

(b) The north pole of the magnet produces an upward magnetic field. The loop sees decrease upward flux as the magnet falls away, and tries to make an upward magnetic field of its own by carrying current counterclockwise, to the right in the resistor.

7-What is the difference between magnetic flux and magnetic field?

Answer : *Magnetic flux measures the "flow" of the magnetic field through a given area of a loop—even though the field does not actually flow. By changing the size of the loop, or the orientation of the loop and the field, one can change the magnetic flux through the loop, but the magnetic field will not change.*

8- A loop of wire is placed in a uniform magnetic field. For what the orientation of the loop is the magnetic flux a maximum? For what orientation is the flux zero?

Answer : *The magnetic flux is $\Phi B = BA\cos\theta$. Therefore the flux is maximum when* ***B*** *is perpendicular to the loop of wire and zero when there is no component of magnetic field perpendicular to the loop. The flux is zero when the loop is turned so that the field lies in the plane of its area.*

9- Defien Faraday's Law of Induction and finds an expression of emf induced, Suppose that a loop enclosing an area $\vec{A}$ Lies in a uniform magnetic field $\vec{B}$

Answer**:** The **emf** induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit. This statement, known as Faraday's law of induction, can be written

$$\boldsymbol{\varepsilon = -\frac{d\phi_B}{dt}}$$

تتولد emf محتثه عندما يتغير الفيض المغناطيسي مع الزمن في الدائره الكهربائيه وهذا ما يعرف بقانون **فراداي للحث**. الاشاره السالبه تشير الى معاكسه المجال المحتث للمجال الاصلي

Where $\boldsymbol{\phi_B = \int BdA}$ Is the magnetic flux through the circuit. If the circuit is a coil consisting of **N** loops all of the same area and if $\boldsymbol{\phi_B}$ Is the magnetic flux through one loop, an **emf** is induced in every loop. The loops are in the series, so their **emf** add; thus, the total induced **emf** in the coil is given by the expression:

حيث ϕ_B الفيض المغناطيسي خلال الدائره الكهربائيه فاذا كانت الدائره عباره عن ملف (coil) مؤلف من عدد N من اللفات واذا كان ϕ_B هو الفيض خلال كل حلقه من الحلقات . اذا ستتولد القوه الدافعه الكهربائيه المحتثه emf في كل حلقه وتضاف (تجمع) ال emf لكل حلقه في الملف . اذا ويمكن التعبير عن مجموع القوه الدافعه الكهربائيه emf بـ

$$\varepsilon = -N\frac{d\phi_B}{dt}$$

Suppose that a loop enclosing an area **A** lies in a uniform magnetic field **B**, as in Figure (1). The magnetic flux through the loop is equal to **BA cosθ**; hence, the induced **emf** can be expressed as:

لنفرض ان حلقة الملف لها مساحه A ضمن الحلقه المغلقه وهي ضمن مجال مغناطيسي B (مجال مغناطيسي خارجي وليس المحتث) كما في الشكل (1). اذا سيكون الفيض المغناطيسي خلال الحلقه **BA cosθ** اذا يمكن التعبير عن القوه الدافعه الكهربائيه المحتثه emf بـ

$$\varepsilon = -N\frac{d}{dt}(BA\cos\theta) \qquad (1)$$

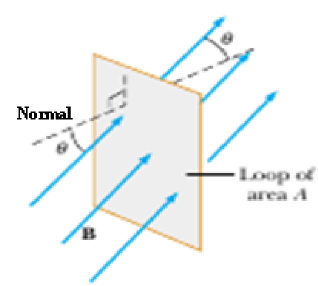


Figure 1

From this expression, we see that an **emf** can be induced in the circuit in several ways:

1.The magnitude of **B** can change with time.
2.The area enclosed by the loop can change with time.
3. The angle **θ** between **B** and the normal to the loop can change with time.
4. Any combination of the above can occur.

من خلال هذه العلاقه (1) يمكن ان نلاحظ تولد القوه الدافعه الكهربائيه المحتثه emf بعدة طرق

1. تغير الفيض مع الزمن
2. تغير المساحه المغلقه بالحلقه مع الزمن
3. تغير الزاويه بين خطوط الفيض و العمود المقام على سطح المساحه
4. او تغير اي من النقاط اعلاه مجتمعة

10- **Example:** A coil of **15 turns** and radius **10cm** surrounds a long solenoid of radius **2cm** and **1x10^3 turns/meter** (show in the Figure) The current in the solenoid changes as **I =(5.00 A)sin(120t)**. Find the induced **emf** in the **15-turn** coil as a function of time.

Solution:

$$\Phi_B = (\mu_o n I) A_{soenoid}$$

$$\Phi_B = -N\frac{d\Phi_B}{dt} = -N\mu_o n(\pi r^2)\frac{dI}{dt}$$

$$\Phi_B = -15\left(4\pi \times 10^{-7} T.\frac{m}{A}\right)(1 \times 10^3 m^{-1})\pi(0.02m)^2\left(\frac{600A}{s}\right)cos120t$$

$$\varepsilon = -14.2cos(120t)mV$$

11- Explain the induced emf and prove that the induced electric field is non-conservative?

<u>To prove that the induced electric field is non conservative?</u>

<u>Answer</u> : We can illustrate this point by considering a conducting loop of radius r situated in a uniform magnetic field that is perpendicular to the plane of the loop, as in Figure1. If the magnetic field changes with time, then, according to Faraday's law, an **emf** $\mathcal{E} = -d\phi_B/dt$ Is induced in the loop The induction of a current in the loop implies the presence of an induced electric field **E**, which must be tangent to the loop because this is the direction in which the charges in the wire move in response to the electric force

يمكن اثبات هذه الحاله من خلال فرضنا ان لدينا موصل على شكل حلقه في مجال مغناطيسي حيث r تمثل نصف قطر الحلقه فاذا تغير الفيض المغناطيسي مع الزمن وحسب قانون فراداي تتولد قوه دافعه كهربائيه محتثه induced emf وذلك يؤدي الى توليد تيار محتث induced current وبالتالي يتولد مجال كهربائي محتث induce electric field يكون اتجاهه مماسا متاثرا بحركة الشحنات في الموصل (الحلقه)

The work done by the electric field in moving a test charge **q** once around the loop is equal to **qƐ**. Because the electric force acting on the charge is **qE**, the work done by the electric field in moving the charge once around the loop is **qE(2πr),** where **2πr** is the circumference of the loop. These two expressions for the work done must be equal; therefore, we see that

اذا سيكون الشغل الازم لتحريك الشحنه حول الحلقه يساوي **qƐ** . ولان القوه المسلطه على الشحنه تساوي **qE** اذا سيكون الشغل المنجز من قبل المجال الكهربائي لتحريك الشحنه حول الحلقه هو

القوه الكهربائيه = الشغل اللازم لتحريك الشحنه

$q\mathcal{E} = qE(2\pi r)$ circumference $2\pi r$

$\mathcal{E} = E.2\pi r$

$\mathcal{E} = -\frac{d\phi_B}{dt}$ Faraday law of induction

$\phi = BA = \pi r^2 B$

$\therefore\ E.2\pi r = -\frac{d}{dt}\,(\pi r^2 B)$

$E = -\frac{r}{2}\frac{dB}{dt} \neq 0$ is no equal to zero

The **emf** for any closed path can be expressed as the line integral of **E. ds** over that path: $\mathcal{E} = \oint \boldsymbol{E.ds}$. In more general cases, **E** may not be constant, and the path may not be a circle. Hence, Faraday's law of induction, $\mathcal{E} = -\boldsymbol{d\phi_B/dt}$, can be written in the general form

القوه الدافعه الكهربائيه لاي مسار مغلق يمكن التعبير عنها بتكامل خطي لـ **E. ds** لكل المسار $\mathcal{E} = \oint \boldsymbol{E.ds}$ وبشكل عام قد يكون المجال الكهربائي غير ثابت و المسار ليس دائريا وعليه يمكن كتابة قانون فراداي للحث بصيغته العامه

$$\oint \boldsymbol{E.ds} = -\frac{\boldsymbol{d\phi_B}}{\boldsymbol{dt}} \qquad \textbf{(2)}$$

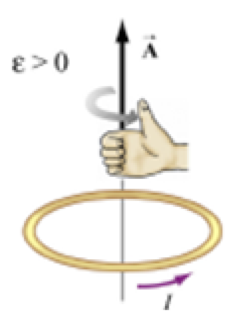
The induced electric field **E** in Equation **1** is a **nonconservative field** that is generated by a changing magnetic field. The field **E** that satisfies Equation**1** cannot possibly be an electrostatic field because if the field were electrostatic, and hence conservative, the line integral of **E. ds** over a closed loop would be zero.

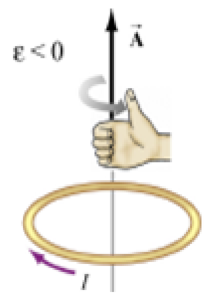
من المعادله (2) نلاحظ ان المجال الكهربائي المحتث ليس محافظا. ذلك بسبب تغير المجال المغناطيسي الاصلي .ولو فرضنا العكس ان المجال الكهربائي في المعادله (2) اذا كان مجال مستقر ثابت و محافظ سيكون التكامل الخطي للـ E. ds لكل مسار الحلقه المغلق يساوي صفرا .

$$\oint \boldsymbol{E}.\boldsymbol{ds} = \begin{cases} \rightarrow = \mathbf{0} \ \ \textbf{conservative} \\ \rightarrow = \mathbf{0} \ \ \textbf{non- conservative} \end{cases}$$

12- clarify the Lenz law and illustrate the induced emf (ε) both experimentally and theoretically ?

Answer:

عمليا **Experimentally**	نضريا **Theoretically**
1 – نجعل الابهام باتجاه متجه المساحه **A** فاذا كان اتجاه دوران الاصابع بنفس اتجاه حركة التيار المحتث **induced current** عندها يكون **induced emf** موجب **ε > 0**  2 - فاذا كان اتجاه دوران الاصابع عكس اتجاه حركة التيار المحتث **induced current** عندها يكون **induced emf** سالب **ε < 0**	1 – نحدد قيمة الفيض المغناطيسي $\Phi_B = BA\cos\theta$ 2 – نجد قيمة مشتقة الفيض $\frac{d\Phi_B}{dt} \begin{cases} \rightarrow > 0 \rightarrow \mathcal{E} < 0 \\ \rightarrow = 0 \rightarrow \mathcal{E} = 0 \\ \rightarrow < 0 \rightarrow \mathcal{E} > 0 \end{cases}$ وهذا يعني ان تغير في الفيض عندما يزداد هذا يؤدي الى تقليل **emf** حتى بالنتيجة تقل **I induced** ويقل معها **B induced**



13- Explain the motional emf in case of supposing the conducting bar moves through a region of uniform magnetic field $\vec{B} = B\hat{K}$ (Pointing into the page) by sliding along two frictionless conducting rails that are at a distance ℓ apart and connected together by a resistor with resistance R?

Answer :

هي فرق الجهد (V) بين طرفي قطعه معدنيه موصله متحركه في مجال مغناطيسي منتظم

$$\boldsymbol{\varepsilon} = \oint(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}).\boldsymbol{d}\vec{\boldsymbol{s}}$$

Consider a conducting bar of length l moving through a uniform magnetic field which points into the page, as shown in Figure 3. Particles with charge q > 0 inside experience a magnetic force $\vec{\boldsymbol{F}} = \boldsymbol{q}\vec{\mathbf{v}} \times \vec{\boldsymbol{B}}$ which tends to push them upward, leaving negative charges on the lower end.

لنفرض ان لدينا موصل على شكل قضيب معدني بطول l يتحرك خلال مجال مغناطيسي منتظم الذي يتجه داخل الصفحه (×) كما مبين في الشكل (3) الشحنات الموجبه في داخل القضيب سوف تتاثر بقوه مغناطيسيه تعمل على دفعها الى اعلى القضيب فيما تندفع الشحنات السالبه الى الاسفل بفعل نفس القوه

The separation of charge gives rise to an electric field $\vec{E}$ inside the bar, which produces a downward electric force $\vec{\boldsymbol{F}} = \boldsymbol{q}\vec{\boldsymbol{E}}$. At equilibrium where the two forces cancel, We have $\boldsymbol{qvB} = \boldsymbol{qE}$, or $\boldsymbol{E} = \boldsymbol{vB}$. Between the two ends of the conductor, there exists a potential difference given by

$$\boldsymbol{V_{ab}} = \boldsymbol{V_a} - \boldsymbol{V_b}$$

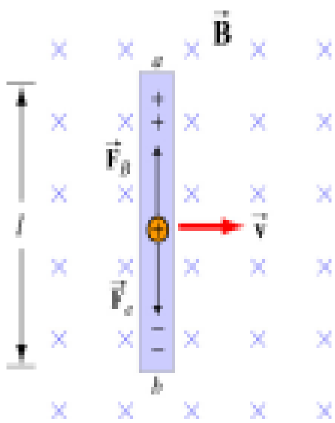
$$\boldsymbol{\varepsilon} = \boldsymbol{E\ell}$$

$$= \boldsymbol{vB\ell}$$

فصل الشحنات بهذه الطريقه يولد مجلا كهربائيا في داخل القضيب المعدني والتي تولد قوه كهربائيه متجه الى الاسفل وبالتالي سوف تلغي القوتين بعضهما لانهما في اتجاهين متعاكسين وسفوف تظهر على الموصل فرق جهد في نهايتي الموصل

Since ε arises from the motion of the conductor, this potential difference is called the motional emf. In general, motional emf around a closed conducting loop can be written as:

$$\boldsymbol{\varepsilon} = \oint (\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}).\boldsymbol{d\vec{s}}$$



Where $d\vec{s}$ is a differential length element?

بما ان ε = induced emf سوف تنشاء من حركه الموصل . سوف نسمي فرق الجهد هذا القوه الدافعه الكهربائيه المتحركه motional emf حيث $d\vec{s}$ تمثل متجه المسافه .

Now suppose the conducting bar moves through a region of uniform magnetic field $\vec{\boldsymbol{B}} = \boldsymbol{B\hat{k}}$ (pointing into the page) by sliding along two frictionless conducting rails that are at a distance $\boldsymbol{\ell}$ apart and connected together by a resistor with resistance **R**, as shown in Figure 4.

لنفرض ان لدينا الموصل يتحرك بواسطه سكتين متوازيتين عديمتي الاحتكاك في مجال مغناطيسي منتضم متجه داخل الصفحه$\vec{B} = B\hat{k}$ وان المسافه بين السكتين هي $\boldsymbol{\ell}$ ومربوطه الى مقاومه **R** كما مبين في الشكل 4 .

Let an external force be applied so that the conductor moves to the right with a constant velocity $\vec{\boldsymbol{v}} = \boldsymbol{v}\hat{\boldsymbol{i}}$. The magnetic flux through the closed loop formed by the bar and the rails is given by:

$$\Phi_{\boldsymbol{B}} = \boldsymbol{BA} = \boldsymbol{B\ell x} \qquad \textbf{(4)}$$

Thus, according to Faraday's law, the induced **emf** is:

$$\boldsymbol{\varepsilon} = -\frac{\boldsymbol{d}\Phi_{\boldsymbol{B}}}{\boldsymbol{dt}} = -\frac{\boldsymbol{d}}{\boldsymbol{dt}}(\boldsymbol{B\ell x}) = -\boldsymbol{BI}\frac{\boldsymbol{dx}}{\boldsymbol{dt}} = -\boldsymbol{B\ell v}$$

Where $\boldsymbol{dx/dt = v}$ is simply the speed of the bar. The corresponding induced current is:

$$\boldsymbol{I} = \frac{|\boldsymbol{\varepsilon}|}{\boldsymbol{R}} = \frac{\boldsymbol{B\ell v}}{\boldsymbol{R}}$$

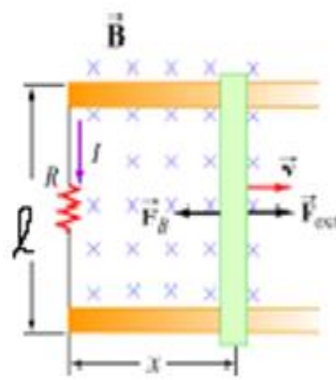


Figure 4

14- A long solenoid has n = 400 turns per meter and carries a current given by I = (30.0 A)(1 – $e^{-1.60t}$). Inside the solenoid and coaxial with it is a coil that has a radius of R= 6.00 cm and consists of a total of N = 250 turns of fine wire (Fig. P31.13). What emf is induced in the coil by the changing current?

Solution:

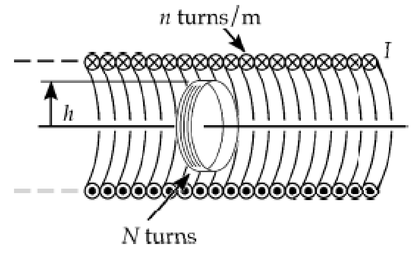


FIG. P31.13

$$B = \mu_0 n I = \mu_0 n (30.0\text{ A})\left(1 - e^{-1.60t}\right)$$

$$\Phi_B = \int B\,dA = \mu_0 n (30.0\text{ A})\left(1 - e^{-1.60t}\right)\int dA$$

$$\Phi_B = \mu_0 n (30.0\text{ A})\left(1 - e^{-1.60t}\right)\pi R^2$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\mu_0 n (30.0\text{ A})\pi R^2 (1.60) e^{-1.60t}$$

$$\varepsilon = -(250)\left(4\pi \times 10^{-7}\text{ N/A}^2\right)\left(400\text{ m}^{-1}\right)(30.0\text{ A})\left[\pi(0.060\,0\text{ m})^2\right]1.60\text{ s}^{-1}e^{-1.60t}$$

$$\varepsilon = \boxed{(68.2\text{ mV})e^{-1.60t}\text{ counterclockwise}}$$

15- coil formed by wrapping 50 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of θ = 30.0° with the direction of the field. When the magnetic field is increased uniformly from 200 μT to 600 μT in time t = 0.400 s, an emf of magnitude ε = 80.0 mV is induced in the coil. What is the total length of the wire?

Solution:

$$\varepsilon = \frac{d}{dt}\left(NB\ell^2\cos\theta\right) = \frac{N\ell^2\Delta B\cos\theta}{\Delta t}$$

$$\ell = \sqrt{\frac{\varepsilon\Delta t}{N\Delta B\cos\theta}} = \sqrt{\frac{\left(80.0\times10^{-3}\text{ V}\right)(0.400\text{ s})}{(50)\left(600\times10^{-6}\text{ T} - 200\times10^{-6}\text{ T}\right)\cos(30.0°)}} = 1.36\text{ m}$$

$$\text{Length } = 4\ell N = 4(1.36\text{ m})(50) = \boxed{272\text{ m}}$$

16- A toroid having a rectangular cross section (a = 2.00 cm by b = 3.00 cm) and inner radius R = 4.00 cm consists of 500 turns of wire that carries a sinusoidal current $I = I_{max}$ sin wt, with I max = 50.0 A and a frequency f = 60.0 Hz. A coil that consists of N = 20 turns of wire links with the toroid, as in Figure P31.17. Determine the emf induced in the coil as a function of time?

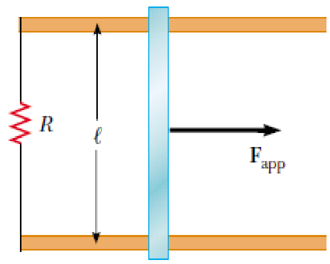


Figure P31.20

Solution:

In a toroid, all the flux is confined to the inside of the toroid.

$$B = \frac{\mu_0 NI}{2\pi r} = \frac{500\mu_0 I}{2\pi r}$$

$$\Phi_B = \int BdA = \frac{500\mu_0 I_{max}}{2\pi}\sin\omega t\int\frac{adr}{r}$$

$$\Phi_B = \frac{500\mu_0 I_{max}}{2\pi}a\sin\omega t\ln\left(\frac{b+R}{R}\right)$$

$$\varepsilon = N'\frac{d\Phi_B}{dt} = 20\left(\frac{500\mu_0 I_{max}}{2\pi}\right)\omega a\ln\left(\frac{b+R}{R}\right)\cos\omega t$$

$$\varepsilon = \frac{10^4}{2\pi}\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(50.0\ \text{A})(377\ \text{rad/s})(0.020\,0\ \text{m})\text{lr}$$

$$= \boxed{(0.422\ \text{V})\cos\omega t}$$

17- Figure P31.20 shows a top view of a bar that can Slide without friction. The resistor is R = 6.00

(and a 2.50-T) The Magnetic field is directed

perpendicularly downward, Into the paper. Let ℓ = 1.20 m. (a) Calculate the applied force required to move the bar to the right at a constant speed of 2.00 m/s. (b) At what rate is the energy delivered

to the resistor?

Solution:

(a) $|F_B| = I|\ell \times B| = I\ell B$

When $I = \dfrac{\varepsilon}{R}$

and $\varepsilon = B\ell v$

we get $F_B = \dfrac{B\ell v}{R}(\ell B) = \dfrac{B^2\ell^2 v}{R} = \dfrac{(2.50)^2(1.20)^2(2.00)}{6.00} = 3.00\text{ N}$.

The applied force is $\boxed{3.00\text{ N to the right}}$.

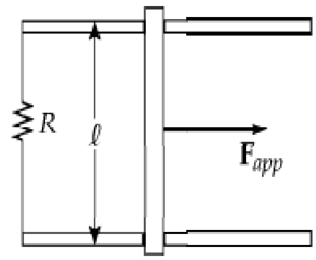


FIG. P31.21

(b) $\mathcal{P} = I^2R = \dfrac{B^2\ell^2v^2}{R} = 6.00\text{ W}$ or $\mathcal{P} = Fv = \boxed{6.00\text{ W}}$

18- For the situation shown in Figure P31.32, the magnetic field changes with time according to the expression $B = (2.00t^3 - 4.00t^2 + 2\ 0.800)$T, and $r_2 = 2R = 5.00$ cm. (a) Calculate the magnitude and direction of the force exerted on an electron located at point $P2$ when $t = 2.00$ s.

(c) At what time is this force equal to zero?

Solution:

(a) $\dfrac{dB}{dt} = 6.00t^2 - 8.00t$ $\qquad |\varepsilon| = \dfrac{d\Phi_B}{dt}$

At $t = 2.00$ s, $\qquad E = \dfrac{\pi R^2(dB/dt)}{2\pi r_2} = \dfrac{8.00\pi(0.025\,0)}{2\pi(0.050\,0)}$

$F = qE = \boxed{8.00 \times 10^{-21}\text{ N}}$

(b) When $6.00t^2 - 8.00t = 0$, $\quad t = \boxed{1.33\text{ s}}$

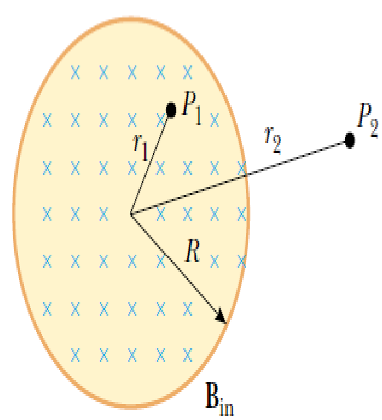


Figure P31.32 Problems 32

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