

Chapter One: Linear Programming

Linear Programming is a new area of applied mathematics, developed in late 1940s, solve a number of problems for the federal government, management science and operation research.

1.1 Linear Programming Problem (LPP) Formulation

In many problems in business and industry we are interested in making decision that will maximize or minimize quantity. For example, a planet manager may want to determine the most economical way of shipping goods from the factory to the markets, a hospital may want design a diet satisfying certain nutritional requirement's a minimum cost, an investor may want to select investments that will maximize profits, in this section we give several examples of LPP and show how mathematical model for them can be formulated.

Example (1.1): company wished to schedule the production of appliance which requiring three different models and its production engineering department has furnished as in table (1.1) data:

Table (1.1)

	Laboure Hours/unit	Material Pound/unit	Profit \$/unit
A	7	4	4
B	3	4	2
C	6	5	3
	150	200	

The supply raw material is restricted to 200 pounds per day. The daily availability of time is 150 hours. Formulate a LPP to determine the daily production rate of the various model in order to maximize the total profit.

Solution:

Let x_1 be the number of A that produces daily.

Let x_2 be the number of B that produces daily.

Let x_3 be the number of C that produces daily.

The total amount of time (Laboure) is:

$$7x_1+3x_2+6x_3$$

The total amount of raw material is:

$$4x_1+4x_2+5x_3$$

Since we have 150 hours as maximum time then:

$$7x_1+3x_2+6x_3 \leq 150$$

And since we have 200 pound as maximum quantity of material, then:

$$4x_1+4x_2+5x_3 \leq 200$$

Of course x_1, x_2, x_3 cannot be negative, so:

$$x_1, x_2, x_3 \geq 0.$$

The total profit is:

$$Z=4x_1+2x_2+3x_3$$

Our problem is finding values of x_1, x_2 and x_3 that will maximize the profit Z is:

$$\text{Max. } Z=4x_1+2x_2+3x_3$$

Subject to the following restriction that must be satisfied x_1, x_2 and x_3 :

$$7x_1+3x_2+6x_3 \leq 150$$

$$4x_1+4x_2+5x_3 \leq 200$$

$$x_1, x_2, x_3 \geq 0.$$

Example (1.2) (Diet Problem): Nutritionist is planning a menu that includes food A and B. Suppose that each once of food A contains 2 units of protein, 1 unit of iron and 1 unit of thiamine. Each once of food B contains 1 units of protein, 1 unit of iron and 3 unit of thiamine. Suppose that each once of food A costs 30 cents, while each once of food B costs 40 cents. The nutritionist wants the meal to provide at least 12 units of protein, at least 9 units of iron, and at least 15 units of thiamine. How

many ounces of each food should be used to minimize the cost of the meal?

Solution: Let

x_1 : number of ounces of food A to be used.

x_2 : number of ounces of food B to be used.

The restrictions inequalities are:

$$2x_1 + 3x_2 \geq 12$$

$$x_1 + x_2 \geq 9$$

$$x_1 + 3x_2 \geq 15$$

of course $x_1, x_2 \geq 0$.

Which want to minimize Z s.t.

$$\text{Min. } Z = 30x_1 + 40x_2$$

s.t.

$$2x_1 + 3x_2 \geq 12$$

$$x_1 + x_2 \geq 9$$

$$x_1 + 3x_2 \geq 15$$

$$x_1, x_2 \geq 0.$$

These examples are typical of LPP, the general form being:

Find values $x_1, x_2, \dots, x_n$, that minimize or maximize

$$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n \quad \dots(1)$$

Subject to:

$$\left. \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n (\leq)(\geq)(=) b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n (\leq)(\geq)(=) b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n (\leq)(\geq)(=) b_m \end{array} \right\} \quad \dots(2)$$

$$x_j \geq 0, j=1, \dots, n. \quad \dots(3)$$

where in system (2), one and only one symbol $\leq$, $=$, $\geq$ occurs in each statement. The linear function in (1) is called objective function. The equalities or inequalities in (2) and (3) are called constraints.

Example (1.3): in designing a new transportation system, a company is considering two types of buses; type A and B. A type A bus can carry 40 passengers and requires 2 mechanics for serving, type B bus can carry 60 passengers and requires 3 mechanics for serving. Suppose that the company must transport at least 300 people daily and in that insurance rules for the size of the garage allow no more than 12 mechanics on the payroll. If each type A bus costs \$20000 and each type B bus costs \$25000, how many buses of each type should be bought to minimize the cost.

Solution: Let

x_1 : number of buses of type A should be bought.

x_2 : number of buses of type B should be bought.

$$\text{Min. } Z = 20000x_1 + 25000x_2$$

s.t.

$$2x_1 + 3x_2 \leq 12$$

$$40x_1 + 60x_2 \geq 300$$

$$x_1, x_2 \geq 0.$$

Example (1.4): A television procedure designs a program based on comedian and time for commercials. The advertiser insists on at least 2 minutes of advertising time, the station insists on no more than 4 minutes of advertising time, and the comedian insists on at least 24 minutes of the comedy program. Also, the total time allotted for the advertising and comedy portions of the program cannot exceed 30 minutes. If it has been determining that each minute of advertising attracts 40000 viewers, each

minute of comedy attracts 45000 viewers, how should the time be divided between advertising and programming to maximize the number of viewers-minutes?

Solution: Let

x_1 : number of advertising time in minutes.

x_2 : number of comedy program time in minutes.

Notes that: $x_1 \geq 2$ and $x_1 \leq 4$, then

$$2 \leq x_1 \leq 4.$$

$$x_2 \geq 24.$$

$$x_1 + x_2 \leq 30,$$

then the problem can be formulated as follows:

$$\text{Max. } Z = 40000x_1 + 45000x_2$$

s.t.

$$2 \leq x_1 \leq 4.$$

$$x_2 \geq 24.$$

$$x_1 + x_2 \leq 30,$$

$$x_1, x_2 \geq 0.$$

Homework (1.1):

HW1: In animal feed procedure mixes two types of grain A and B. each unit of grain A contains 2 grams of fat, 1 gram of protein and 80 calories. each unit of grain B contains 3 grams of fat, 3 grams of protein and 60 calories. Suppose that the procedure wants each unit of the final product to yield at least 18 grams of fat, at least 12 grams of protein and at least 480 calories. If each unit of A cost 10 cents and each unit of B costs 12 cents, how many of each type of grain should the producer use to minimize the cost?

Solution: Let

x_1 : number of units of grain A should be used.

x_2 : number of units of grain B should be used.

$$\text{Min. } Z = 10x_1 + 12x_2$$

$$2x_1 + 3x_2 \geq 18$$

$$x_1 + 3x_2 \geq 12,$$

$$80x_1 + 60x_2 \geq 480$$

HW2: A trust fund is planning to invest up to \$6000 in two types of bonds: A and B, Bond A is safer than bond B and carries a dividend of 8%, and bond B carries a dividend of 10%, Suppose that the funds rules state that no more than \$4000 may be invested in bond B, while at least \$1500 must be invested in bond A. How much should be invested in each type of bond to maximize the funds return?

HW3: Three products are processed through three different operations. The times (in minutes) required per unit of each product. The daily capacity of operations (in minutes) and the profit per unit sold of each product (in dollars) are as in table (1.2):

Table (1.2)

Operation	P1	P2	P3	Op operation
	Time per unit (min.)			min./day
1	1	2	1	430
2	3	0	2	460
3	1	4	0	420
Profit (\$)	3	2	5	\$/unit

The sum of the all-unused capacity of all three operations must not exceed 10 min/day. The market study indicates that the ratio of the

number of units of P1 to the number of units of P2 and P3 must be at least equal to 0.4. formulate this problem as a LPP.

HW4: The protein Diet club serves a luncheon consisting of two dishes. A and B suppose that each unit of A has 1g. of fat 1g. of carbohydrate and 4g of protein, whereas each unit of B has 2g of fat, 1g of carbohydrate and 6g of protein. If the dietician planning the luncheon wants to provide no more than 10g of fat or more than 7g of carbohydrate, how many units of A and how many units of B should be served to maximize the amount of protein consumed?