

### 1.4.2 The Pivot Operation

Before we start solving, we must choose the pivot element, when it chosen, we apply row operations so that the pivot element is replaced by 1 and all other entries in the same column become 0's.

**Example (1.14):** in table (1.6) we have the following data:

Table (1.6)

↓

| Basic Variable (BV) | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Z | Solution |
|---------------------|-------|-------|-------|-------|---|----------|
| $S_1$               | 2     | ④     | 1     | 0     | 0 | 120      |
| $S_2$               | 2     | 2     | 0     | 1     | 0 | 80       |
| Z                   | -3    | -4    | 0     | 0     | 1 | 0        |

**Solution:** The pivoting element is in the intersection of row1 and column2. Now we try to replace the pivoting element 4 by 1 and make the other entries of same column by 0's.

We notice that  $x_2$  will be called the entering new basic variable and  $S_1$  will be called the leaving non basic variable.

Now we will choose another pivoting element, (row2 and column1), where  $x_1$  will be the entering new basic variable and  $S_2$  will be the leaving non basic variable (see table (1.7)).

Table (1.7)

↓

| BV    | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Z | Solution |
|-------|-------|-------|-------|-------|---|----------|
| $x_2$ | 1/2   | 1     | 1/4   | 0     | 0 | 30       |
| $S_2$ | ①     | 0     | -1/2  | 1     | 0 | 20       |
| Z     | -1    | 0     | 1     | 0     | 1 | 120      |

Then obtain the last tableau (see table (1.8)).

Table (1.8)

| BV    | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Z | Solution |
|-------|-------|-------|-------|-------|---|----------|
| $x_2$ | 0     | 1     | 1/2   | -1/2  | 0 | 20       |
| $x_1$ | 1     | 0     | -1/2  | 1     | 0 | 20       |
| Z     | 0     | 0     | 1/2   | 1     | 1 | 140      |

Then  $x_1=20$ ,  $x_2=20$ ,  $S_1=0$ ,  $S_2=0$  and  $Z=140$ .

### 1.4.3 Selecting the Pivot Element

1. Selecting the pivot column by locating the most negative entry in the objective row. If all the entries in this column are negative, the problem is unbounded and there is no solution.
2. Divide each entry in the last column (solution column) by the corresponding entry (from the same row) in the pivot column (ignore any rows in which the pivot column entry is less than or equal to 0) the row in which the smallest positive ratio is obtained is the pivot row.
3. The pivot element is the entry at the intersection of the pivot row and the pivot column (see table (1.9)).

Table (1.9)

| BV    | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Z | Sol. |
|-------|-------|-------|-------|-------|---|------|
| $S_1$ | 2     | ④     | 1     | 0     | 0 | 120  |
| $S_2$ | 2     | 2     | 0     | 1     | 0 | 80   |
| Z     | -3    | -4    | 0     | 0     | 1 | 0    |
| $x_2$ | 1/2   | 1     | 1/4   | 0     | 0 | 30   |
| $S_2$ | ①     | 0     | -1/2  | 1     | 0 | 20   |
| Z     | -1    | 0     | 1     | 0     | 1 | 120  |
| $x_2$ | 0     | 1     | 1/2   | -1/2  | 0 | 20   |
| $x_1$ | 1     | 0     | -1/2  | 1     | 0 | 20   |
| Z     | 0     | 0     | 1/2   | 1     | 1 | 140  |

Since there are no negative entries in the object row, then the problem is been solved,  $x_1=20$ ,  $x_2=20$ ,  $Z=140$ .

**Homework (1.4):** Solve the following LPP using simplex method.

(1).  $Max Z=5x_1 + 7x_2$

s.t.

$$2x_1 + 3x_2 \leq 12, x_1 \geq 0$$

$$3x_1 + x_2 \leq 12, x_2 \geq 0$$

(2).  $Max Z=2x_1 + x_2 + 3x_3$

s.t.

$$x_1 + 2x_2 + x_3 \leq 25,$$

$$3x_1 + 2x_2 + 3x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

**Example (1.15):** Mikes Famous toy trucks specializes in making four kinds of trucks. Three machines are used in the production of these trucks. The time, (in hours) each machine works to make each type of truck and the profit for each truck are given in table (1.10). The maximum time available per week for each machine mentioned in the same table. How many of each type truck should be produced to maximize the profit?

Table (1.10)

| Truck                   | Metal Casting/h | Paint Spry/h | Packaging/h | Profit/\$ |
|-------------------------|-----------------|--------------|-------------|-----------|
| Delivery Truck          | 2               | 1            | 0.5         | 0.5       |
| Dump Truck              | 2.5             | 1.5          | 0.5         | 1.0       |
| Garbage Truck           | 2               | 1            | 1           | 1.5       |
| Gasoline Truck          | 2               | 2            | 1           | 2.0       |
| Available time per week | 4000            | 1800         | 1000        |           |

**Solution:** Let

$x_1$  = Number of delivery trucks.

$x_2$  = Number of Dump trucks.

$x_3$  = Number of Garbage trucks.

$x_4$  = Number of Gasoline trucks.

$$\text{Max } Z = 0.5x_1 + x_2 + 1.5x_3 + 2x_4$$

s.t.

$$2x_1 + 2.5x_2 + 2x_3 + 2x_4 \leq 4000$$

$$x_1 + 1.5x_2 + x_3 + 2x_4 \leq 1800$$

$$0.5x_1 + 0.5x_2 + x_3 + x_4 \leq 1000$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$2x_1 + 2.5x_2 + 2x_3 + 2x_4 + S_1 = 4000$$

$$x_1 + 1.5x_2 + x_3 + 2x_4 + S_2 = 1800$$

$$0.5x_1 + 0.5x_2 + x_3 + x_4 + S_3 = 1000$$

$$-0.5x_1 - x_2 - 1.5x_3 - 2x_4 + Z = 0$$

Table (1.11) show the implementation of simplex method.

Table (1.11)

|   |       |       |       |       |       |       |       |       |      |
|---|-------|-------|-------|-------|-------|-------|-------|-------|------|
| ↓ |       |       |       |       |       |       |       |       |      |
|   | BV    | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $S_1$ | $S_2$ | $S_3$ | Sol. |
|   | $S_1$ | 2     | 2.5   | 2     | 2     | 1     | 0     | 0     | 4000 |
| ← | $S_2$ | 1     | 1.5   | 1     | 2     | 0     | 1     | 0     | 1800 |
|   | $S_3$ | 0.5   | 0.5   | 1     | 1     | 0     | 0     | 1     | 1000 |
|   | Z     | -0.5  | -1    | -0.5  | -2    | 0     | 0     | 0     | 0    |
| ↓ |       |       |       |       |       |       |       |       |      |
|   | $S_1$ | 1     | 1     | 1     | 0     | 1     | -1    | 0     | 2200 |
|   | $x_4$ | 0.5   | 0.75  | 0.5   | 1     | 0     | 0.5   | 0     | 900  |
| ← | $S_3$ | 0     | -0.25 | 0.5   | 0     | 0     | 0.5   | 1     | 100  |
|   | Z     | 0.5   | 0.5   | -0.5  | 0     | 0     | 1     | 0     | 1800 |
|   |       |       |       |       |       |       |       |       |      |
|   | $S_1$ | 1     | 1.5   | 0     | 0     | 1     | 0     | -2    | 2000 |
|   | $x_4$ | 0.5   | 1     | 0     | 1     | 0     | 1     | 1     | 800  |
|   | $x_3$ | 0     | -0.5  | 1     | 0     | 0     | -1    | 2     | 200  |
|   | Z     | 0.5   | 0.25  | 0     | 0     | 0     | 0.5   | 1     | 1900 |

The maximum profit is \$1900, for  $x_1 = 0$ ,  $x_2 = 0$ ,  $x_3 = 200$ ,  $x_4 = 800$ .

Notice that  $S_1 = 2000$ , then the machine No. (1) wasting 2000 hours from the available 4000 hours, it may be possible to release this machine for other duties.

#### 1.4.4 The Unbound Case

If all the entries in a column of a tableau are 0's or negative, that mean the problem is unbounded maximum solution does not exist.

**Example (1.16):** Consider the table (1.12):

Table (1.12)

| BV    | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Sol. |
|-------|-------|-------|-------|-------|------|
| $S_1$ | -1    | 1     | 1     | 0     | 2    |
| $S_2$ | 1     | -1    | 0     | 1     | 2    |
| Z     | -1    | -1    | 0     | 0     | 0    |

When there are two equal smallest negative entries in the last row, you may choose either column say column1, then the tableau (1.12) becomes the tableau (1.13):

Table (1.13)

| BV    | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Sol. |
|-------|-------|-------|-------|-------|------|
| $S_1$ | 0     | 0     | 1     | 1     | 4    |
| $x_1$ | 1     | -1    | 0     | 1     | 2    |
| Z     | 0     | -2    | 0     | 1     | 2    |

Notice that column2 is 0 or negative value, then Z has no maximum and its unbounded, see figure (1.6).

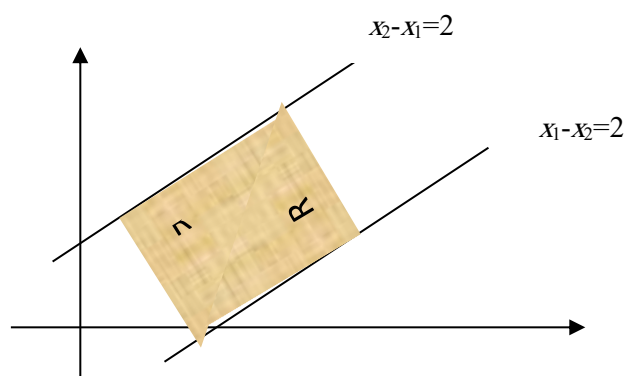


Figure (1.6): the graph of unbounded LPP.

**Homework (1.5):** Solve the following LPP by using simplex method.

(1).  $\text{Max } Z = x_1 + 5x_2$

s.t.

$2x_1 + x_2 \leq 10, x_1 \geq 0$

$x_1 + 2x_2 \leq 10, x_2 \geq 0$

(2).  $\text{Max } Z = 5x_1 + 7x_2$

s.t.

$$x_1 + 2x_2 \leq 2, \quad x_1 \geq 0$$

$$2x_1 + x_2 \leq 2, \quad x_2 \geq 0$$

$$(3). \text{ Max } Z = 2x_1 + x_2 + x_3$$

s.t.

$$-2x_1 + x_2 - 2x_3 \leq 4, \quad x_1 \geq 0$$

$$x_1 - 2x_2 + x_3 \leq 2, \quad x_2 \geq 0$$

$$x_3 \geq 0$$

$$(4). \text{ Max } Z = 2x_1 + x_2 + 3x_3$$

s.t.

$$x_1 + 2x_2 + x_3 \leq 25$$

$$3x_1 + 2x_2 + 3x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

$$(3). \text{ Max } Z = 2x_1 + x_2 + x_3$$

s.t.

$$-2x_1 + x_2 - 2x_3 \leq 4, \quad x_1 \geq 0$$

$$x_1 - 2x_2 + x_3 \leq 2, \quad x_2 \geq 0$$

$$x_3 \geq 0$$

$$(5). \text{ Max } Z = x_1 + 2x_2 + 4x_3 - 4x_4$$

s.t.

$$5x_1 + 4x_3 + 6x_4 \leq 20$$

$$4x_1 + 2x_2 + 2x_3 + 8x_4 \leq 40$$

$$x_1, x_2, x_3, x_4 \geq 0$$