

Assignment Model

2.1 Introduction

"The best person for job" is an efficient description of what the assignment model seeks to accomplish. The situation can be illustrated by the assignment of works to jobs. The objective of the model is to determine the optimum (least-cost) assignment of and n jobs is represented in table (2.1).

Table (2.1): The Assignment model Table

Workers	Jobs					
		1	2	...	n	
	1	c_{11}	c_{12}	...	c_{1n}	1
	2	c_{21}	c_{22}	...	c_{2n}	1
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	n	c_{n1}	c_{n2}	...	c_{nn}	1
		1	1	...	1	

The element c_{ij} represents the cost of assignment worker i to job j ($i, j = 1, 2, \dots, n$). The assignment model is actually special case of the transportation model in which the works represents the sources, and the jobs represent the destinations.

Now we will discuss the most important solving methods for the assignment model.

2.2 Permutation Method (PM)

In this method we will seek all the possible states of the model.

Example (2.1): Solve the following assignment by using PM (table (2.2)).

Table (2.2): PM data

	1	2	3
1	15	10	9
2	9	15	10
3	10	12	8

Solution: Table (2.3) shows all the possible permutation of assignment distribution.

Table (2.3): All the possible permutation of assignment distribution.

	1	2	3			
1.	1	2	3	→	15+15+8	Z=38
2.	1	3	2	→	15+10+12	Z=37
3.	2	1	3	→	10+9+8	Z=27
4.	2	3	1	→	10+10+10	Z=30
5.	3	1	2	→	9+9+12	Z=30
6.	3	2	1	→	9+15+10	Z=34

See that 1-2, 2-1, 3-3, Z=27.

2.3 Least-Cost Method (LCM)

This method depends on finding the minimum cost in assignment table, after assigning the worker to the job, cross the row and the column of the cost value, then repeating the process unit completing all the assignment model.

Example (2.2): call Example (2.1).

Solution: First we identify 3-3 since it's the most minimum cost= 8, then cross row 3 and column 3. In the same process we choose 2-1 with cost 9, lastly 1-2 with cost 10 (see table (2.4)).

Table (2.4): Assignment Process.

	1	2	3
1	15	(10)	9
2	(9)	15	10
3	10	12	(8)

The final result of assignment is 1-2, 2-1, 3-3 with: $Z = 10 + 9 + 8 = 27$.

In this manner we can introduce another two methods:

1. **Top-to-Bottom Least-Cost Method (TBLCM)**: This method applies starting from top to bottom using LCM.
2. **Bottom-to-Top Least-Cost Method (BTLCM)**: While this method applies starting from bottom. To top using LCM.

Table (2.5) shows the Assignment Process for two above methods.

Table (2.5): Assignment Process.

(a). TBLCM

	1	2	3			1	2	3
1	15	10	(9)		1	15	(10)	9
2	(9)	15	10		2	(9)	15	10
3	10	(12)	8		3	10	12	(8)

1-3, 2-1, 3-2

$$Z = 9 + 9 + 12 = 30$$

(b). BTLCM

1-2, 2-1, 3-3

$$Z = 10 + 9 + 8 = 27$$

Notes:

- 1- LCM Is more efficient then TBLCM and BTLCM.
- 2- LCM its related methods are not necessary to give optimum solution.

3- PM considered as brute force (exhaustive search) attack.

Example (2.3): Solve the following assignment model using PM, LCM, TBLCM and BTLCM (Table (2.6)).

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

Solution:

(1) **PM:**

- | | | | |
|-----|--------------------|---|------|
| 1. | 1-1, 2-2, 3-3, 4-4 | → | Z=24 |
| 2. | 1-1, 2-2, 3-4, 4-3 | → | Z=23 |
| 3. | 1-1, 2-3, 3-2, 4-4 | → | Z=21 |
| 4. | 1-1, 2-3, 3-4, 4-2 | → | Z=25 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 23. | 1-4, 2-3, 3-1, 4-2 | → | Z=24 |
| 24. | 1-4, 2-3, 3-2, 4-1 | → | Z=26 |

The best (minimum) cost is Z=21 for 1-1, 2-3, 3-2, 4-4.

(2) **LCM:** Table (2.7).

The best solution is: 1-1, 2-3, 3-2, 4-4 $Z=1+10+5+5=21$		1	2	3	4
	1	1	4	6	3
	2	9	7	10	9
	3	4	5	11	7
	4	8	7	8	5

(3) **TBLCM:** Table (2.8).

The best solution is: 1-1, 2-2, 3-4, 4-3 $Z=1+7+7+8=23$		1	2	3	4
	1	1	4	6	3
	2	9	7	10	9
	3	4	5	11	7
	4	8	7	8	5

(4) **BTLCM**: Table (2.9).

The best solution is: 1-3, 2-2, 3-1, 4-4 $Z=6+7+4+5=22$		1	2	3	4
	1	1	4	6	3
	2	9	7	10	9
	3	4	5	11	7
	4	8	7	8	5

2.4 The Hungarian Method (HM)

Step1: Identify each rows minimum, and subtract it from all the entries of the row.

Step2: For the new matrix, identify each column minimum and subtract it from all the entries of the col.

Step3: If no feasible assignment can be secured from **step1** and **2**:

- Draw the min. number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.
- Select the smallest uncovered element, and subtract it from every uncovered element, then add it to every element at the intersection of two lines.
- If no feasible assignment can be found among the resulting zero entries, repeat **step1**.

Step4: Identify the optimal assignment as the one associated with zero elements of the matrix obtained in **step3**.

Example (2.4): Solve the following assignment using the Hungarian method (Table (2.10)).

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

Solution:

	1	2	3	4
1	0	3	5	2
2	2	0	3	2
3	0	1	7	3
4	3	2	3	0

→

	1	2	3	4
1	0	3	2	2
2	2	0	0	2
3	0	1	4	3
4	3	2	0	0

	1	2	3	4
1	0	2	1	1
2	3	0	0	2
3	0	0	3	2
4	4	2	0	0

→

	1	2	3	4
1	0	2	1	1
2	3	0	0	2
3	0	0	3	2
4	4	2	0	0

The optimum solution is: 1-1, 2-3, 3-2, 4-4, $Z = 3 + 2 + 4 + 3 + 9 = 21$

Example (2.5): Solve the following assignment model using Hungarian method Table (Table (2.11)).

	1	2	3	4	5
1	3	8	2	10	3
2	8	7	2	9	7
3	6	4	2	7	5
4	8	4	2	3	5
5	9	10	6	9	10

Solution:

	1	2	3	4	5
1	1	6	0	8	1
2	6	5	0	7	5
3	4	2	0	5	3
4	6	2	0	1	3
5	3	4	0	3	4

→

	1	2	3	4	5
1	0	5	0	7	0
2	5	4	0	6	4
3	3	1	0	4	2
4	5	1	0	0	2
5	2	3	0	2	3

	1	2	3	4	5
1	0	5	1	7	0
2	4	3	0	5	3
3	2	0	0	3	1
4	5	1	1	0	2
5	1	2	0	1	2

→

	1	2	3	4	5
1	0	5	2	7	0
2	3	2	0	4	2
3	2		1	3	1
4	5	1	2	0	2
5	0	1	0	0	1

The optimum solution is: 1-5, 2-3, 3-2, 4-4, 5-1, $Z = 3 + 2 + 4 + 3 + 9 = 21$

Homework (2.1):

(1) Solve the following assignment model using H. M. (Table (2.12)).

	1	2	3	4	5
1	3	9	2	3	7
2	6	1	5	6	6
3	9	4	7	10	3
4	2	5	4	2	1
5	9	6	2	4	5

(2) Jo shop needs to assign four jobs it received to 4 works. The varying skills of the works give rise to varying costs for performing the jobs. Table (2.12) summarizes the cost data of the assignments. The data indicate that worker 1 cannot work on job 3, and worker 3 cannot work on job 4. Determine the optimal assignment.

Table (2.12): The cost data of the assignments.

	1	2	3	4
1	50	50	--	20
2	70	40	20	30
3	90	30	50	--
4	70	20	60	70