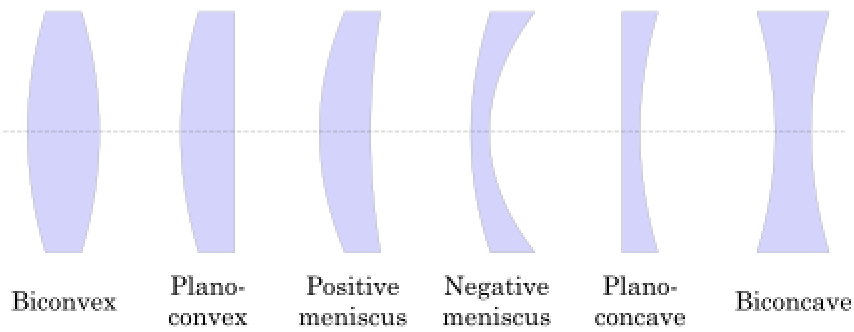
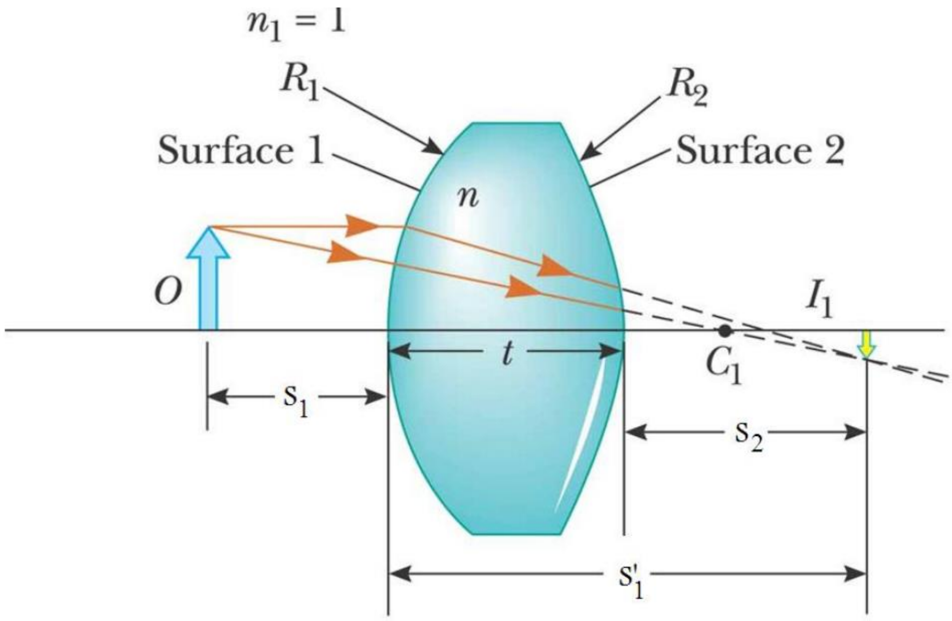
chapter four

THIN LENSES

Types of lenses



A thin lens may be defined as one whose thickness is considered small in comparison with the distances generally associated with its optical properties. Such distances are, for example, radii of curvature of the two spherical surfaces, primary and secondary focal lengths, and object and image distances.



The combination of various surfaces of thin lenses will determine the signs of the corresponding spherical radii.

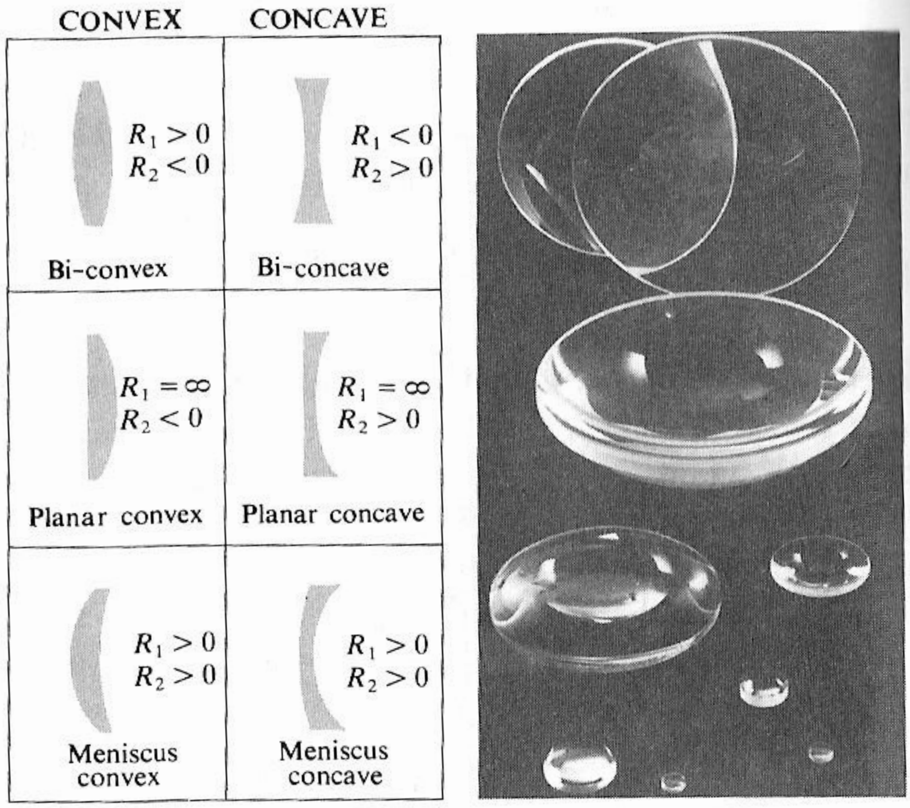


Figure 1 Cross sections of various centered spherical simple lenses. The surface on the left is #1 since it is encountered first. Its radius is R_1. (Photo courtesy of Melles Griot.)

4.1 FOCAL POINTS AND FOCAL LENGTHS

Diagrams showing the refraction of light by an equiconvex lens and by an equiconcave lens are given in figure 4.2. **The axis in each case is a straight line through the geometrical center of the lens and perpendicular to the two faces at the points of intersection**. For spherical lenses this line which joins the centers of curvature of the two surfaces. Ray diagrams shown in the figure 4.2 illustrates the primary and secondary focal points F and F' and the corresponding focal lengths f and f' of thin lenses.

<u>The primary focal point (F) of convex lens:</u> an axial point having the property that any ray coming from it , travels parallel to the axis after refraction.

<u>The secondary focal point (F') of convex lens:</u> an axial point having the property that any incident ray traveling parallel to the axis will, after refraction, proceed toward.

<u>The primary focal point (F) of concave lens:</u> an axial point having the property that any ray proceeding toward it travels parallel to the axis after refraction.

<u>The secondary focal point (F') of concave lens:</u> an axial point having the property that any incident ray traveling parallel to the axis will, after refraction, appear to come from, F'.

<u>Focal length:</u> The distance between the center of a lens and either of its focal points, these distances denoted by f and f' in the below figure.

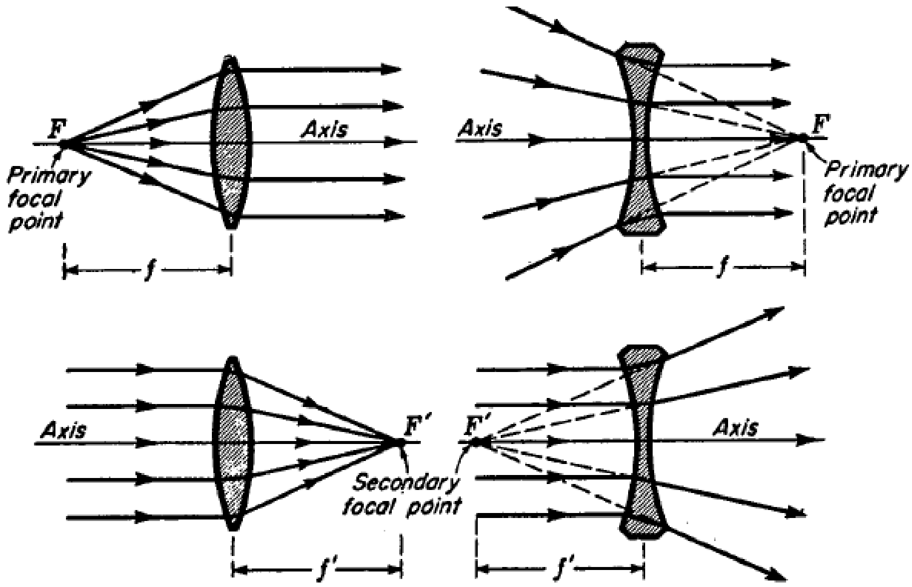
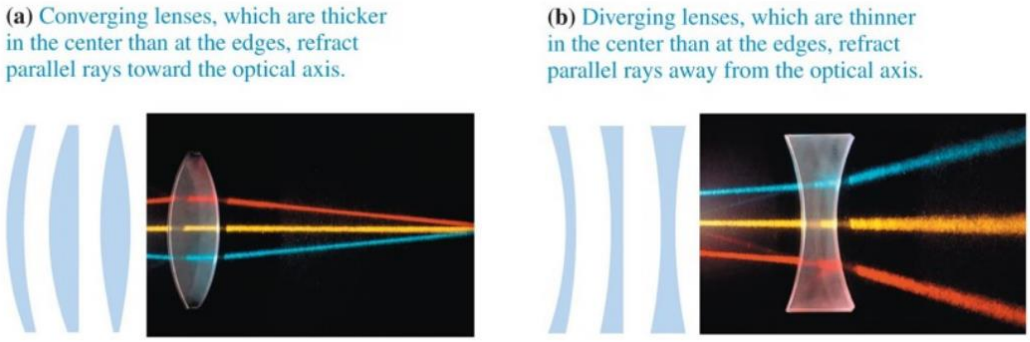


Figure 4.2: Ray diagrams illustrating the primary and secondary focal points F and F' and
the corresponding focal lengths f and f' of thin lenses.



4.2 CONJUGATE POINTS AND PLANES

If the principle of the reversibility of light rays is applied to figure 4.3, we observe that *Q'M'* becomes the object and *QM* becomes its image. The object and image are therefore **conjugate**. Any pair of object and image points such as *M* and *M'* in figure 4.3 are called conjugate points, and planes through these points perpendicular to the axis are called **conjugate planes.**

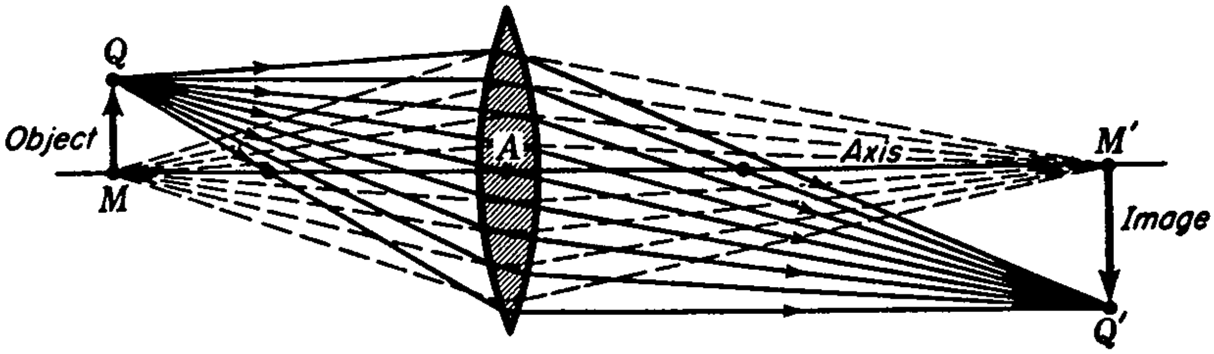


Figure 4.3 Image formation by an ideal thin lens. All rays from an object point *Q* which pass through the lens are refracted to pass through the image point *Q'*.

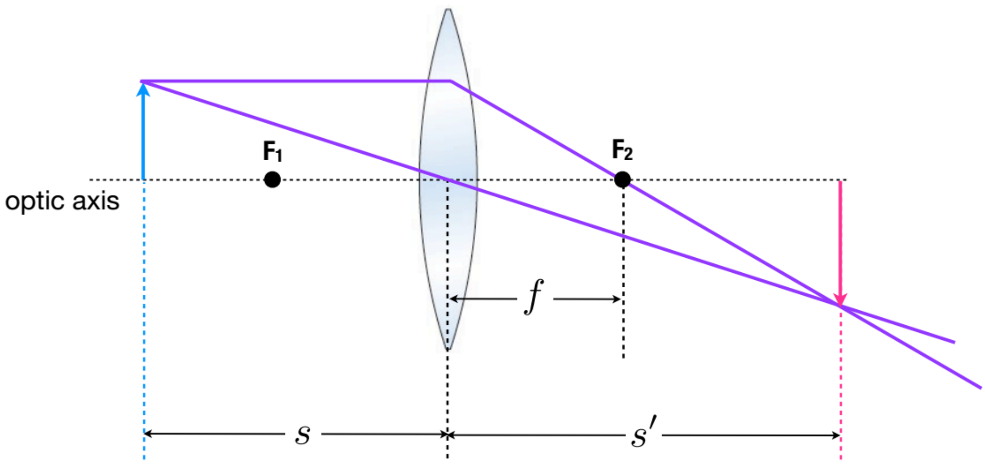
If we know the focal length of a thin lens and the position of an object, there are

three methods of determining the position of the image*:* **(1) graphical construction, (2) experiment, and (3) use of the lens formula**

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \tag{4.1}$$

Here s is the object distance, s' is the image distance, and f is the focal length, all measured to or from the center of the lens. This lens equation will be derived later in this chapter.

➜ we draw the rays as though they refract at the center - this is OK for ‘thin’ lenses



$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}} \qquad \boxed{m \equiv \frac{y'}{y} = -\frac{s'}{s}}$$

4.3 Sign Conventions for Thin Lenses

1: object distance (s) is positive if object is in front of lens and is a negative if object is in back of lens.

2: image distance (s') is positive if image is in back of lens and is negative if image is in front of lens.

3: r_1 and r_2 are positive if center of curvature is in back of lens.

4: r_1 and r_2 are negative if center of curvature is in front of lens.

5: focal length (***f***) is positive if the lens is converging and is negative if the lens is diverging.

Table (1) **Meanings Associated with the Signs of Various Thin Lens and Spherical Interface Parameters**

Quantity	Sign +	Sign −
S or u	Real object	Virtual object
s_i	Real image	Virtual image
f	Converging lens	Diverging lens
y_o	Erect object	Inverted object
y_i	Erect image	Inverted image
M_T	Erect image	Inverted image

4.4 THE PARALLEL-RAY METHOD

a: For Convex Lens

The parallel-ray method is illustrated in figure 4.4. Consider the light emitted from the point *Q* on the object. Of the rays emanating from this point in different directions the one *(QT)* traveling parallel to the axis will by definition of the **focal point** be refracted to pass through *F'*. The ray *QA*, which goes through the lens center where the faces are parallel, is undeviated and meets the other ray at some point *Q'*. These two rays are sufficient to locate the tip of the image at *Q'*, and the rest of the image lies in the conjugate plane through this point. All other rays from *Q* will also be brought to a focus at *Q'*. As a check, we note that the ray *QF* which passes through the primary focal point will by definition of *F* be refracted parallel to the axis and will cross the others at Q' as shown in the figure. The numbers I, 2, 3, etc., in figure 4.2 indicate the order in which the lines are customarily drawn.

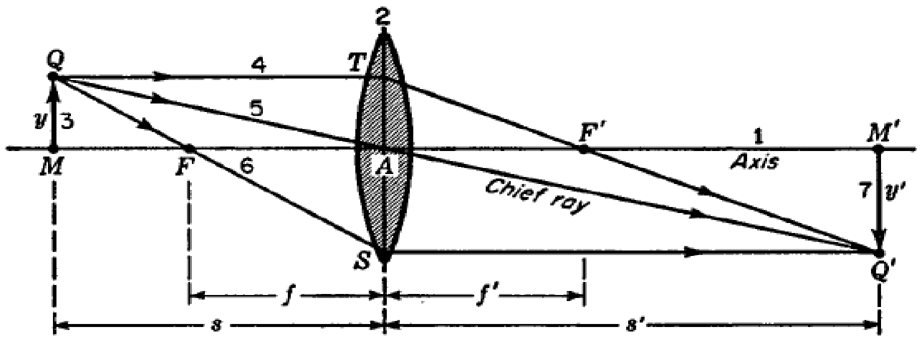


Figure 4.4a: The parallel-ray method for graphically locating the image formed by a thin lens.

b: For Concave Lens

With the negative lens shown in figure 4.4b the image is virtual for all positions of the object, is always smaller than the object, and lies closer to the lens than the object. As is seen from the diagram, rays diverging from the object point Q are made more divergent by the lens. To the observer's eye at E these rays appear to be coming from the point Q' on the far side of but close to the lens. In applying the lens formula to a diverging lens it must be remembered that the focal length j is negative.

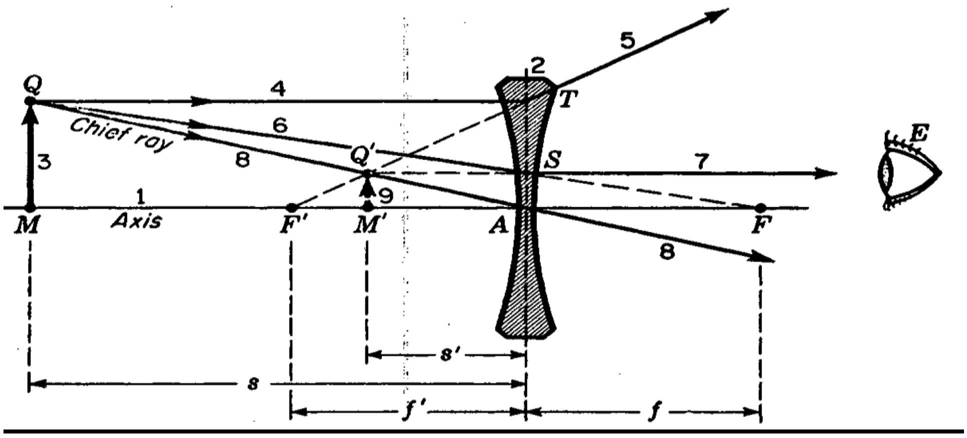


Figure 4.4b: The parallel-ray method for graphically locating the image formed by a concave lens.

4.5 LATERAL MAGNIFICATION

A simple formula for the image magnification produced by a single lens can be derived from the geometry of figure 4.4a. By construction it is seen that the right triangles *QMA* and *Q'M'A* are similar. Corresponding sides are therefore proportional to each other, so that

$$\frac{M'Q'}{MQ} = \frac{AM'}{AM} \tag{4.2}$$

where *AM'* is the image distance ***s'*** and *AM* is the object distance ***s***. Taking upward directions as positive, $y = MQ$, and $y' = -M'Q'$; so we have by direct substitution $y'/y = -s'/s$. The lateral magnification is therefore

$$m = \frac{y'}{y} = -\frac{s'}{s} \tag{4.3}$$

When ***s*** and ***s'*** are both positive, as in figure 4.4a, the negative sign of the magnification signifies an inverted image.

4.6 IMAGE FORMATION

When an object is placed on one side or the other of a converging lens and beyond

the focal plane, an image is formed on the opposite side. If the object is moved closer to the primary focal plane, the image will be formed farther away from the secondary focal plane and will be larger, i.e., magnified. If the object is moved farther away from *F*, the image will be formed closer to *F'* and will be smaller.

In figure 4.5 all the rays coming from an object point *Q* are shown as brought to a focus *Q'*, and the rays from another point *M* are brought to a focus at *M'*. Such ideal conditions and the formulas given in this chapter hold only for paraxial rays, i.e., rays close to lens axis and making small angles with it.

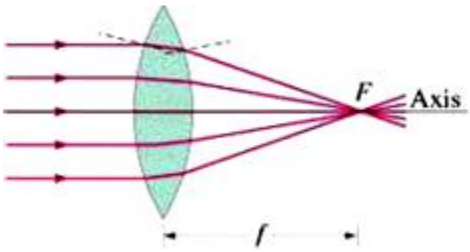
Table 2 **Images of Real Objects Formed by Thin Lenses**

Convex

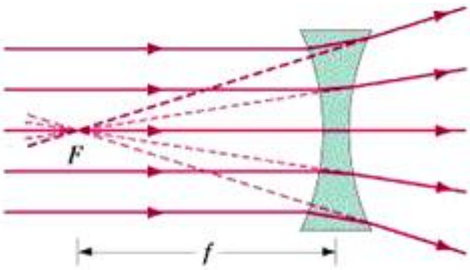
Object	Image			
Location	Type	Location	Orientation	Relative Size
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\|s_i\| > s_o$	Erect	Magnified

Concave

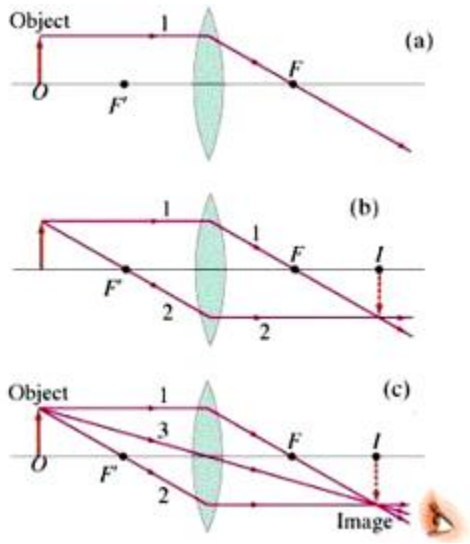
Object	Image			
Location	Type	Location	Orientation	Relative Size
Anywhere	Virtual	$\|s_i\| < \|f\|$, $s_o > \|s_i\|$	Erect	Minified



convex



concave lens



1) construction graphcally to determne state of image form in conves lens

Images of Real Objects Formed by Thin Lenses

Convex

Object Location	Image Type	Image Location	Image Orientation	Image Relative Size
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\lvert s_i \rvert > s_o$	Erect	Magnified

Concave

Object Location	Image Type	Image Location	Image Orientation	Image Relative Size
Anywhere	Virtual	$\lvert s_i \rvert < \lvert f \rvert$, $s_o > \lvert s_i \rvert$	Erect	Minified

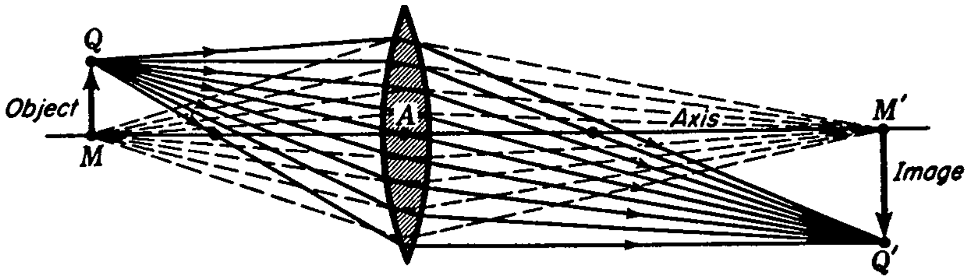


Figure 4.5: Image formation by an ideal thin lens. All rays from an object point *Q* which pass through the lens are refracted to pass through the image point *Q'*.

4.7 LENS MAKERS' FORMULA

If a lens is to be ground to some specified focal length, the refractive index of the glass must be known. Supposing the index to be known, the radii of curvature must be so chosen as to satisfy the equation

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

(4.4)

As the rays travel from left to right through a lens, **all convex surfaces are taken as having a positive radius and all concave surfaces a negative radius**. For an equiconvex lens, r_1 for the first surface is positive and r_2 for the second surface negative. Substituting the value of $1/f$ from equation(4.1), we can write

$$\frac{1}{s} + \frac{1}{s'} = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

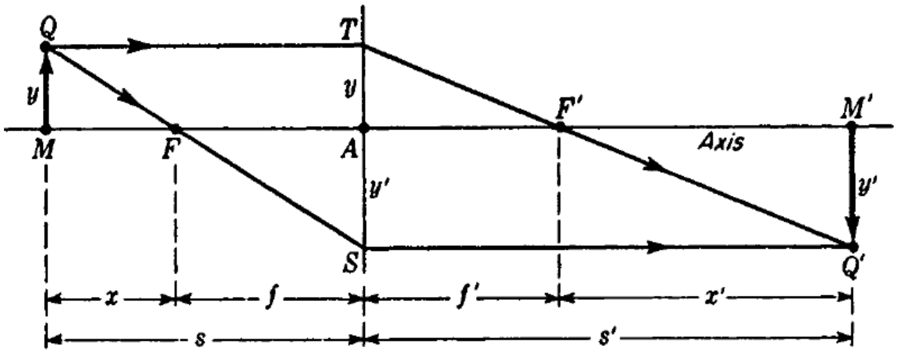
(4.5)

4.8 DERIVATION OF THE LENS FORMULA

A diagram for derivation the equation 4.1 (**lens formula**) is presented in figure 4.6 , which shows only two rays leading from the object of height *y* to the image of height *y'*. Let sand *s'* represent the object and image distances from the lens center and *x* and *x'* their respective distances from the focal points *F* and *F'*. From similar triangles *Q'TS* and *F' TA* the proportionality between corresponding sides gives

$$\frac{y - y'}{s'} = \frac{y}{f'}$$

(4.6)



Figure(4.6): The geometry used for the derivation of thin-lens formulas.

$$\frac{y - y'}{s} = \frac{-y'}{f}$$

(4.7)

Note that *y* - *y'* is written instead of *y* + *y'* because *y'*, by the convention of signs,

is a negative quantity. From the similar triangles *QTS* and *FAS,* The sum of these two equations is

$$\frac{y - y'}{s} + \frac{y - y'}{s'} = \frac{y}{f'} - \frac{y'}{f}$$

(4.8)

Since $f = f'$,the two terms on the right can be combined and *y* - *y'* canceled out,

yielding the desired equation,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad (4.9)$$

This equation is called Gaussian form or the lens equation of thin lenses, can be used to relate the image distance and object distance for a thin lens.

Another form of the lens formula is the Newtonian form, is obtained in an analogous way from two other sets of similar triangles, *QMF* and *FAS* on the one hand and *TAF'* and *F'M'Q'* on the other. We find

$$\frac{y}{x} = \frac{-y'}{f} \quad \text{and} \quad \frac{-y'}{x'} = \frac{y}{f} \tag{4.10}$$

Multiplication of one equation by the other gives

$$xx' = f^2$$

In the Gaussian formula the object distances are measured from the center of lens, while in the Newtonian formula they are measured from the focal points. Object distances *(s* or *x)* are positive if the object lies to the left of its reference point *(A* or *F,* respectively), while image distances *(s'* or *x')* are positive if the image lies to the right of its reference point *(A* or *F'*, respectively).

The lateral magnification as given by Eq. (4c) corresponds to the Gaussian form.

When distances are measured from focal points, one should use the Newtonian form, which can be obtained directly from equation (4.10)

$$m = \frac{y'}{y} = -\frac{f}{x} = -\frac{x'}{f} \tag{4.11}$$

In the more general case where the medium on the two sides of the lens is different, it will be shown in the next section that the primary and secondary focal distances *f* and *f'* are different, being in the same ratio as the two refractive indices. The newtonian lens formula then takes the symmetrical form

$$xx' = ff'$$

The result is that the object and image must be on the opposite sides of their respective focal points.

4.9 DERIVATION OF THE LENS MAKERS' FORMULA

The geometry required for this derivation is shown in figure 4.7. Let *n, n'*, and *n''*represent the refractive indices of the three media as shown, f_l and f_1'; the focal lengths for the first surface alone, f_2' and f_2'' the focal lengths for the second surface alone.

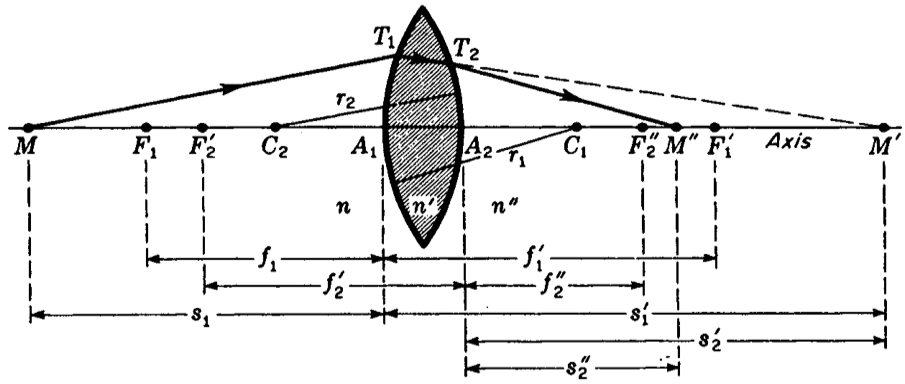


Figure 4.7: Each surface of a thin lens has its own focal points and focal lengths, as well as separate object and image distances.

The oblique ray MT_1 is incident on the first surface as though it came from an axial object point M at a distance s_1 from the vertex A_1 . At T_1 the ray is refracted according to Eq. (3b) and is directed toward the conjugate point *M':*

$$\frac{n}{s_1} + \frac{n'}{s_1'} = \frac{n' - n}{r_1}$$

(4.12)

Arriving at T_2 , the same ray is refracted in the new direction T_2M''. For this second surface the object ray $T_1 T_2$ has for its object distance s_2', and the refracted ray gives an image distance of s_2''. When the following equation determined in chapter 3 is applied to second refracting surface,

$$\frac{n'}{s_2'} + \frac{n''}{s_2''} = \frac{n'' - n'}{r_2}$$

(4.13)

If we now assume the lens thickness to be negligibly small compared with the

object and image distances, we note the image distance s_1' for the first surface becomes equal in magnitude to the object distance s_2' for the second surface. Since M' is a virtual object for the second surface, the sign of the object distance for this surface is negative. As a consequence we can set $s_1' = - s_2'$ and write

$$\frac{n'}{s_1'} = -\frac{n'}{s_2'}$$

If we now add equations (4.12) and (4.13) and substitute this equality, we obtain

$$\frac{n}{s_1} + \frac{n''}{s_2''} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2}$$

(4.14)

If we now call s_1 the object distance and designate it s as in figure (4.8) and call

s_2''. the image distance and designate it s'', we can write equation (4.14) as

$$\frac{n}{s} + \frac{n''}{s''} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2}$$

(4.15)

This is the general formula for a thin lens having different media on the two sides. By setting s or s'' equal to infinity. When this is done, we obtain

$$\frac{n}{f} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2} = \frac{n''}{f''}$$

(4.16)

In words, the focal lengths have the ratio of the refractive indices of the two media n and n'' , see figure(4.8)

$$\frac{f}{f''} = \frac{n}{n''}$$

(4.17)

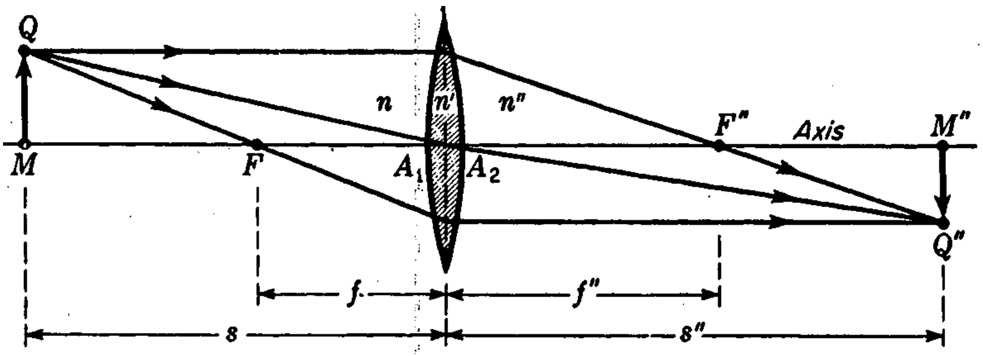


Figure 4.8 : When the media on the two sides of a thin lens have different indices, the primary and secondary focal lengths are not equal and the ray through the lens center is deviated.

If the medium on both sides is the same, $n = n''$, equation (4.15) reduces to

$$\frac{n}{s} + \frac{n''}{s''} = (n' - n)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (4.18)$$

Note that the minus sign in the last factor arises when n'' and n' are reversed for the removal of like terms in the last factor of equation (4.15).

Finally, if the surrounding medium is air $(n = 1)$, we obtain the lens **makers' Formula**

$$\frac{1}{s} + \frac{1}{s''} = (n' - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

4.10 POWER OF LENS

The power of a lens is the measure of its ability to produce convergence of a parallel beam of light. A convex lens of large focal length produces a small converging effect of the rays and a convex lens of a small focal length produce a large converging effect. Concerning, the concave lens, it produces divergence. The power of convex and for concave takes +ve and –ve respectively.

If the distances are measured in meter, the unit of power of lens is called diopter (D). the power of lens may be calculated by the relation

$$P = \frac{1}{f}$$

(4.19)

where P is the power of lens and f is the focal lens.

Now, the power of lens by depending on the equation (4.19) can be written as

$$V + V'' = P_1 + P_2$$

(4.20)

where

$$V = \frac{n}{s} \qquad V'' = \frac{n''}{s''} \qquad P_1 = \frac{n' - n}{r_1} \qquad P_2 = \frac{n'' - n'}{r_2}$$

V and V'' are called reduced vengeance because they are direct measures of the convergence and divergence of the object and image wave front.

- The divergent wave from the object s is positive, and so is its, V.
- For convergent wave from the object s is negative , and so is its, V.

- For divergent wave from the image s'' is negative , and so is its, V''.
- For convergent wave from the image s'' is negative , and so is its, V''.

Equation (4.20) can be written as

$$V + V'' = P \tag{4.20}$$

where P is the power of the lens and is equal to the sum of the powers of the two surfaces:

$$P = P_1 + P_2$$

4.10 THIN-LENS COMBINATIONS

Consider two converging lenses spaced some distance apart as shown in figure 4.9. Here an object Q_1M_1 is located at a given distance s_1 in front of the first lens, and an image $Q_2'M_2'$ is formed some unknown distance s_2' from the second lens. We first apply the parallel – ray method to find this image distance and then show how to calculate it by the use of the thin-lens formula.

The first step in applying the parallel – ray method is to disregard the presence of

the second lens and find the image produced by the first lens alone. In the diagram

the parallel-ray method, as applied to the object point Q_1 locates a real and inverted image at Q_1'. Any two of the three incident rays 3, 5, and 6 are sufficient for this purpose. Once Q_1' is located, we know that all the rays leaving Q_1 will, upon refraction by the first lens, be directed toward Q_1' Making use of this fact, we construct a fourth ray by drawing line 9 back from Q_1' through A_2 to W. Line 10 is then drawn in connecting W and Q_1.

The second step is to imagine the second lens in place and to make the following

changes. Since ray 9 is seen to pass through the center of lens 2, it will emerge without deviation from its previous direction. Since ray 7 between the lenses is parallel to the axis, it will upon refraction by the second lens pass through its secondary focal point F'_2. The intersection of rays 9 and 11 locates the final image point Q'_2 . Q_1 and Q'_1 are conjugate points for the first lens, Q_2 and Q'_2 are conjugate points for the second lens, and Q_1 and Q'_2 are conjugate for the combination of lenses. When the image $Q'_2M'_2$ is drawn in, corresponding pairs of conjugate points on the axis are M_1 and M'_1, M_2 and M'_2, and M_1 ,M'_2.

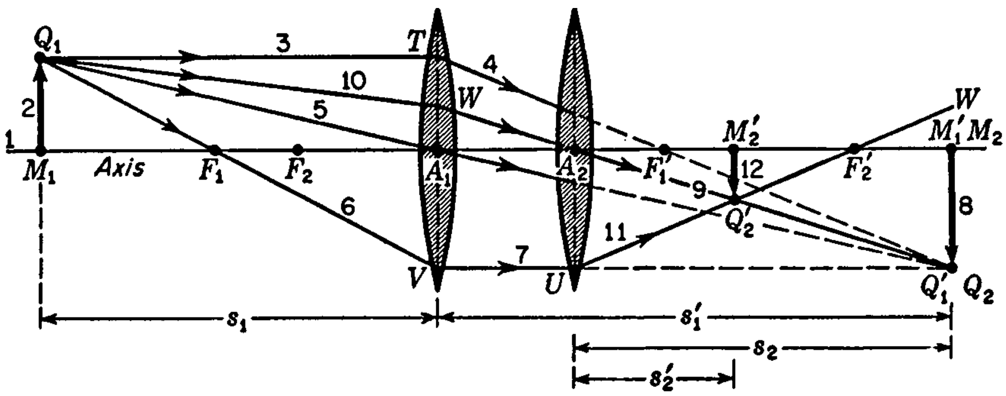


Figure 4.9: The parallel-ray method for graphically locating the final image formed by two
thin lenses.

Problems

1: Two thin converging lenses of focal lengths (f = + 3cm) and (f=+4cm) respectively are placed in air and separated by a distance of 2 cm. an object is placed 4cm in front of the first lens. Find the position and the nature of the image and its lateral magnification.

2 : If an object is located 6.0 cm in front of a lens of focal length + 10.0 cm, where will the image be formed?

3 : An object is placed 12.0cm in front of a diverging lens of focal length 6.0 cm. Find the image.

4 : A plano-convex lens having focal length of 25.0 cm is to be made of glass of refractive index n = 1.520. Calculate the radius of curvature of the grinding and polishing tools that must be used to make this lens.

5: The radii of both surfaces of an equiconvex lens of index 1.60 are equal to 8.0 cm. Find its power.