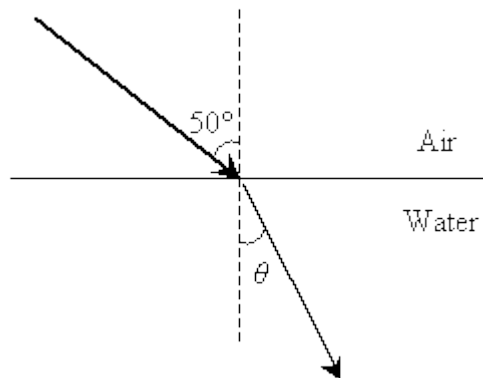


## Lect.2 -Chapter2

**1-Principle of Ray Reversibility:** Any actual ray of light in an optical system, if reversed in direction, will retrace the same path backward.

Practice Problem: A ray is incident from air ( $n \approx 1.00$ ) and passes into water ( $n \approx 1.33$ ) as shown below. What is the angle  $\theta$ ? (The normal to the surface is shown as a dashed line.)



**Solution:** The law of refraction is expressed as follows, where the angles  $\theta_1$  and  $\theta_2$  are measured from the normal:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

In this case, the ray starts in air ( $n_1 = 1.00$ ) at an angle  $\theta_1 = 50^\circ$  from the normal and passes into water ( $n_2 = 1.33$ ), refracting with an angle  $\theta_2 = \theta$  from the normal.

$$1 \cdot \sin(50^\circ) = 1.33 \sin(\theta)$$

$$0.766 = 1.33 \sin \theta$$

$$\sin \theta = \frac{0.766}{1.33} \approx 0.576$$

$$\theta = \sin^{-1} 0.576 \approx 35.2^\circ$$

Thus, the refracted ray bends toward the normal by about  $35^\circ$  when passing from air (the material with the lower index) to water (the material with the higher index).

### Sample Problem 2:

Light traveling through an optical fiber ( $n=1.44$ ) reaches the end of the fiber and exits into air. (a) If the angle of incidence on the end of the fiber is  $30^\circ$ , what is the angle of refraction outside the fiber? (b) How would your answer be different if the angle of incidence were  $50^\circ$ ?

### Solution:

(a) Since the light is now traveling from the fiber into air, we will call the fiber material 1 and air material 2. Thus,  $n_1 = 1.44$ ,  $n_2 = 1.00$ , and  $\theta_1 = 30^\circ$ . Snell's Law then becomes

$$(1.44) \sin 30^\circ = 1.00 \sin \theta_2.$$

$$\sin \theta_2 = (1.44/1.00) \sin 30^\circ = 1.44 (0.500) = 0.720$$

$$\theta_2 = \sin^{-1} (0.720) = 46^\circ.$$

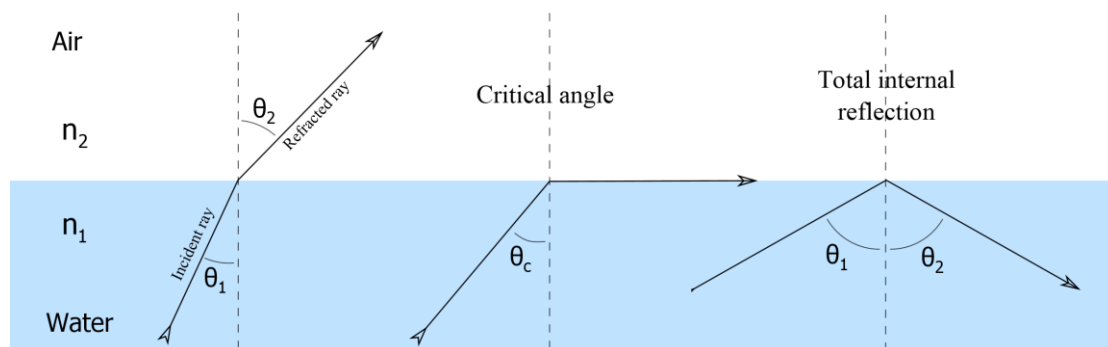
Notice that this time, the angle of refraction is larger than the angle of incidence. The light is bending away from the normal as it enters a rarer material.

(b) Replacing the angle of incidence with  $50^\circ$  gives

$$\sin \theta_2 = (1.44/1.00) \sin 50^\circ = 1.44 (0.766) = 1.103$$

This equality cannot be met, so **light cannot exit the fiber** under these conditions.

### Critical angle -Total internal reflection



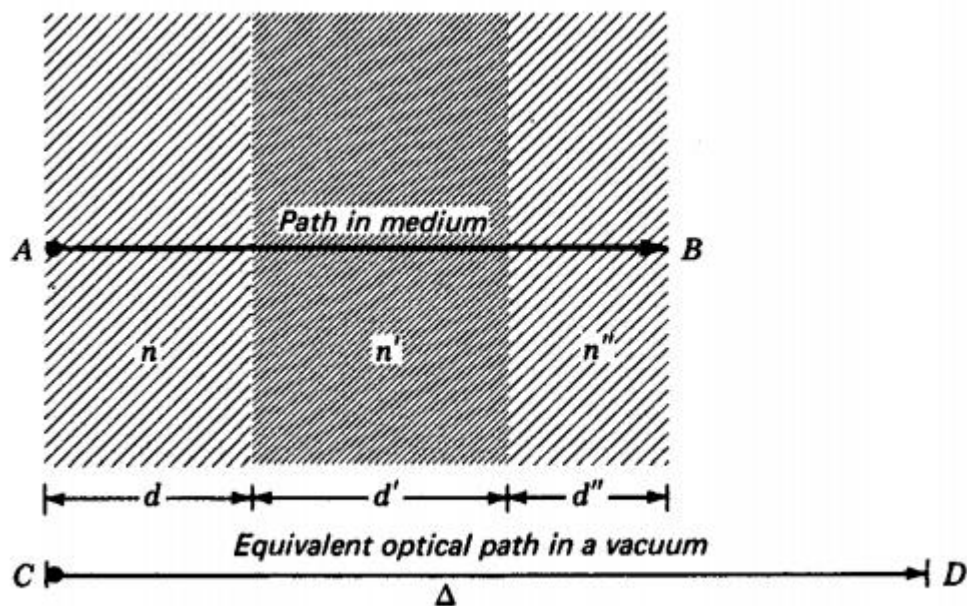
$$n_1 \sin \theta_i = n_2 \sin \theta_t, \quad \theta_c = \theta_i$$

$$\sin \theta_c = \frac{1}{1.5} \sin(\theta_i) \quad , \quad \sin(\theta_t) = \sin 90,$$

$$\theta_c = \sin^{-1} \frac{1}{1.5} \quad , \quad \text{then } \theta_c = 41.8$$

### Optical Path

The path that light takes in traversing an optical medium or system is often called the **optical path**. The [optical path length](#) as defined in [optics](#) is the length of the path multiplied by the [index of refraction](#) of the medium



The path  $d$  of a ray of light through any medium at any time product to velocity:

$$d = vt, \text{ since refractive index } n = \frac{c}{v}$$

$$d = \frac{c}{n} t \quad \text{or} \quad n d = ct$$

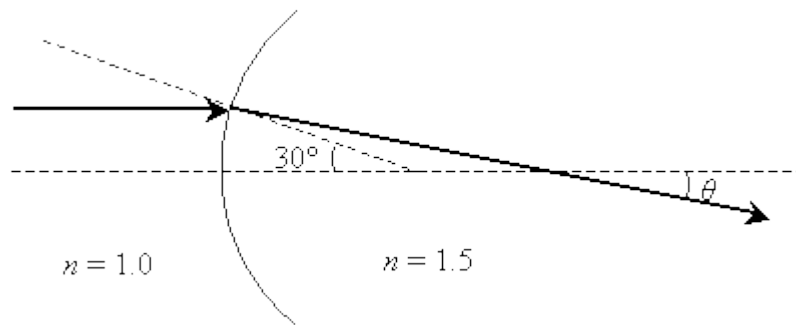
The product  $nd$  is called the *optical path*  $\Delta$ :

$$\Delta = nd$$

The optical path represents the distance light travels in a vacuum in the same time it travels a distance  $d$  in the medium. If a light ray travels through a series of optical media of thickness  $d, d', d'', \dots$  and refractive indices  $n, n', n'', \dots$ , the total optical path is just the sum of the separate values:

- $$\Delta = nd + n'd' + n''d'' + \dots \quad (1)$$

**Homework(1):** What is the angle  $\theta$  in the diagram below? (Assume the normal at the point of incidence intersects the horizontal dashed line at an angle of  $30^\circ$ .)



H.w (2): problems

-

1- What is the angle of refraction when a ray from air with an angle of incidence of  $25^\circ$  is incident to water? Draw the ray diagram

Answer:  $18.5^\circ$

2- Find the critical angle for a light ray that is incident from water to air.

Answer:  $48.8^\circ$

3- The index of refraction for crown glass for red light is 1.514.

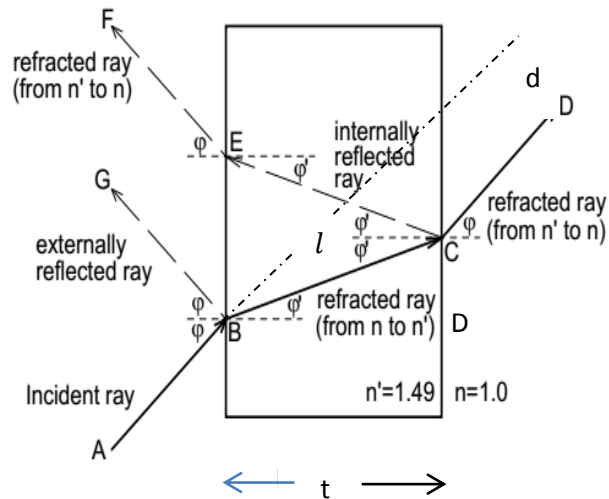
What is the speed of red light in crown glass?

Answer:  $1.98 \times 10^8$  m/s

## 2- Plane –parallel plate

The laws of reflection and refraction can be used to examine the passage of light through a plate with plane parallel surfaces. Figure 2 below shows an incident ray and the reflected and refracted rays at the boundaries between the air and the acrylic. The light ray emerges from the plate parallel to the original ray but laterally displaced from it.

Incident ray AB partly (about 5%) reflects externally to generate the reflected ray BG. The rest of the incident light refracts to ray BC. At the other end of the plate, ray BC internally reflects to give ray CE and refracts to produce ray CD, the exiting light. Ray CE behaves similarly and yields a refracted ray and an internally reflected ray which in turn gives a refracted and reflected component and so on. Note that because the sides of the plate are parallel,



When light travel a glass plate with plane surface in  $\Delta BCE$

$$\sin(\phi - \phi') = \frac{l}{d}; \quad d = l \sin(\phi - \phi')$$

$$\sin(\phi - \phi') = \sin \phi \cos \phi' - \sin \phi' \cos \phi$$

$$d = l (\sin \phi \cos \phi' - \sin \phi' \cos \phi) \text{-----(1)}$$

$$\text{in } \Delta BCD ; \quad \cos \phi = \frac{t}{l} \text{-----(2)} \quad ; \quad l = \frac{t}{\cos \phi'} \text{-----(3)}$$

sub eq(3) into (1)

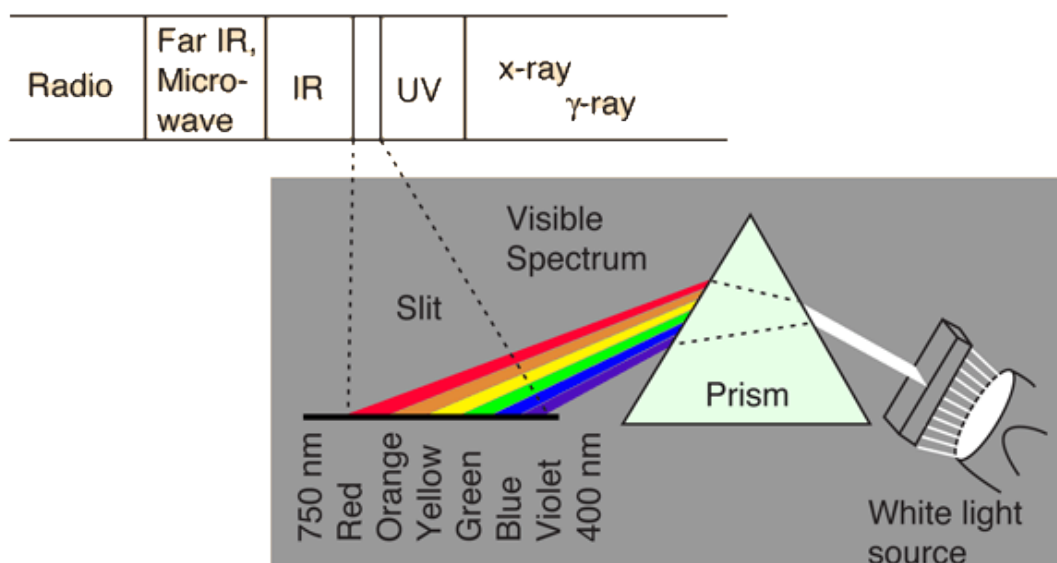
$$d = t \left( \sin \phi - \frac{\sin \phi' \cos \phi}{\cos \phi'} \right) \text{ applying snell's law } \frac{n}{n'} \sin \phi$$

$n \sin \phi = n' \sin \phi'$  ; the lateral distance may be written :

$$d = t \sin \phi \left( 1 - \frac{n}{n'} \frac{\cos \phi}{\cos \phi'} \right)$$

## Color dispersion

The refractive index of materials varies with the wavelength (and [frequency](#)) of light. This is called dispersion and causes [prisms](#) and [rainbows](#) to divide white light into its constituent spectral [colors](#).

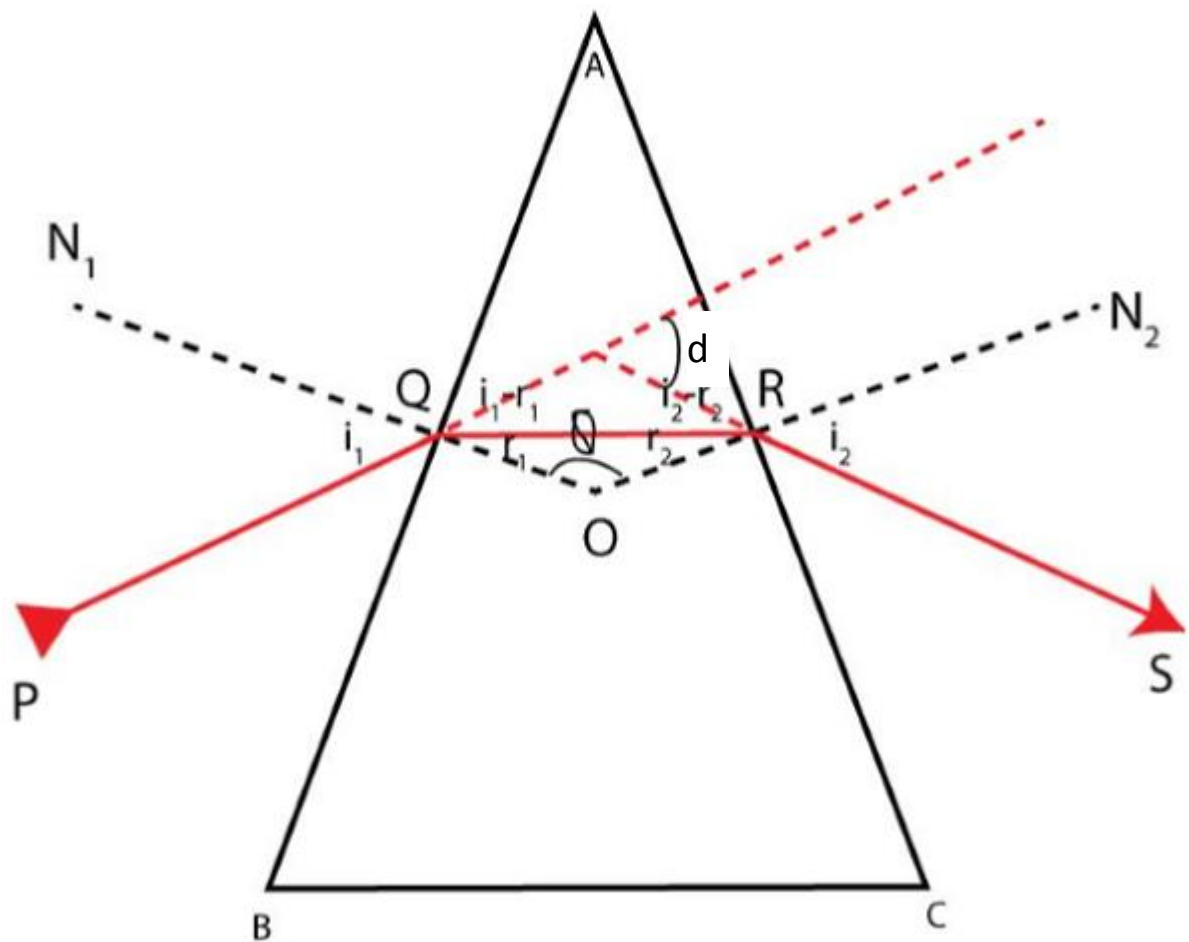


White light may be separated into its [spectral colors](#) by dispersion in a prism.

## Minimum Deviation:

When the path of light ray through the prism is symmetrical, then angle of deviation is minimum.

Let us consider a section of prism ABC with PQ as the incident ray OR as the refracted ray and RS as the emergent ray as shown in figure –



From figure we can observe that –

### Refracted ray

When the incident ray strikes the prism it is refracted and it known as refracted ray. Refracted ray lies within the prism.

### Emergent ray

When the refracted ray strikes the other surface of the prism it is again refracted and moves out of the prism and is known as emergent ray.

### Angle of deviation

Angle between the incident ray and emergent ray is known as angle of deviation.

### Angle of prism

The angle between the two inclined refracting surfaces is known as angle of prism.

refracted ray and RS as the emergent ray as shown in figure –

$$\angle A + \angle Q = 180^\circ$$

And

$$r_1 + r_2 + \angle Q = 180^\circ$$

Hence, Angle of prism is given by –

$$A = r_1 + r_2$$

From figure we can observe that –

$$D_{\min} = (i_1 - r_1) + (i_2 - r_2)$$

$$D_{\min} = (i_1 + i_2) - (r_1 + r_2)$$

$$D_{\min} = i_1 + i_2 - A$$

Refractive index of material of prism is given by –

$$n = \frac{\sin i}{\sin r}$$

For small angle  $\sin i \approx i$

$$n = \frac{i}{r}$$

$$\therefore i = nr$$

Hence, Angle of deviation is given by –

$$D_{\min} = nr_1 + nr_2 - A$$

$$\therefore D_{\min} = n(r_1 + r_2) - A$$

$$\therefore D_{\min} = nA - A$$

$$\therefore D_{\min} = A(n - 1)$$

In a prism when angle of incidence is increased gradually, the angle of deviation first decreases then increases. In this process the minimum value of deviation achieved is minimum deviation.

When a prism is in the condition of minimum deviation, the angle of incidence  $i_1$  is equal to the angle of emergence  $i_2$

Similarly, angle  $r_1 = r_2$



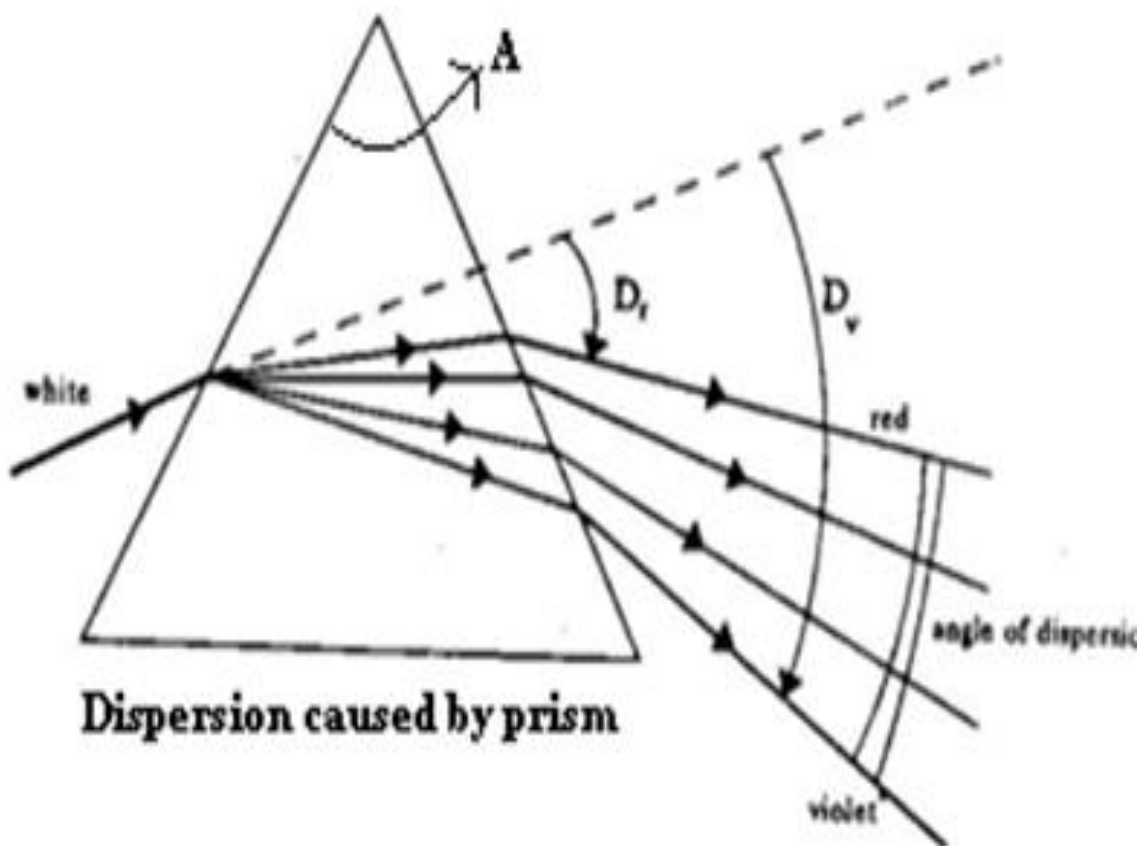
Then,

$$A = 2r \text{ or } r = A/2$$

$$D_m = 2i - A \text{ or } i = D_m + A/2$$

$$n = \sin i / \sin r$$

$$\therefore n = \frac{\sin(D_m + \frac{A}{2})}{\sin(\frac{A}{2})}$$



The refractive index of a medium is different for light rays of different wavelengths. Larger the wavelength, the lesser is the refractive index. As a result light of different wavelengths deviate differently, splitting white light into its constituent colours. This is called dispersion.

i.e  $n_{\text{red}} < n_{\text{violet}} \quad \lambda_{\text{red}} > \lambda_{\text{violet}}$

For a small refracting angle A, the deviation D is

Given by:

$$D \approx (n - 1) A$$

The angle between the red and violet rays called angle of dispersion is given by:

$$\phi = D_v - D_r = (n_v - n_r) A$$