Chapter Two

***Electrostatic***

## 2 – 1: Electric Charge

It is well known that rubbing a hard rubber on a piece of wool endows the rubber an ability to pick up small pieces of paper. Actually the process of rubbing will make the charges, specifically electrons, to transform from the wool to the rubber. Hence, the wool becomes positively charged while rubber can behave negatively charged.

Did the sum or the total number of charges change before and after? or the net change i.e., conserved in a closed system.

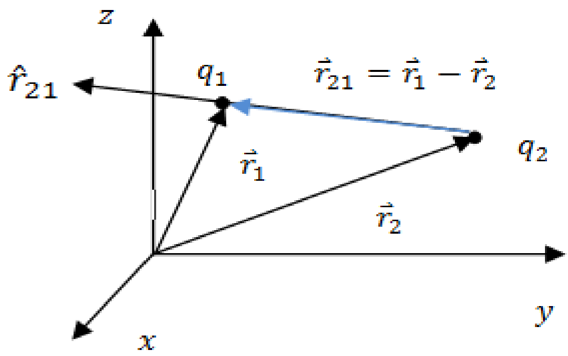
## 2 – 2: Coulombs′ Law

During the late of eighteenth century many observation concerns with the electric charge have been recorded. The results of these behaviors can be summarized by the following three statements;

1. There are two and only two kinds of charges which called now positive and negative.
2. Two point charges exert on each other by a forces act along the line joining them and which are inversely proportional to the square of the distance between them, $\left(F \propto \frac{1}{r^2}\right)$ . . . *inverse-square* force
3. These forces are directly proportional to the product of the charges $(F \propto q_1 q_2)$ and they are being repulsive for like charges and attractive for unlike charges.

Charles A. De Coulomb has been translating the last two statements into an experimental relation known as *Coulombs' law* (the same mathematical form as Newton's law of gravity $\boxed{\vec{F}_1 = G\frac{M_1 M_2}{r_{21}^2}\hat{r}_{21}}$). The mathematical expression for this law *in vector notation* is given;

$$\boxed{\vec{F}_1 = \frac{1}{4\pi \epsilon_o}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}}$$




$$\vec{F}_1 = \frac{1}{4\pi\epsilon_o} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} \cdot (\vec{r}_1 - \vec{r}_2)$$

Where $\vec{F}_1$ is the force on charge $q_1$ by charge $q_2$, $r_{21} = |\vec{r}_{21}| = |\vec{r}_1 - \vec{r}_2|$ is the *distance* between the two charges, $\vec{r}_{21}$ is the two charges *separation vector*, $\hat{r}_{21}$is a *unit separation vector* directed from $q_2$ toward $q_1$. In terms of SI unites, the proportionality constant equals to; $k = \frac{1}{4\pi\epsilon_o} \approx 9 \times 10^9 N.m^2/c^2$.

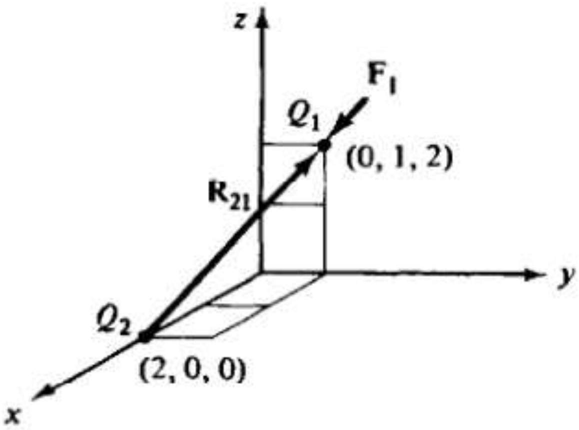
$\epsilon_o$ is the electrical ***permittivity*** of free space (electrical properties for air is nearly the same as free space) which equal to: $8.854 \times 10^{-12} c^2/N.m^2$. $\epsilon_o$ may be defined in terms of the speed of light in vacuum $c$ and the electrical ***permeability*** of free space ($\mu_o = 4\pi \times 10^{-7} Kg.m/c^2$) as follows;

$$c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$$

## Example:

Find the force that act on charge $q_1 = +20\mu c$ due to a charge $q_2 = -300\mu c$ where $q_1$ locating at (0,1,2)m and $q_2$ at (2,0,0)m.

## Solution:




At 1st , we have to compute the separation vector between two charges, the two position vectors are:

$$\vec{r}_1 = (0)\hat{\imath} + (1)\hat{\jmath} + (2)\hat{k} = \hat{\jmath} + 2\hat{k}$$

$$\vec{r}_2 = (2)\hat{\imath} + (0)\hat{\jmath} + (0)\hat{k} = 2\hat{\imath}$$

$$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$$

$$= -2\hat{\imath} + \hat{\jmath} + 2\hat{k}$$

$$r_{21} = |\vec{r}_{21}| = \sqrt{(-2)^2 + (1)^2 + (2)^2} = 3$$

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{r_{21}} = \frac{-2\hat{\imath} + \hat{\jmath} + 2\hat{k}}{3}$$

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_o} \cdot \frac{q_1 q_2}{{r_{21}}^2} \cdot \frac{\vec{r}_{21}}{r_{21}}$$

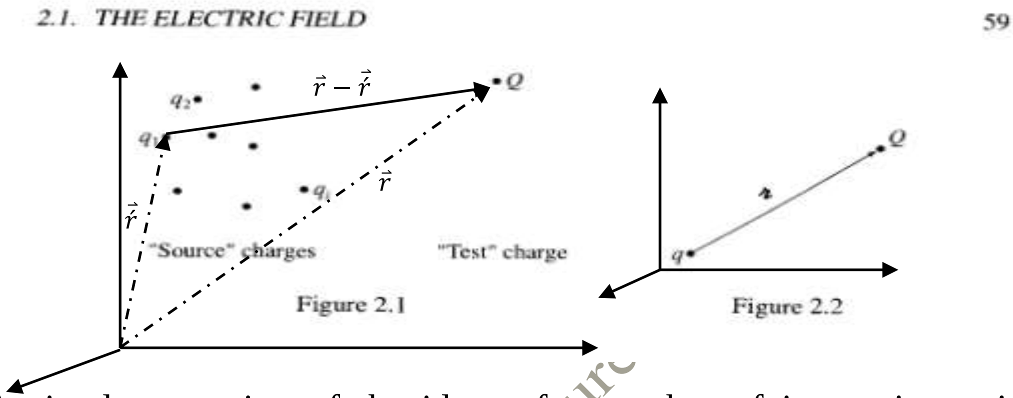
$$= 9 \times 10^9 \times \frac{(20 \times 10^{-6}).(-300 \times 10^{-8})}{3^2} \cdot \frac{-2\hat{\imath} + \hat{\jmath} + 2\hat{k}}{3}$$

$$= 6\left(\frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{3}\right) \text{ N}$$

The force *magnitude* is 6N and the direction is such that $q_1$ is *attracted* to $q_2$.

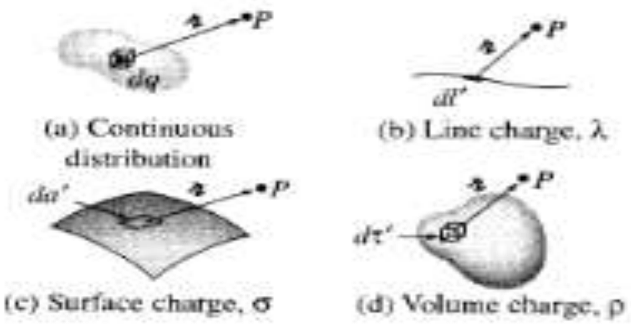
- For more than one point charge exerting by forces on a one point charge $q$, the resultant force on it is simply the *vector sum* (linear superposition) of the individual force by $\acute{q}_1$ exerts on $q$ plus the force of $\acute{q}_2$, plus force by $\acute{q}_3$, etc. For this situation, Coulombs″ law given as the following;

$$\vec{F}_q = \frac{q}{4\pi\epsilon_o}\sum_{i=1}^{n}\acute{q}_\imath\,\frac{(\vec{r}-\vec{\acute{r}})}{\left|\vec{r}-\vec{\acute{r}}\right|^3} \quad . \;.\; . \quad (2-2)$$




- A simple extension of the ideas of $n$-number of interacting point charges is the interaction of a point charge $q$ with a small element of source (***continues*** *charge distribution)* dq́. For this situation equation (2-1) became;

$$\vec{F}_q = \frac{q}{4\pi\epsilon_o}\int\frac{(\vec{r}-\vec{\acute{r}})}{\left|\vec{r}-\vec{\acute{r}}\right|^3}.\,d\acute{q} \qquad . \;.\; . \quad (2-3)$$




In fact, this continues charge distribution $d\acute{q}$ may describe by;

1. A volume charge density (charge per volume) which defined by;

$$\rho(\acute{r}) = \lim_{\Delta\acute{V}\to 0}\frac{\Delta\acute{q}}{\Delta\acute{V}} = \frac{d\acute{q}}{d\acute{V}} \quad . \;.\; . \quad (2-4a)$$

2. A surface charge density (charge per area) which defined by;

$$\sigma(\acute{r}) = \lim_{\Delta\acute{s}\to 0}\frac{\Delta\acute{q}}{\Delta\acute{s}} = \frac{d\acute{q}}{d\acute{s}} = \frac{d\acute{q}}{d\acute{a}} \quad . \ . \ . \ (2-4b)$$

3. Line charge density which defined by;

$$\lambda(\acute{r}) = \lim_{\Delta\acute{l}\to 0}\frac{\Delta\acute{q}}{\Delta\acute{l}} = \frac{d\acute{q}}{d\acute{l}} \quad . \ . \ . \ (2-4c)$$

Now, the combination of equation (2-2) and (2-3) with the aid of (2-4) we get;

$$\boxed{\begin{aligned}\vec{F}_q = \frac{q}{4\pi\epsilon_o}\sum_{i=1}^{n}\acute{q}_\iota\,\frac{(\vec{r}-\vec{\acute{r}})}{\left|\vec{r}-\vec{\acute{r}}\right|^3} + \frac{q}{4\pi\epsilon_o}\int_V \frac{(\vec{r}-\vec{\acute{r}})}{\left|\vec{r}-\vec{\acute{r}}\right|^3}\rho(\acute{r})d\acute{\tau} \\ + \frac{q}{4\pi\epsilon_o}\int_s \frac{(\vec{r}-\vec{\acute{r}})}{\left|\vec{r}-\vec{\acute{r}}\right|^3}\sigma(\acute{r})d\acute{a} \quad . \ . \ . \ (2-5)\end{aligned}}$$

Actually, last equation describe the force $\vec{F}$that a last charge $q$ experience when it located at a point $\vec{r}$ from a charge distribution consist of $n$ point charges $(q_1, q_2, q_3, \ldots, q_n)$ located at the points $(\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_n)$, respectively. Also a volume charge distribution characterized by the surface charge density $\sigma(\acute{r})$ on the surfaces.

*H.W:*

*What does the meaning of combining equation (2-2) and (2-3)?*

Example:

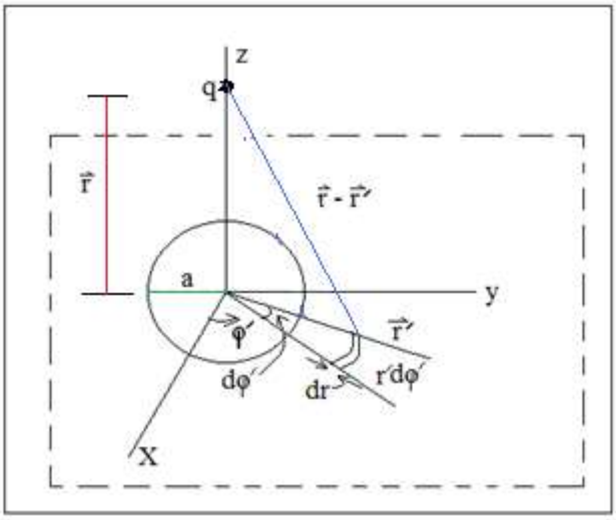
Find the force on a charge $q$ lying on the $z$ axis above the center of circular hole of radius $a$ in an infinite uniformly charged flat plane occupying the x-y plane carrying surface charge density.

Solution:

لاجل تحديد نوع قانون كولوم الواجب استخدامه نلاحظ شكل الشحنه وهنا هو توزيع مشحون متصل على شكل مستوي متجانس، علية فانه ليس شحنه او شحنات نقطية انما يتطلب عمليه تكامل للحل. ولاجل تحديد نوع الاحداثيات التي يجب استخدامها لحل المسألة فاننا نلاحظ موقع نقطة الكشف وشكل الشحنة، حيث تقع نقطة الكشف على محور z وشكل الشحنة هو لوح

مستوي لكن يحتوي فتحة دائرية. ان هذين الاحداثيان هما (z,r) وبذلك نستنتج بأن الاحداثيات هي اسطوانية فقط.

$$\vec{F}_q = \frac{q}{4\pi\epsilon_o}\int_s \frac{(\vec{r}-\vec{\acute{r}})}{\left|\vec{r}-\vec{\acute{r}}\right|^3} d\acute{q}$$




The point charge is lying at the position vector $\vec{r} = z\hat{k}$ . . . (1) $where$ $x = 0, \quad y = 0$

While, the charged flat (plan) $(x, y)$ is located at the position vector $\vec{\acute{r}} = \acute{x}\hat{\imath} + \acute{y}\hat{\jmath}$

where $z = 0$

Using the transformation relations (from Cartesian to Cylindrical):

$$\vec{\acute{r}} = \acute{r}cos\acute{\varphi}\hat{\imath} + \acute{r}sin\acute{\varphi}\hat{\jmath} \quad . . . (2)$$

Thus, by subtracting eq.(2) from eq.(1), we get the detector to source separation vector:

$$\vec{r} - \vec{\acute{r}} = z\hat{k} - \acute{r}cos\acute{\varphi}\hat{\imath} - \acute{r}sin\acute{\varphi}\hat{\jmath} \quad . . . (3)$$

Separation vector magnitude:

$$\left|\vec{r}-\vec{\acute{r}}\right| = \sqrt{\acute{r}^2 sin^2\acute{\varphi} + \acute{r}^2 cos^2\acute{\varphi} + z^2} = (z^2 + \acute{r}^2)^{1/2} \quad . . . (4)$$

$d\acute{q} = \sigma(\acute{r}, \acute{\varphi})d\acute{s}$ (Surface charge density) استخدمنا علامة "برايم" مع العنصر التفاضلي للشحنة للتمييز عن الشحنة النقطية

$= \sigma\, (d\acute{r})(\acute{r}d\acute{\varphi})$

$$\vec{F}_q = \frac{q}{4\pi\epsilon_o}\int_s \frac{(\vec{r}-\vec{\acute{r}})}{|\vec{r}-\vec{\acute{r}}|^3}d\acute{q}$$

$$= \frac{q}{4\pi\epsilon_o}\int_a^\infty\int_0^{2\pi}\frac{(z\hat{k}-\acute{r}cos\acute{\varphi}\hat{\imath}-\acute{r}sin\acute{\varphi}\hat{\jmath})}{(z^2+\acute{r}^2)^{\frac{3}{2}}}.\sigma\acute{r}d\acute{r}d\acute{\varphi}$$

But: $\int_0^{2\pi}cos\acute{\varphi}d\acute{\varphi} = \int_0^{2\pi}sin\acute{\varphi}d\acute{\varphi} = zero$, thus;

$$\vec{F}_q = \frac{q}{4\pi\epsilon_o}2\pi\int_a^\infty\frac{\sigma z\hat{k}}{(z^2+\acute{r}^2)^{3/2}}\acute{r}d\acute{r}$$

$$\vec{F}_q = \frac{q}{4\epsilon_o}\sigma z\hat{k}\int_a^\infty(z^2+\acute{r}^2)^{-\frac{3}{2}}.2\acute{r}d\acute{r}$$

$$= \frac{q\sigma z\hat{k}}{4\epsilon_o}.\frac{(z^2+\acute{r}^2)^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)}\Bigg|_a^\infty$$

$$= \frac{-q\sigma z\hat{k}}{2\epsilon_o\sqrt{z^2-a^2}}\Bigg|_a^\infty$$

$$= \frac{-q\sigma z}{2\epsilon_o\sqrt{z^2-a^2}}.\hat{k}$$

H.W:

1. Discuss what will happen for $\vec{F}_q$ at the limits when;

   a. $z \to \infty$

   b. $z \to 0$

And hence give the Physical meaning.

2. Find the force that exerted on a point charge $q$ located at $\vec{r}$ in the $x-y$ plane by a long line charge $\lambda$ uniformly distributed along a thin wire lying along the $z$ axis.

- **Helmholtz' Theorem** *states that a vector field can be specified almost completely (up to the gradient of an arbitrary scalar field) if both its* ***divergence*** *and* ***curl*** *are specified everywhere.*

## 2 - 3: The Electric field:

An electric field is said to exist in the region of space around a charge (or charged object) and time-varying magnetic fields. This charged object is the source charge. When another charged object, the test charge, enters this electric field, an *electric force* acts on it as a *field force*. The test charge serves as a *detector* of the field. While, the presence of the test charge is not necessary for the field to exist, where the existence of an electric field is a property of the source charge

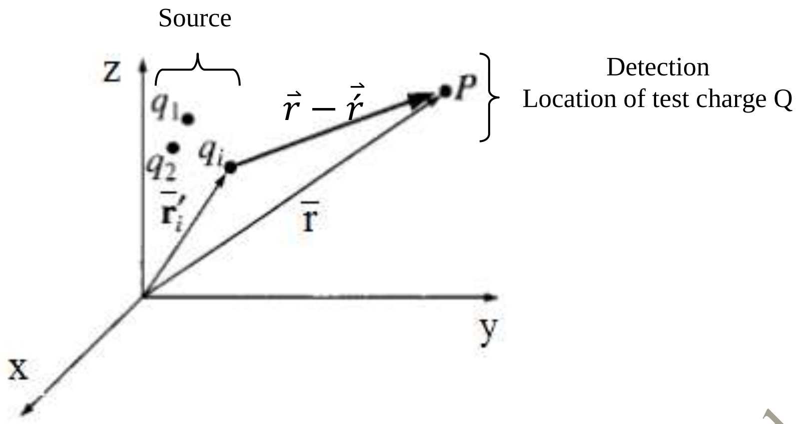
The electric field at a point is defined as the limit of the ratio: *the force on the test charge placed at a point, to the charge of the test charge, the limit being taken as the magnitude of the test charge goes to zero*;

$$\vec{E} = \lim_{q\to 0}\frac{\vec{F}_q}{q} . \quad . \quad . \quad (2-6) \quad Electrostatic\ field\ definition$$

In fact the limiting process is included in the definition of $\vec{E}$ to ensure that the test (sensing) charge q does not affect the charge distribution that produce $\vec{E}$.

From eqs. (2-4) & (2-5), we may set up the general form of the electric field at a point located at $\vec{r}$ as in the following form;

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_o}\sum_{i=1}^{n} q_i \frac{(\vec{r}-\vec{\acute{r}}_i)}{|\vec{r}-\vec{\acute{r}}_i|^3} + \frac{1}{4\pi\epsilon_o}\int_V \frac{(\vec{r}-\vec{\acute{r}})}{|\vec{r}-\vec{\acute{r}}|^3}\rho(\acute{r})d\acute{V} + \frac{1}{4\pi\epsilon_o}\int_S \frac{(\vec{r}-\vec{\acute{r}})}{|\vec{r}-\vec{\acute{r}}|^3}\sigma(\acute{r})d\acute{a} \quad . \quad . \quad . \quad (2-7)$$




## Notes:

The last equation is very general, where in most cases one or more of the terms will not be needed.

The defined electric field, may be calculated at each point in the vicinity of the charge distribution (source charge distribution). Thus $\vec{E} = \vec{E}(\vec{r})$ is a *vector function* or *vector field*.

## H.W:

Does the electric field depend on the value of the sensing charge? Why? What is the Physical meaning for that?

## Example:

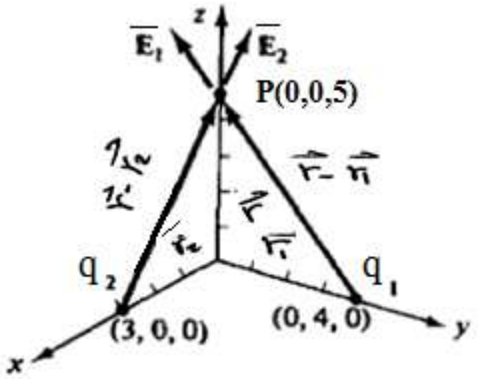
Find $\vec{E}$ at the point $P(0,0,5)$ due to $q_1 = 0.35\mu C$ located at (0,4,0)m and another source charge $q_1 = -0.55\mu C$ located at (3,0,0)m.

## Solution:

According to equation (2-7); *the net electric field produced at any point by a system of charges is equal to the vector sum of all individual fields - the superposition principle*. i.e:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_o}\sum_{i=1}^{n} q_i \frac{(\vec{r}-\vec{r}'_i)}{|\vec{r}-\vec{r}'_i|^3}$$

$$= \frac{1}{4\pi\epsilon_o}\left\{q_1\frac{(\vec{r}-\vec{r}'_1)}{|\vec{r}-\vec{r}'_1|^3} + q_2\frac{(\vec{r}-\vec{r}'_2)}{|\vec{r}-\vec{r}'_2|^3}\right\} = \vec{E}_1 + \vec{E}_2$$




Detection point P for the test charge is located at the vector: $\vec{r} = 5\hat{k}$

Source point charge 1 located at the vector: $\vec{r_1} = 4\hat{j}$

Source point charge 2 located at the vector: $\vec{r_2} = 3\hat{\imath}$

Separation vector between P and $q_1$ is: $\vec{r} - \vec{r_1} = 5\hat{k} - 4\hat{j}$

Separation vector between P and $q_2$ is: $\vec{r} - \vec{r_2} = 5\hat{k} - 3\hat{\imath}$

$|\vec{r} - \vec{r_1}| = \sqrt{(-4)^2 + (5)^2} = \sqrt{41}$

$|\vec{r} - \vec{r_2}| = \sqrt{(-3)^2 + (5)^2} = \sqrt{34}$

$$\vec{E}_1(\vec{r} - \vec{r_1}) = \frac{0.35 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12}} \cdot \frac{-4\hat{j} + 5\hat{k}}{\sqrt{41}}$$

$$= -48\hat{j} + 60\hat{k} \quad \text{(N/C) or (V/m)}$$

$$\vec{E}_2(\vec{r} - \vec{r_2}) = \frac{-0.55 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12}} \cdot \frac{-3\hat{j} + 5\hat{k}}{\sqrt{34}}$$

$$= 74.9\hat{\imath} - 124.4\hat{k} \quad \text{(N/C) or (V/m)}$$

$$\vec{E}(\vec{r}) = \vec{E}_1 + \vec{E}_2$$

$$= -48\hat{j} + 60\hat{k} + 74.9\hat{j} - 124.4\hat{k}$$

$$= 74.9\hat{\imath} - 48\hat{j} - 64.9\hat{k}$$

H.W:

a) Prove that $N/C = V/m$

b) Find the electric field at a distance $z$ above the midpoint of a straight line segment of length $2L$, which carries a uniform line charge $\lambda$.

## 2 – 4: Electrostatic Potential جهد الكهربائية الساكنة

Ref.: "Classical Electrodynamics, 3ed edition" J.D. Jackson, p.30.

When $V$ represents a scalar function, such as the scalar potential, we explained that: $\vec{\nabla} \times (\vec{\nabla} V) = 0$. In fact, the electric field relates with $V$ in simple relation, such that it satisfying that identity. To prove that, we have to take the curl of equation (2-7) as follows;

وفقا لنظرية هلمز، اي مجال (متجة) تتم معرفة كل خصائصة بدقة فقط بعد حساب تباعده والتفافة في كل مكان

$$\vec{\nabla} \times \vec{E}(\vec{r}) = \vec{\nabla} \times \left\{ \frac{1}{4\pi\epsilon_o} \sum_{i=1}^{n} q_i \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} + \frac{1}{4\pi\epsilon_o} \int_V \frac{(\vec{r} - \vec{\acute{r}})}{|\vec{r} - \vec{\acute{r}}|^3} \rho(\acute{r}) d\acute{V} + \frac{1}{4\pi\epsilon_o} \int_S \frac{(\vec{r} - \vec{\acute{r}})}{|\vec{r} - \vec{\acute{r}}|^3} \sigma(\acute{r}) d\acute{a} \right\}$$

$$\vec{\nabla} \times \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_o} \sum_{i=1}^{n} q_i \vec{\nabla} \times \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} + \frac{1}{4\pi\epsilon_o} \int_V \vec{\nabla} \times \frac{(\vec{r} - \vec{\acute{r}})}{|\vec{r} - \vec{\acute{r}}|^3} \rho(\acute{r}) d\acute{r} + \frac{1}{4\pi\epsilon_o} \int_S \vec{\nabla} \times \frac{(\vec{r} - \vec{\acute{r}})}{|\vec{r} - \vec{\acute{r}}|^3} \sigma(\acute{r}) d\acute{a} \quad . \ . \ . \ (2-8)$$

Using the following identity, (which has been given in table (1-1) in the text book as $(\vec{\nabla} \times (\varphi \vec{F}))$;

مؤثر الكرل لحاصل ضرب داله عددية $U$ في دالة اتجاهية $\vec{A}$

$$\vec{\nabla} \times (\varphi \vec{F}) = \varphi\, \vec{\nabla} \times \vec{F} + \vec{\nabla}\varphi \times \vec{F}$$

Let $\varphi = \frac{1}{|\vec{r}-\vec{r}'|^3}$, and $\vec{F} = \vec{r} - \vec{r}'$, then the curl term in the last equation can be written as;

$$\vec{\nabla} \times \frac{1}{|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}') = \frac{1}{|\vec{r}-\vec{r}'|^3}.\vec{\nabla} \times (\vec{r}-\vec{r}') + \left\{\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|^3}\right\} \times (\vec{r}-\vec{r}')$$

According to the two identities given in problems (1-13) and (1-16) from text book, and the H.W. in grad subject given in ch1;

$$\left.\begin{aligned} \vec{\nabla} \times \vec{r} &= 0 \\ \vec{\nabla} f(r) &= \frac{\vec{r}}{r}\frac{df}{dr} \\ \vec{\nabla}\frac{1}{r^3} &= -\frac{3\vec{r}}{r^5} \end{aligned}\right\}$$

for the separation vector $\vec{r} - \vec{r}'$ we have;

$$\left.\begin{aligned} \vec{\nabla} \times (\vec{r}-\vec{r}') &= 0 \\ \vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|^3} &= -\frac{3(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^5} \end{aligned}\right\} \ldots (\#)$$

these two equations together, with that $(\vec{r}-\vec{r}') \times (\vec{r}-\vec{r}') = 0$, we reach to the fact that;

$$\vec{\nabla} \times \left\{\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right\} = 0$$

therefore equation (2-8) becomes;

| |
|---|
| $\vec{\nabla} \times \vec{E}(\vec{r}) = 0 \;...\; (2-9)$ Vanishing of Electrostatic field |

وهي معادلة تصف المجال الكهربائي بدلاله الموقع (position vector $\vec{r}$) وبثلاث ابعاد.

Q. Show that (proof that) $\vec{E}(\vec{r})$ produced by a collection of: several point charges, volume charge distribution, and surface charge distribution is vanishing.

**H.W:**

Apply Stockes theorem onto the electric field $\vec{E}(\vec{r})$ vanishing property: $\vec{\nabla} \times \vec{E}(\vec{r}) = 0$, What is the Physical meaning for the result?

Ans.-The line integral of $\vec{E}(\vec{r})$ around any closed surface equals to zero, i.e. $\vec{E}(\vec{r})$ is independent on path.

Curl-less fields, (eq.2-9), have several properties: (Griffiths p.52)

a) $\vec{\nabla} \times \vec{E} = 0$ everywhere المجال الكهربائي لا دوراني irrotational

b) $\int_a^b \vec{E}.d\vec{l}$ is independent of path.

c) $\oint \vec{E}.d\vec{l} = 0$ for any closed loop, i.e. $\vec{E}$ is a conservative field.

d) $\vec{E}$ is the gradient of some scalar, $\vec{E}(\vec{r}) = -\vec{\nabla}U$.

Then according to property (d), this vector field can be written as the gradient of a scalar potential, $U$, i.e.:

$$\vec{\nabla} \times \vec{E} = 0 \Leftrightarrow \vec{E}(\vec{r}) = -\vec{\nabla}U$$

$$\boxed{\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}) = -\vec{\boldsymbol{\nabla}}\boldsymbol{U}(\vec{\boldsymbol{r}}) \quad ...(2-10) \quad \text{Field} - \text{scalar Potantial relation}}$$

The scalar function $U(\vec{r})$ is called *scalar electrostatic potential*.

بواسطة المعادلة اعلاة فان بالامكان حساب المجال الكهربائي (وهو كميه اتجاهية تمتلك ثلاث مركبات) بشكل اسهل من خلال حساب الجهد الكهربائي العددي (الذي هو كمية عددية وبمركبه واحدة) باستخدام مؤثر الانحدار.

Eq.2-10 could be re-write as:

$$\boxed{\vec{\nabla}U(\vec{r}) = -\vec{E}(\vec{r}) \quad ... \quad \text{differential form for electrostatic potential}}$$

In order to determine $U(\vec{r})$, we must include eq.2-9 in the field general equation (2-7) (معادلة المجال), and applying the identity: $\vec{\nabla}\frac{1}{r} = -\frac{\vec{r}}{r^3}$ according to the following procedure;

equation (2-7) could be re-write as:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_o}\left\{\sum_{i=1}^{n} q_i\left(-\vec{\nabla}\frac{1}{|\vec{r}-\vec{\acute{r}}|}\right) + \int_V \left(-\vec{\nabla}\frac{1}{|\vec{r}-\vec{\acute{r}}|}\right)\rho(\acute{r})\,d\acute{V} + \int_S \left(-\vec{\nabla}\frac{1}{|\vec{r}-\vec{\acute{r}}|}\right)\sigma(\acute{r})\,d\acute{a}\right\}$$

Since the gradient process involves derivation with respect to $\vec{\acute{r}}$, thus the last equation may written as;

$$\vec{E}(\vec{r}) = -\vec{\nabla}\left[\frac{1}{4\pi\epsilon_o}\left\{\sum_{i=1}^{n}\frac{q_i}{|\vec{r}-\vec{\acute{r}}|} + \int_V \frac{\rho(\acute{r})d\acute{V}}{|\vec{r}-\vec{\acute{r}}|} + \int_S \frac{\sigma(\acute{r})d\acute{a}}{|\vec{r}-\vec{\acute{r}}|}\right\}\right]$$

$$\boxed{\therefore\ U(\vec{r}) = \frac{1}{4\pi\epsilon_o}\sum_{i=1}^{n}\frac{q_i}{|\vec{r}-\vec{r}_i|} + \frac{1}{4\pi\epsilon_o}\int_V \frac{\rho(\acute{r})d\acute{V}}{|\vec{r}-\vec{\acute{r}}|} + \frac{1}{4\pi\epsilon_o}\int_S \frac{\sigma(\acute{r})d\acute{a}}{|\vec{r}-\vec{\acute{r}}|}\quad ...\ (2-11)\qquad scalar\ electrostatic\ potential}$$

Equation (2-11) represent the ***scalar electrostatic potential*** at a point located at $(\vec{r})$, due to;

- A system of $n$ *discrete point charges* $(q_1, q_2, q_3, \dots, q_4)$ located at $(r_1, r_2, r_3, \dots, r_4)$.
- A *continues charge distribution* consists of:

1. A *volume charge distribution* located at $\vec{\acute{r}}$.
2. A *surface charge distribution* located at $\vec{\acute{r}}$.

Electrostatic potential could be deduced by another procedure as follows; integrating of equation (2-10) yields;

$$\int_{ref}^{r} E(\vec{r})\,.\,d\vec{r} = -\int_{ref}^{r} \vec{\nabla}\,U\,.\,d\vec{r}$$

$$= -\int_{ref}^{r} d\vec{U} = -U(\vec{r})|_{ref}^{r}$$

$$= -\big(U(\vec{r}) - U(ref)\big)$$

When U is zero at the ref. point, we have:

$$\int_{ref}^{r} E(\vec{r}) \,.\, d\vec{r} = -U(\vec{r})$$

$$\rightarrow U(\vec{r}) = -\int_{ref}^{r} E(\vec{r}) \,.\, d\vec{r} \qquad \text{Integral form of electrostatic potential} \qquad ... \quad (2-12)$$

**Cases:** absolute potential

If the electric field $\vec{E}$ produced due to only a one point charge $q_1$ located at a separation vector of $\vec{r}-\vec{r}_1$ from a detection point P. Then the absolute potential for $q_1$, could be calculated by eq.(2-12), as:

$$\vec{E}(\vec{r}) = \frac{q_1}{4\pi\epsilon_o} \frac{(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} \quad ... \quad (2-13)$$

substitution of equation (2-13) in eq. (2-12) yields;

$$U(\vec{r}) = -\frac{q_1}{4\pi\epsilon_o} \frac{1}{|\vec{r}-\vec{r}_1|^3} \int_{ref}^{r} (\vec{r}-\vec{r}_1) \,.\, d\vec{r}$$

By rule of dot product:

$$= -\frac{q_1}{4\pi\epsilon_o} \frac{1}{|\vec{r}-\vec{r}_1|^3} \int_{ref}^{r} |\vec{r}-\vec{r}_1| . |d\vec{r}| . cos\theta$$

Consider that $q$ is moved from infinity to the point P, such that $ref. = \infty$, then

$$= -\frac{q_1}{4\pi\epsilon_o} \frac{d\vec{r}}{|\vec{r}-\vec{r}_1|^2}\Bigg|_{\infty}^{r}$$

Which means that the potential at ref. is zero, $U_{ref.} = 0$, thus:

$$U(\vec{r}) = \frac{1}{4\pi\epsilon_o} \frac{q_1}{|\vec{r}-\vec{r}_1|} \; (Volt) \qquad absolute\ potantial \qquad ... \; (2-14)$$

The scalar potential at a fixed point P due to a point charge $q_1$ on a distance $|\vec{r}-\vec{r}_1|$ away.

H.W:

1. Find $U(\vec{r})$ when the source charge point $q$ is located at the origin.
2. Find $U(\vec{r})$ when the source being a *charge distribution* which distributed through a volume $V$.
3. Find $U(\vec{r})$ when the source being a *charge distribution* distributed within a surface $S$.

## 2 – 4: Electrostatic potential energy,$E_p$: الطاقة الكامنة للكهربائية الساكنة

$$W(\vec{r}) = E_p = -\int_{ref}^{r} \vec{F}(\vec{r}).d\vec{r} \quad . . . \quad (2-15)$$

هي الشغل الذي يجب ان ينجزه مجال كهربائي ساكن (على شكل قوة كهربائية $\vec{F}$ = (r) على شحنة اختبارية تنتقل خلال ذلك المجال من نقطة مرجعية $r_{ref}$ الى نقطه اخرى r ، (q$\vec{E}$(r) اي انه طاقه كامنة في ذلك المجال. الاشارة السالبة تشير الى ان الشغل المبذول على الشحنة هو ضد تأثير المجال الكهربائي. ويؤخذ بنظر الاعتبار الاتي:

1. لكون الشحنة المستخدمة اختبارية ذلك يعني أنها صغيرة بحيث ان تحريكها لايغير المجال وان شحنتها موجبة، بالاتفاق.
2. نقطة المرجع تقع في المالانهاية وهناك فأن الجهد يكون صفر.
3. في حال عدم وجود مؤثر ثالث فان قانون حفظ الطاقه يكون: $E_T = E_p + E_k$

حيث $E_p$ تمثل طاقة المجال الكامنة و $E_k$ الطاقة الحركية للجسيم المتنقل.

Where $W(\vec{r})$ is the electrostatic potential energy at a space point $(\vec{r})$ relative to the reference point at which the electrostatic potential energy is arbitrary taken to be zero.

$$dW(\vec{r}) = -\vec{F}.d\vec{r}$$

$$= -q\vec{E}.d\vec{r}$$

$$w = -q\int_{ref}^{r} \vec{E}.d\vec{r}$$

$\frac{w}{q} = -\int_{ref}^{r} \vec{E}.d\vec{r}$ (N/C) potential energy per unit charge

Comparing with eq.2-12, lead to conclude that:

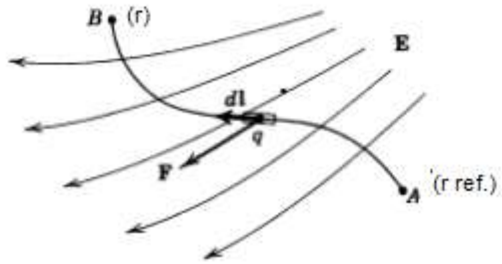
*" Electrostatic scalar potential ≡Electrostatic Potential energy per charge "*

Remembering that $\vec{F} = q\vec{E}$ and comparing equs. (2-15) and (2-12) we can directly realized that;

$$U(\vec{r}) = \frac{W(\vec{r})}{q} \quad . . . \ (2-16) \text{ scalar potential definition}$$

Eqn. (2-16) defines that; the electrostatic potential is just the potential energy per unit charge.

الجهد الكهربائي او فرق الجهد الكهربائي بين نقطتين في الفضاء هو الشغل لوحدة الشحنة الذي يجب ان يبذل لتحريك شحنه صغيرة "اختبارية" من الموقع الاول الى الثاني.




**H.W:**

- What will be the mathematical form of the electrostatic potential due to;

1. Charge distributed through a volume V.
2. = = = a surface S.
3. = = = a line L.

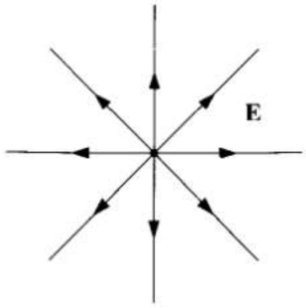
- Explain field forces exerted on: electron, proton (-q) and neutron (∓q) moves with a constant speed within an electrostatic field, separately.

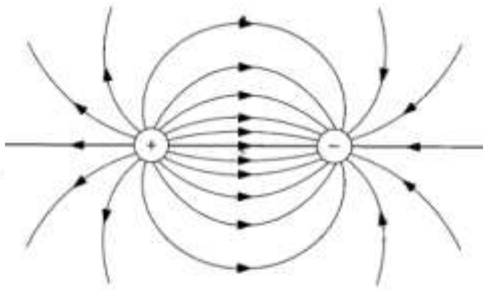
## 2 – 6: Flux and Flux Density

The electric (electrostatic) flux ($\psi$) is defined as the number of electrostatic force lines that originates on positive charge and terminate on negative charge. This definition strictly speaking that one Coulomb of electric charge gives rise to one Coulomb of electric flux. Hence;

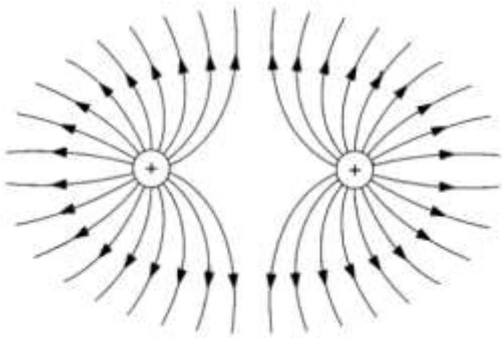
$$\psi = q \qquad (Coul.)$$

While the electric flux $\psi$ is a scalar quantity, there is a vector quantity related to $\psi$ called *density of flux* and denoted by $\vec{D}$.



Equal but opposite charges



Equal charges

The flux density at a point p is defined as the amount of flux that crosses the deferential area $(n/4\pi dr^2)$. i.e.;

$$\vec{D} = \frac{d\psi}{d\vec{s}} \qquad (C/m^2)$$

or

$$\vec{D}.d\vec{s} = d\psi$$

$$\boldsymbol{\psi} = \int_s \vec{\boldsymbol{D}}.\boldsymbol{d\vec{a}} = \int_s \vec{\boldsymbol{D}}.\hat{\boldsymbol{n}}\boldsymbol{da} \quad . \ . \ . \quad (2-17)$$

Where $d\vec{a}$ is the vector surface element, of magnitude $da$ and direction $\hat{n}$

where $d\mathbf{S}$ is the vector surface element, of magnitude $dS$ and direction $\mathbf{a}_n$. The unit vector $\mathbf{a}_n$ always taken to point out of $S$, so that $d\Psi$ is the amount of flux passing from the interior of $S$ to exterior of $S$ through $dS$.

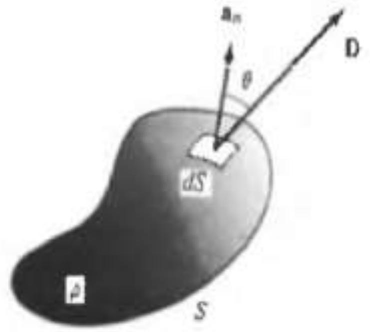
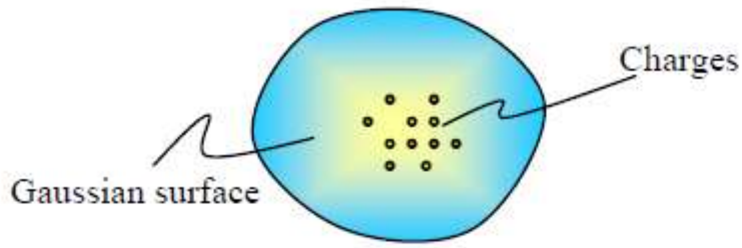



**Fig. 3-4**

## 2 – 7: Gauss's Law (حساب المجال الكهربائي لشحنة غير متجانسة) قانون كاوس

This law relates between the integral of the normal component of the electric field over a closed surface and the total charge enclosed by the surface. "the flux of the electric field over the Gaussian surface equals to the net charge enclosed by that surface."

Gaussian surface is a closed surface, e.g. sphere, cube, cylinder, etc. And, Gauss's law tells how the fields at the Gaussian surface are related to the charges contained within that surface.




قانون كاوس هو صيغة جديدة من قانون كولوم. الفيض الكلي المخترق لسطح خيالي "سطح كاوسي" يحيط بشحنة كهربائية يساوي صافي قيمة الشحنة المحاطة بهذا السطح.

$$\oint_s \vec{E}\,.\,d\vec{a} = \oint_s \vec{E}\,.\,\hat{n}da = \frac{1}{\varepsilon_\circ} q_{enc.} \rightarrow \quad \text{Integral form of Gauss's law}$$

حيث $\hat{n}$ هو اتجاه المساحة السطحية $da$ والذي يمكن ان يتم اعتماده ضمنيا بالصيغة: $d\vec{a}$

or:

$$\vec{\nabla}\,.\,\vec{E} = \frac{\rho}{\varepsilon_\circ} \longrightarrow \quad \text{Differential form of Gauss's law}$$

## 2 – 7 – 1: The Integral Form

In order to derive the Gauss's law in its integral form, we have to fallow the following procedure. The electric field at a point $\vec{r}$ due to a point charge q located at the origin, $\acute{r} = 0$, is;

$$E(\vec{r}) = \frac{q}{4\pi\varepsilon_\circ}\frac{\vec{r}}{|\vec{r}|^3} \quad \dots (2-18)$$

Consider the surface integral of the normal component of $\vec{E}$ over a closed surface that encloses the origin and consequently the charge q,

$$\oint_S \vec{E}.d\vec{a} = \frac{q}{4\pi\varepsilon_\circ}\mathrm{d}\Omega$$

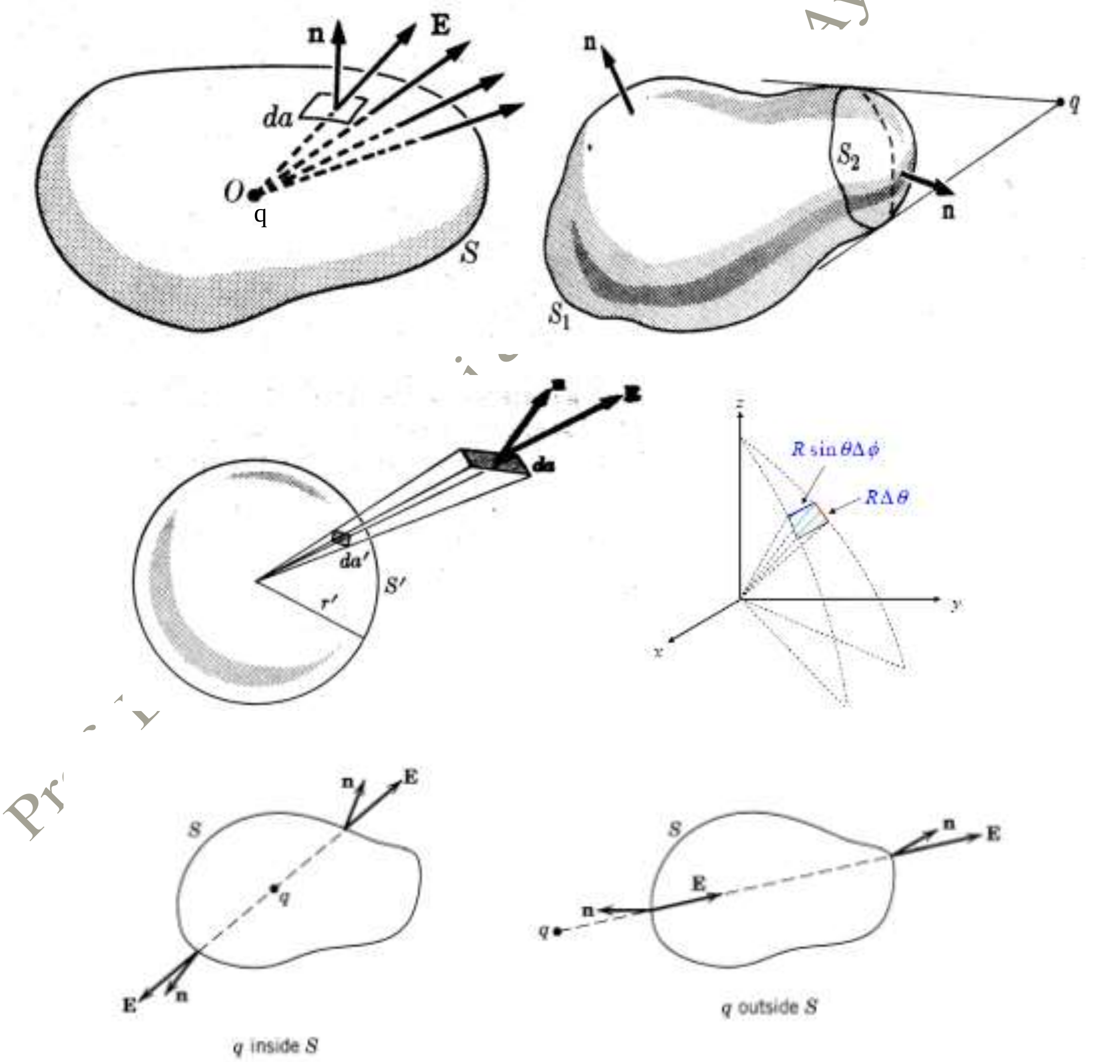



Figure 1.2 Gauss's law. The normal component of electric field is integrated over the closed surface $S$. If the charge is inside (outside) $S$, the total solid angle subtended at the charge by the inner side of the surface is $4\pi$ (zero).

For the outer case: cos?-cos(180-?)=0

$$d\Omega = \frac{cos\theta}{r^2}da$$

fig.2-2

which is

$$\oint_s \vec{E}.d\vec{a} = \frac{q}{4\pi\varepsilon_\circ}\oint_s \left(\frac{\hat{n}}{r^2}\right).(r^2 sin\theta d\theta d\varphi\hat{n}) \quad ...(2-19)$$

$$= \frac{q}{4\pi\varepsilon_\circ}\oint_0^{\pi} sin\theta d\theta \oint_0^{2\pi} d\varphi$$

$$= \frac{q}{4\pi\varepsilon_\circ}[-cos\theta]_0^{\pi}[2\pi - 0]$$

$$\oint_s \vec{E}.\vec{n}\ da = \frac{q}{\varepsilon_\circ} \quad ... \text{(2-20)}$$

Which is the integral form of Gauss's law.

The quantity $(\vec{r}/r^3).\vec{n}\ da$, in eq.2-19, represent the projection of the element area da on the plane perpendicular to r. However when it divided by $r^2$ it will represent the solid angle subtended by da which is written $d\Omega$ .thus:

$\oint_s \frac{\vec{r}.\vec{n}}{r^3}\ da = 4\pi$ solide angle in steradian units $d\Omega = \frac{cos\theta}{r^2}da$

One can easily prove that when the charge $q$ being outside the closed surface *S* is the same flux will be *zero*.

element of surface area. If the electric field **E** at the point on the surface due to the charge $q$ makes an angle $\theta$ with the unit normal, then the normal component of **E** times the area element is:

$$\mathbf{E}\cdot\mathbf{n}\,da = \frac{q}{4\pi\epsilon_0}\frac{\cos\theta}{r^2}\,da \tag{1.7}$$

Since **E** is directed along the line from the surface element to the charge $q$, $\cos\theta\,da = r^2\,d\Omega$, where $d\Omega$ is the element of solid angle subtended by $da$ at the position of the charge. Therefore

$$\mathbf{E}\cdot\mathbf{n}\,da = \frac{q}{4\pi\epsilon_0}\,d\Omega \tag{1.8}$$

If we now integrate the normal component of **E** over the whole surface, it is easy to see that

$$\oint_S \mathbf{E}\cdot\mathbf{n}\,da = \begin{cases} q/\epsilon_0 & \text{if } q \text{ lies inside } S \\ 0 & \text{if } q \text{ lies outside } S \end{cases} \tag{1.9}$$

This result is Gauss's law for a single point charge. For a discrete set of charges, it is immediately apparent that

$$\oint_S \mathbf{E}\cdot\mathbf{n}\,da = \frac{1}{\epsilon_0}\sum_i q_i \tag{1.10}$$

where the sum is over only those charges *inside* the surface $S$. For a continuous charge density $\rho(\mathbf{x})$, Gauss's law becomes:

$$\oint_S \mathbf{E}\cdot\mathbf{n}\,da = \frac{1}{\epsilon_0}\int_V \rho(\mathbf{x})\,d^3x \tag{1.11}$$

where $V$ is the volume enclosed by $S$.

Equation (1.11) is one of the basic equations of electrostatics. Note that it depends upon

the inverse square law for the force between charges,
the central nature of the force, and
the linear superposition of the effects of different charges.

Clearly, then, Gauss's law holds for Newtonian gravitational force fields, with matter density replacing charge density.

It is interesting to note that, even before the experiments of Cavendish and Coulomb, Priestley, taking up an observation of Franklin that charge seemed to reside on the outside, but not the inside, of a metal cup, reasoned by analogy with Newton's law of universal gravitation that the electrostatic force must obey an inverse square law with distance. The present status of the inverse square law is discussed in Section I.2.

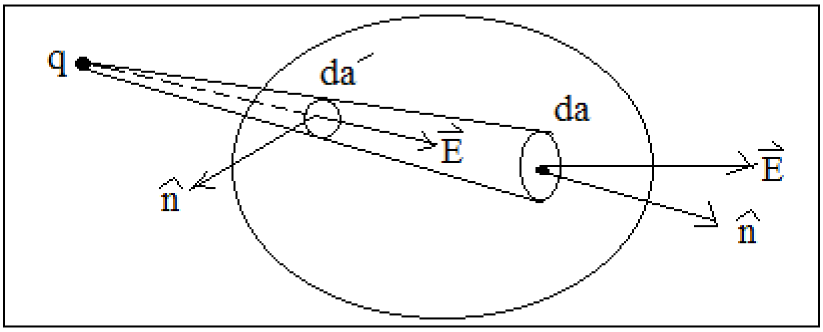
## *1.4 Differential Form of Gauss's Law*

Gauss's law can be thought of as being an integral formulation of the law of electrostatics. We can obtain a differential form (i.e., a differential equation) by



H.W.

1. Prove mathematically that $\oint_s \vec{E}.\vec{n}\,da = 0$ when the charge q being outside the surface S.



2. Prove that, when the surface S encloses n charges the Gauss's law will take the following form

$$\oint_s \vec{E}.\vec{n}\ da = \frac{1}{\varepsilon_\circ}\sum_{i=1}^{n} q_i$$

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i = \frac{1}{\varepsilon_\circ}\sum_{i=1}^{n}\left(\oint_s \vec{E}_i.\vec{n}\ da\right)$$
$$= \frac{1}{\varepsilon_\circ}\sum_{i=1}^{n} q_i$$

## 2 – 7 – 2: The Differential Form

Remember that, the volume charge density is

$$\rho = \frac{dq}{dV}$$

and hence $dq = \rho\, dV$. Thus the total charges that may distribute within the volume v is;

$$q = \int_d \rho\, dV \quad \ldots (2-21)$$

The substitution of equation (2-21) into (2-20) yields;

$$\oint_s \vec{E}.\hat{n}\, da = \frac{1}{\varepsilon_\circ}\int_V \rho\, dV \quad \ldots . (2-22)$$

**H.W.** What does equation (2-22) state?

From the divergence theorem we have;

$$\oint_s \vec{F}.\hat{n} da = \int_V \vec{\nabla}.\vec{F}\, dV$$

Thus; $$\oint_s \vec{E}.\hat{n}\, da = \int_V \vec{\nabla}.\vec{E} dV \quad \ldots (2-23)$$

From equations (2-22) and (2-20); Since equation (2-23) must be valid for any volume v. therefore;

$$\int_V \vec{\nabla}.\vec{E} dV = \frac{1}{\varepsilon_\circ}\int_V \rho\, dV$$

$$\boldsymbol{\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_\circ}} \quad \ldots (2-24)$$

which the *differential form* of Gauss's law.

**Note:** The relation between the electrical field $\vec{E}$ and the electric flux density can be thought from the following relation;

$$\vec{\boldsymbol{D}} = \boldsymbol{\varepsilon}_{\circ}\vec{\boldsymbol{E}}$$

Consequently, the integral and differential forms of Gauss's forms of Gauss's law can be expressed in terms of $\vec{D}$ respectively as follows;

$$\left.\begin{aligned} \oint_s \vec{D}\,.\hat{n}\,da &= q \\ \vec{\nabla}.\vec{D} &= \rho \end{aligned}\right\} \qquad (2\text{-}25)$$

## 2 – 8: Application of Gauss's Law

Gauss's law providing a very easy way for calculating electric field in certain highly symmetric situations of considerable physical interest. However, in order to make Gauss's law be useful way in calculating the electric field, it must be possible to choose a Gaussian surface that satisfying the following conditions;

1. The surface is closed.

e.

al.

There are three classes of charge distributions which lend themselves to the application of Gauss' law:

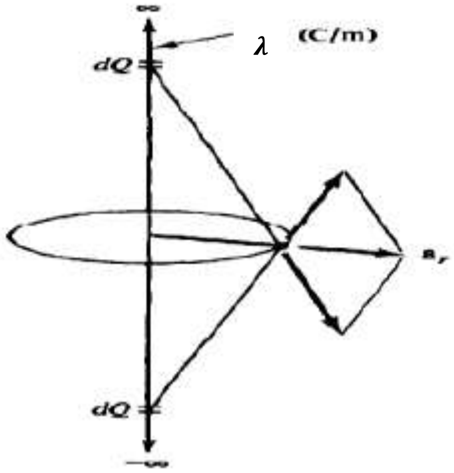
1. Infinite sheets of charge
2. Infinite cylindrical charge distribution, or infinite line charge.
   Coordinate system: Cylindrical
   Gaussian surface: Cylinder
3. Spherically symmetric charge distribution
   Coordinate system: spherical
   Gaussian surface: sphere

**Example**: (Schaums Outline, p.35)

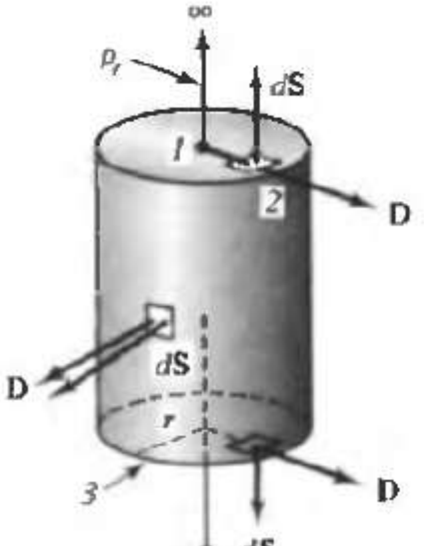
Use a special Gaussian surface to find $\vec{E}$ that deduced due to a uniform line charge $\lambda$ (Coul/m) of length L.

Solution:

Assume that the line charge to be coinciding with z- axis of cylindrical coordinates, fig.




From the cylindrical symmetry it can be seen that $\vec{E}$ can only have an $r$-component and this component can only depend on $r$. Thus, the special Gaussian surface for this situation is a closed *right circular cylinder* whose axis is the $z$-axis.



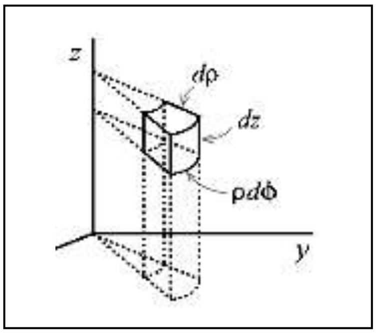



Appling equation (2-20) yields;

$$\oint_{s_1} \vec{E}\,.\,\hat{n}\,da + \oint_{s_2} \vec{E}\,.\,\hat{n}\,da + \oint_{s_3} \vec{E}\,.\,\hat{n}\,da = \frac{q}{\varepsilon_\circ}$$

$$\oint_{s_1} E\,.\cos\frac{\pi}{2}.\,da + \oint_{s_2} E\,.\cos 0\ \, da + \oint_{s_3} E\,.\cos\frac{\pi}{2}\ da = \frac{q}{\varepsilon_\circ}$$

$$\oint_{s_2} E\,.\,da = \frac{q}{\varepsilon_\circ}$$




$E\oint_{s_2} da = \frac{q}{\varepsilon_\circ}$ , $E$ constant because $r$ is constant

$$E\oint_{s_2} \boldsymbol{r}\,\boldsymbol{d\varphi} dz = \frac{q}{\varepsilon_\circ}$$

$$E\int_0^{2\pi} d\varphi \int_0^L dz = \frac{q}{r\varepsilon_\circ}$$

$$E(2\pi L) = \frac{q}{r\varepsilon_\circ}$$

But: $q = \lambda L$, thus;

$$E = \frac{\lambda}{2\pi\varepsilon_\circ r}$$

H.W.

1. When will $\vec{E}$ and $\hat{n}$ be parallel and anti-parallel?
2. What is the value of $\Psi$ and $\vec{D}$ in above example?
3. Does the line charge is positive or negative.
4. Using Gauss's law, find the electric field produced at a space point $\vec{r}$ due to a point charge Q located at the origin. ***Schaum,p.34.***



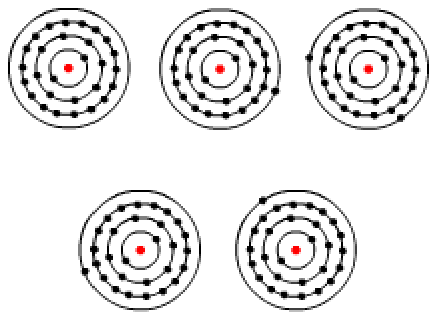
**2 – 9: Electrical Conductors, Insulators and Semiconductors**: (text:p.30, Schaum p.78, others texts)

**Atomic structure explanation:**

ينظر في نقل الموضوع الى اول فقرة في الفصل السادس لـ د.حسن

*In metals*

A metal consists of a lattice of atoms, each with a shell of electrons. This is also known as a positive ionic lattice, where the outer electrons (three or less valence electrons which are located in the outer orbit of an atom) are slightly (covalent) bonded to their nuclei. I.e. a specific electron could associate (leave its own nucleus) with all atoms nuclei in one lattice. *They are sometimes described as an electron gas*. When an electrical potential difference (a voltage) is applied across the metal, the electrons drift from one end of the conductor to the other (electrons drift opposite to the applied field and positive ions is vice versa). Thus, electrical conductivity depends only on electron mobility; electric conductivity $\boldsymbol{\sigma = \rho_e \mu_e}$.



$^{29}Cu$, e=29, $4^{th}$ energy levels, it has only 1 electron in outer shell.

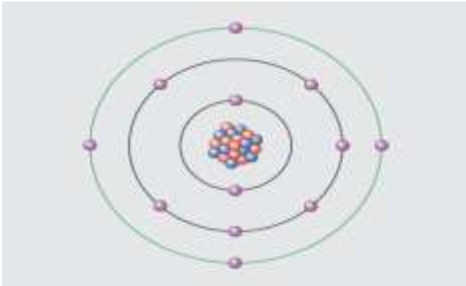
*In insulators*

Insulators resist the flow of electricity. they contain seven or eight valence electrons. Examples of insulators are: rubber, plastic, glass, and wood.

An insulator has all its electrons tightly bonded to the nucleus and so it takes very large forces of either heat or potential to dislodge them. Remembering that this type of substances not absolutely electrical insulator, where the application of a large enough potential difference (the materials *breakdown voltage*) on a low dimensional such a matter, the lattice will deflects (reconstructed) and current could be flow.

*In semiconductors*

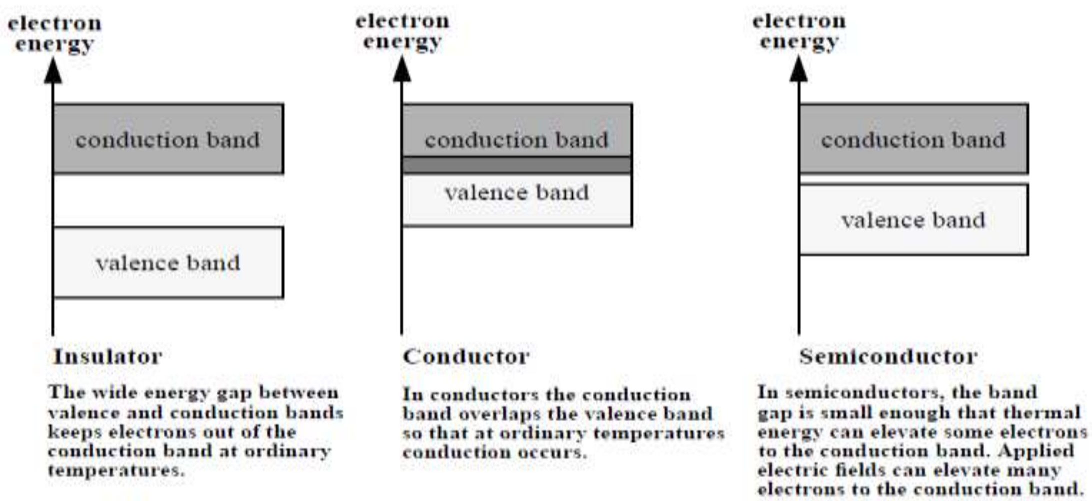
Semiconductors are materials that are neither good conductors nor good insulators. They contain four valence electrons. When heated, their resistance decreases. Two common materials are silicon and germanium Semiconductors contain four valence electrons.

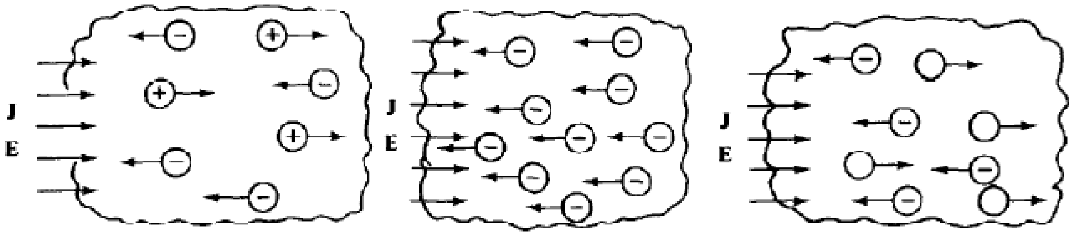


Semiconductors fall between these two groups. They do not normally pass current easily at room temperature, having a resistivity, somewhere between conductors and insulators. They have properties however which make them very useful in electronic devices.

Electric conductivity $\sigma = \rho_e \mu_e + \rho_h \mu_h$








$\sigma = \rho_- \mu_- + \rho_+ \mu_+$

(*a*) Liquid or gas

$\sigma = \rho_e \mu_e$

(*b*) Conductor

$\sigma = \rho_e \mu_e + \rho_h \mu_h$

(*c*) Semiconductor

*In Superconductors*

Which are special materials which when cooled below their critical temperature (100K or less) become perfect conductors. Superconductors are at one of the last great frontiers of scientific discovery. The limits of superconductivity not yet been reached, and the theories that explain superconductor behavior are constantly under review.

*Ionic liquids/electrolytes/Gases*

In these types of matter there are generally present both positive and negative ions some singly charged and others doubly and possibly in different masses. In electrolytes, electrical conduction happens not by band electrons or holes, but by full atomic species (ions) traveling, each carrying an electrical charge.

○ **Band theory simplified**

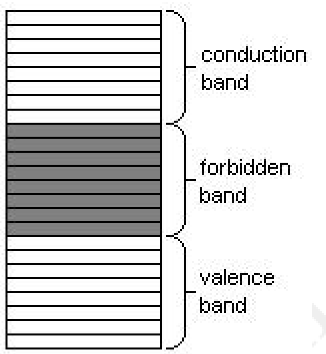
Quantum mechanics states that the energy of an electron in an atom cannot be any arbitrary value. Rather, there are fixed energy levels which the electrons can occupy, and values in between these levels are impossible. The energy levels are grouped into two bands: the valence band and the conduction band (the latter is generally above the former).

| shell # (code) | set of orbitals letter |
|---|---|
| 1 (K) | s |
| 2 (L) | s,p |
| 3 (M) | s,p,d |
| 4 (N) | s,p,d,f |
| 5 (O) | s,p,d,f,g |
| 6 (P) | s,p,d,f,g,h |
| … | s,p,d,f,g,h,i… |

1s(2), 2s(2), 2p(6), 3s(2), 3p(6), 4s(2), 3d(10), 4p(6), 5s(2), 4d(10)[3]… (2,8,18,32,50, .

Electrons in the conduction band may move freely throughout the substance in the presence of an electrical field.

In insulators and semiconductors, the atoms in the substance influence each other so that between the valence band and the conduction band there exists a forbidden band of energy levels, which the electrons cannot occupy. In order for a current to flow, a relatively large of energy amount must be furnished to an electron for it to leap across this forbidden gap and into the conduction band. Thus, even large voltages can yield relatively small currents.



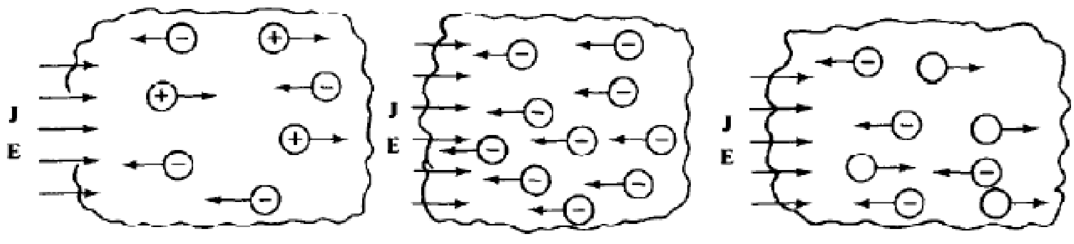
Electron energy levels in an insulator

In metals, the Fermi level lies in the conduction band (see Band Theory, below) giving rise to free conduction electrons. However, in semiconductors the position of the Fermi level is within the band gap, approximately half-way between the conduction band minimum and valence band maximum for intrinsic (undoped) semiconductors. This means that at 0 kelvins, there are no free conduction electrons and the resistance is infinite. However, the resistance will continue to decrease as the charge carrier density in the conduction band increases.

In terms of atomic structure there are no free electrons to carry charge. This means that the energy difference between the conduction bands and valences bands (band gap) is wide. For insulators like diamonds or quartz this is in the 5-8eV range, such that very few carriers have enough thermal energy to overcome the band gap.

The difference between the three groups of materials lies in the number of easily detached electrons within the atomic structure. The electrons concerned are more or less loosely held in the outermost electron shells called valence shells and so these electrons are called valence electrons.

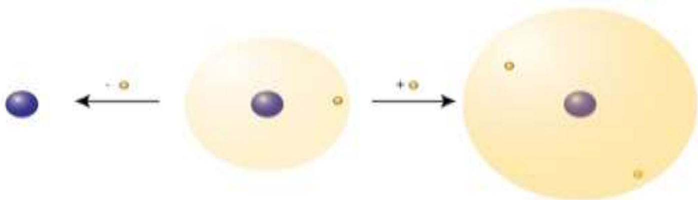





$\sigma = \rho_- \mu_- + \rho_+ \mu_+$ (*a*) Liquid or gas

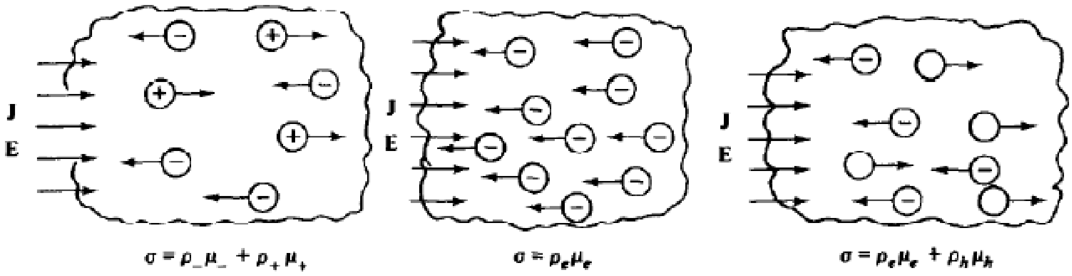
$\sigma = \rho_e \mu_e$ (*b*) Conductor

$\sigma = \rho_e \mu_e + \rho_h \mu_h$ (*c*) Semiconductor



| No. of protons | 1 | 1 | 1 |
|---|---|---|---|
| No. of electrons | 0 | 1 | 2 |
| Charge | +1 | 0 | -1 |
| Notation | $H^+$ | H | $H^-$ |
| Classification | cation | neutral (not an ion) | anion |

An ion is an atom or molecule in which the total number of electrons is not equal to the total number of protons, giving it a net positive or negative electrical charge.




(*a*) Liquid or gas (*b*) Conductor (*c*) Semiconductor