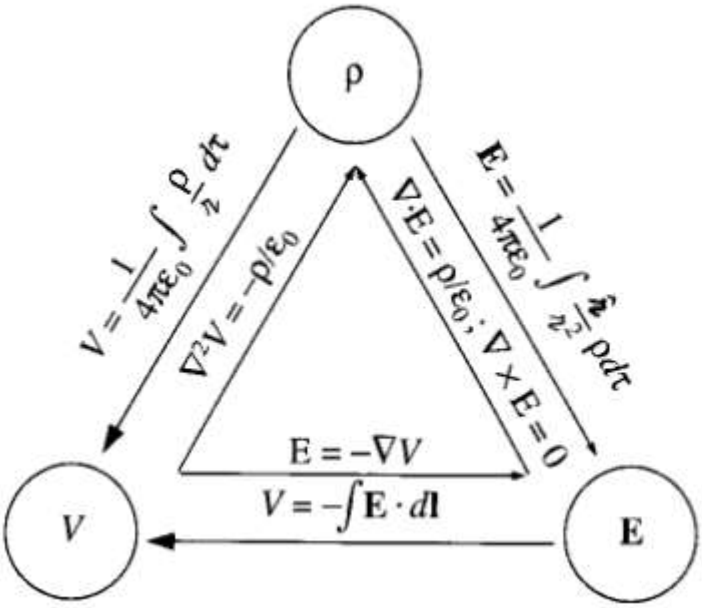
Chapter three

Electrostatic Boundary Value Problems

As given in chapter two, the electrostatic field has two characteristic properties which are;

- $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_\circ}$ Diff. form of Gauss law
- $\vec{\nabla} \times \vec{E} = 0$ Vanishing of Electrostatic field

وبذلك فأن خصائص مجال الكهربائية الساكنة قد اتضحت وفقا لنظرية هلمز، ومن المعادلة الاخيرة (وبالاستفادة من خاصية رياضية) فلقد وجب ان يمثل المجال الانحدار لدالة غير متجة هي الجهد العددي، اي: $\vec{E} = -\vec{\nabla}U$



As shown in chapter two, the procedure by which the electric field $\vec{E}$ is determined can be accomplished either by Coulomb's law or Gauss's law when the charge distribution is known, or using $\vec{E} = -\vec{\nabla}\,U$ when the potential U is known throughout the region.

But in most practical situations neither the charge distribution nor the potential distribution is known.

In this chapter, we shall consider practical electrostatic problems where only electrostatic conditions (charge and potential) at some boundaries are known and it's desired to find $\vec{E}$ and U throughout the region. Such problems, however, are usually solved by Poisson's or Laplace's equation.

<u>3-1 Poisson's Equation</u> معادلة بواسون

Poisson's and also Laplace's equation are easily derived from Gauss's low for a *linear medium*. It has been shown in previous chapter that Gauss's low can be expressed as;

$$\vec{\nabla}.\vec{D} = \vec{\nabla}.\left(\epsilon\vec{E}\right) = \rho \quad \dots (2-26)$$

Also, it is proved that

$$\vec{E} = -\vec{\nabla}U \quad \dots (2-10)$$

Substituting equation (2-10) in(2-26) yields ;

$$\vec{\nabla}.\epsilon(-\vec{\nabla}U) = \rho \quad (3-1)$$

For a *homogenous* medium equation (3-1) becomes;

$$\boxed{\boldsymbol{\nabla^2 U = -\frac{\rho}{\epsilon}} \qquad \dots (3-2)}$$

Equation (3-2) called ***Poisson's equation***.

<u>3-2 Laplace's Equation</u> معادلة لابلاس

In fact Laplace's equation is a special case for Poisson's equation which occurs when the region under consideration being *free from charge* (*i.e.* = 0). Thus equation (3-2) becomes;

$$\boxed{\boldsymbol{\nabla^2 U = 0} \quad \dots (\boldsymbol{3-3})}$$

Last equation is ***Laplace's equation*** for *homogenous medium*.

For an *inhomogeneous* medium the Laplace's equation is equation (3-1) when the right-hand side vanishes ($\rho = 0$).

According to ideas of chapter one, Laplace's equation in *Cartesian, cylindrical and spherical* coordinates respectively is given by;

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}=0 \ . \ . \ . \ (3-3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial U}{\partial \varphi^2}+\frac{\partial^2 U}{\partial z^2}=0 \ \ . \ . \ . \ (3-4)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right)+\frac{1}{r^2\sin\theta}\ \frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial U}{\partial \theta}\right)+\ \frac{1}{r^2\sin^2\theta}\frac{\partial U}{\partial \varphi^2}=0 \ . \ . \ (3-5)$$

Depending on whether the potential is $U(x\,,y\,,z)$, $U(r\,,\varphi,z)$ and $U(r,\theta,\varphi\ )$.

Laplace's equation is of primary importance is solving electrostatic problems, involving a set of *conductors material* at different potentials. Examples of such problems include capacitors and vacuum tube diode.

H.W:

Find the mathematical form of *Poisson's equation* in Cartesian, cylindrical and spherical coordinates.

3-3: Laplace's Equation Solution in One Dimension:

3-3-1: Cartesian Coordinates

$$\nabla^2 U=0$$

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}=0$$

$$\frac{\partial^2 U}{\partial x^2}=0$$

$$\frac{d^2 U}{dx^2}=0$$

$$\frac{d}{dx}(\frac{dU}{dx})=0$$

$$\frac{dU}{dx}=a$$

$$U=a\int dx$$

$$\boxed{U(x) = ax + b \quad . \ . \ . \quad (3-6)}$$

Where a and b are constants to be determined according to the imposed boundary condition. Equation (3-6) describe *equipotential surfaces* which are plate located at $x =$ constant.

Example:

Two *conductor* plates have been set up such that the first one is locted at $z = 0$ with $U = 0$ and the secoud one located at $z = d$ with $U = 100V$, find the *electeic field* and the *flux density* deduced between the two plates, assuming that this region is *charge –free*.

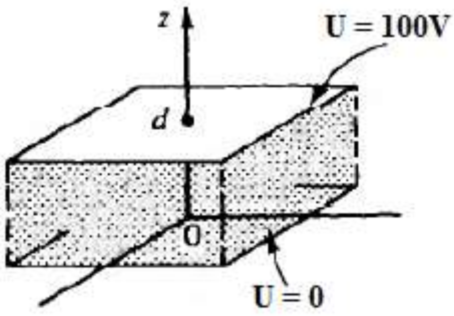
Solution: $\rho = 0$, where the plates are both conductors, thus the key equation is Laplace eq.:

$$\nabla^2 \mathrm{U} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$

$U_x = U_y = constant$, thus the 1st and 2nd derivatives for the 1st and 2nd terms in last eq. will vanishes. Laplace equation will take the form:

$$\frac{\partial^2 U}{\partial z^2} = 0$$

$$\frac{d^2 U}{dz^2} = 0$$



$$\int \frac{d}{dz}\left(\frac{dU}{dz}\right) = 0$$

$$\therefore \frac{dU}{dz} + c = 0 \rightarrow \int dU = (-c) \int dZ$$

$$U = \int adz = az + b$$

$$\boxed{\therefore \boldsymbol{U = az + b}}$$

المعادلة الاخيرة تمثل دالة الجهد غير المتجه في اي موقع بين المستويين في المسألة، الان نطبق الشروط الحدودية المعطاة في المسألة لغرض حساب الثوابت في المعادلة:

1. $U(z = 0) = 0 \rightarrow a(0) + b = 0$

$$\therefore b = 0$$

2. $U(z=d)=100 \rightarrow a(d)+b=100$

$$\therefore a=\frac{100}{d}$$

Thus: $$\mathrm{U(z)}=\left(\frac{100}{d}\right)\mathrm{z}$$

الان لحساب دالة المجال بين اللوحين بالاستفادة من المعادلة الاخيرة، نطبق العلاقة التبادلية بين الجهد والمجال (الصيغة التفاضلية للجهد):

$$\vec{E}=-\vec{\nabla}\mathrm{U}$$

للاحداثيات المتعامدة في مسألتنا:

$$\vec{E}=-\left\{\frac{\partial U}{\partial x}\hat{\imath}+\frac{\partial U}{\partial y}\hat{\jmath}+\frac{\partial U}{\partial z}\hat{k}\right\}$$

$$=-\frac{\partial}{\partial z}\left(100\frac{z}{d}\right)\hat{k}$$

$$=-\frac{100}{d}\hat{k} \quad (V/m)$$

لحساب كثافة الفيض:

$$\vec{D}=\epsilon\vec{E}$$

$$\therefore \vec{D}=-\frac{1000}{d}\epsilon\hat{k} \qquad (C/m^2)$$

Where; at the first plate:

$$D_n=-\frac{\epsilon 100}{d}\left(\frac{c}{m^2}\right)$$

$D_n=+\frac{\epsilon 100}{d}\left(\frac{c}{m^2}\right)$ at the second plate.(why?)

3-3-2 Cylindrical Coordinate:

As an example assume that U is a function only for r, *i.e.* $U(r,\varphi,z)=U(r)$. For this case Laplce's equation given in equation (3-4) reduces to the form;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)=0$$

$$\rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{dU}{dr}\right)=0$$

$$\int \frac{d}{dr}\left(r\frac{dU}{dr}\right)=0$$

$$\therefore r\frac{dU}{dr}=a$$

$$\frac{dU}{dr}=ar^{-1}\rightarrow \int dU=a\int\frac{dr}{r}$$

$$U=\int\frac{a}{r}dr=a\ln r+b$$

$$\boxed{\therefore U(r)=a\ln r+b \qquad . \quad . \quad . \quad (3-7)}$$

Equation (3-7) describe an *equipotential surfaces* which are cylinders of $r=$ constant.

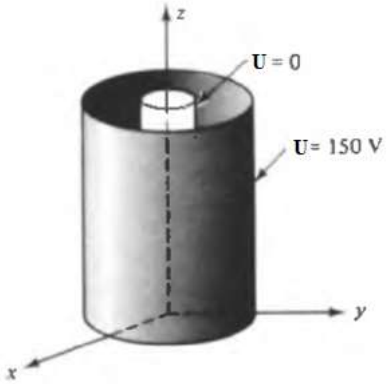
<u>Example</u>: Find the *potential* function and the *electric field intensity* for the region between two concentric right circular cylinders, where $U=U_1$ at $r=r_1$ and $U=0$ at $r=r_2$, where $r_2>r_1$.

Solution: the potential is cons. with φ and z,

Laplace equation reduces to:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)=0$$

$$U(r)=a\ln r+b$$



Applying the boundary conditions;

1. 1.$U(r_2)=a\ln r_2+b=0$

$$b=-a\ln(r_2)$$

2. $U(r_1)=a\ln(r_1)-a\ln(r_2)=U_1$

$$a\left(\ln\left(\frac{r_1}{r_2}\right)\right) = U_1$$

$$\therefore a = \frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)}$$

$$\therefore b = -\frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)} .\ln(r_2)$$

$$\rightarrow U(r) = \frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)} .\ln(r) - \frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)} .\ln(r_2)$$

$$= \frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)}\{\ln(r) - \ln(r_2)\}$$

$$\boxed{U(r) = \frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)}\ln\left(\frac{r}{r_2}\right)}$$

$$\vec{E} = -\vec{\nabla}U$$

$$= -\left\{\frac{\partial U}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial U}{\partial \varphi}\hat{\varphi} + \frac{\partial U}{\partial z}\vec{k}\right\}$$

$$= -\frac{dU}{dr}\hat{r}$$

$$= -\frac{d}{dr}\left\{\frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)}\ln\left(\frac{r}{r_2}\right)\right\}\hat{r}$$

$$= -\frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)} .\frac{d}{dr}\ln\left(\frac{r}{r_2}\right)\hat{r}$$

$$= -\frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)} .\frac{1}{r/r_2} .\frac{1}{r_2}\hat{r}$$

$$\boxed{\therefore \vec{E} = -\frac{U_1}{\ln\left(\frac{r_1}{r_2}\right)} .\frac{\hat{r}}{r}}$$

H.W Find U and $\vec{E}$ and $\vec{D}$ for the above example for the case when $U_1 = 150$, $r_1 = 1mm$ and $r_2 = 20mm$.

Example: In cylindrical coordinates, two planes of a constant φ are located as in the figure. Find the expression for $\vec{E}$ between the two planes assuming a potential of $100V$ for $\varphi = \alpha$ and a $zero$ potential for that at $\varphi = 0$.

Solution:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \varphi^2} + \frac{\partial^2 U}{\partial z^2} = 0$$

$$\frac{1}{r^2}\frac{\partial^2 U}{\partial \varphi^2} = 0$$

$$\frac{1}{r^2}\frac{d^2 U}{d\varphi^2} = 0$$

$$\frac{d}{d\varphi}\left(\frac{dU}{d\varphi}\right) = 0$$

$$\frac{dU}{d\varphi} = a$$

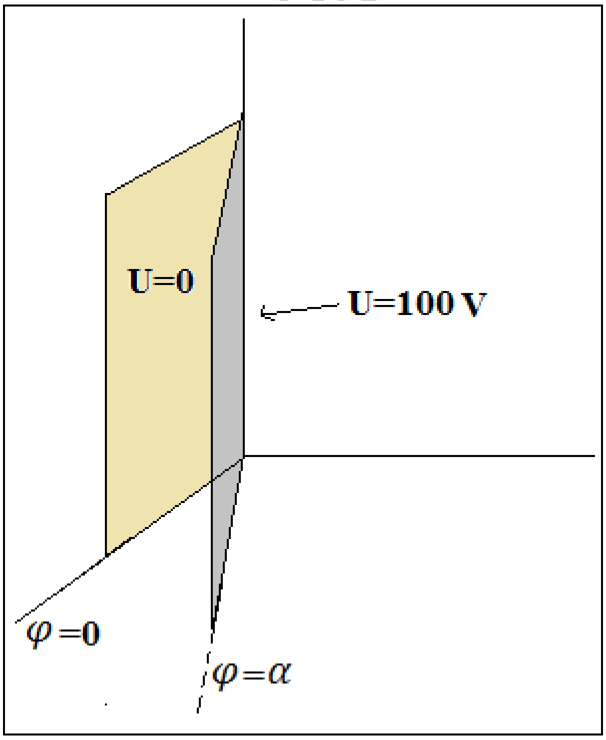
$$U = a\varphi + b$$

$$U|_{\varphi=0} = 0$$

$$\therefore b = 0$$

$$U|_{\varphi=\alpha} = 100V$$

$$\therefore 100 = a\alpha \rightarrow a = \frac{100}{\alpha}$$



$$\therefore U = \frac{100}{\alpha}\varphi \quad (V)$$

$$\vec{E} = -\vec{\nabla} U = -\frac{1}{r}\frac{d}{d\varphi}\left(100\frac{\varphi}{\alpha}\right)\hat{\varphi} =$$

$$\boxed{\therefore \vec{E} = -\frac{100}{r\alpha}\hat{\varphi} \quad (V/m)}$$

3-3-3 Spherical coordinates:

Laplace's equation in this coordinates is written as in equation (3-5).

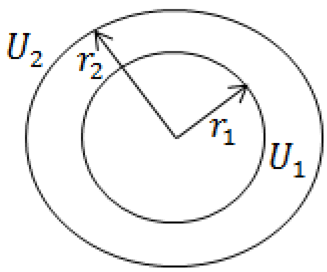
$$\nabla^2 U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) + \frac{1}{r^2\, sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial U}{\partial\theta}\right) + \frac{1}{r^2\, sin\theta}\frac{\partial^2 U}{\partial\varphi^2} = 0$$

$$. . . (3-5)$$

The equation describes U when it varies with $(r,\theta\,,\varphi\,)$. As an example if we assume that U is vary only with r, *i.e.* $U(r,\theta\,,\varphi\,) = U(r)$.

Consequently equation (3-5) reduces to the form;

$$\frac{1}{r^2}\frac{d}{dr}\left(r\frac{dU}{dr}\right) = 0$$



Two concentric spheres

$$\frac{d}{dr}\left(r\frac{dU}{dr}\right) = 0$$

$$r\frac{dU}{dr} = a$$

$$\frac{dU}{dr} = \frac{a}{r}$$

$$\boxed{U = -\frac{a}{r} + b}$$

$$\vec{E} = -\vec{\nabla}U = -(\frac{dU}{dr}\hat{r} + \frac{1}{r}\frac{\partial U}{\partial\theta}\hat{\theta} + \frac{1}{r\, sin\theta}\frac{dU}{d\varphi}\hat{\varphi})$$

$$= -\frac{dU}{dr}\hat{r}$$

$$\boxed{\vec{E} = \frac{-a}{r^2}\hat{r}}$$

Example:

Find the potential between two concentric conducting spheres of radii $0.1\ m$ and $2.0\ m$ at potential $U = 0$ and $U = 100$, respectively.

Solution:

Since U is not a function for θ and φ, Laplace's equation reduces to:

$$\nabla^2 U = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{\partial U}{\partial r}\right) = 0$$

As shown above the general solution for this differential equation is;

$$U(\vec{r}) = -\frac{a}{r} + b$$

Thus;

$$1. U(0.1) = -\frac{a}{0.1} + b = 0 \rightarrow b = \frac{a}{0.1}$$

$$2. U(2.0) = -\frac{a}{2.0} + b = -\frac{a}{2.0} + \frac{a}{0.1} = 100$$

$$\rightarrow a = 10.53\ and\ b = 105.3$$

Substituting the value of a and b in the potential general form:

$$\boxed{U(\vec{r}) = \frac{-10.53}{r} + 105.3 \quad (V)}$$

$$\vec{E} = -\vec{\nabla} U$$

$$= -\frac{dU}{dr}\hat{r}$$

$$\boxed{\therefore \vec{E} = -\frac{10.53}{r^2}\hat{r}\ \left(\frac{V}{m}\right)}$$

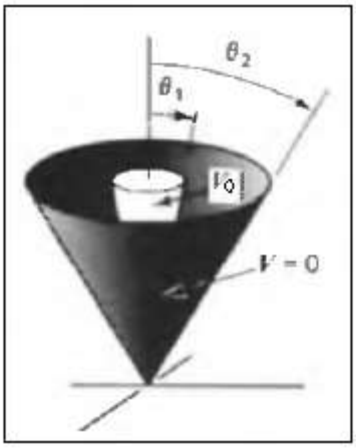
Example:

Two conducting cones, (θ_1 and θ_2) of infinite extent, are separated by an infinitesimal gab, at $r = 0$. If $U(\theta_1) = 0$ and $U(\theta_2) = V_\circ$, find U and $\vec{E}$ between the two cones.

Solution:

We are taken about two cones, *i.e.* if they are do whole period, they will construct two spheres, thus we need to solve the problem using the spherical coordinates. It is seen that U vary only with θ and constant with r and φ. Thus Laplace's equation reduces to the following form;

$$\nabla^2 U(\theta) = \frac{1}{r^2\, sin\theta}\frac{d}{d\theta}\left(sin\theta\frac{dU}{d\theta}\right) = 0$$



$$\frac{d}{d\theta}\left(sin\theta\frac{dU}{d\theta}\right) = 0$$

$$sin\theta\frac{dU}{d\theta} = a$$

$$dU = a\frac{d\theta}{sin\theta}$$

$$U(\theta) = a\int\frac{d\theta}{sin\theta}$$

Notes from the Calculus (Math.):

1. $\frac{1}{sin\theta}(= csc\theta) \neq sin^{-1}\theta(= ArcSin\theta)$
2. $\frac{1}{cos\theta}(= sec\theta) \neq cos^{-1}\theta(= ArcCos\theta)$
3. $sin\theta = 2sin\frac{\theta}{2}cos\frac{\theta}{2}$
4. $tan\theta = \frac{sin\theta}{cos\theta}$
5. $\int\frac{du}{u} = ln|u| + c$
6. $\int\frac{d\theta}{sin\theta} = \int csc\theta d\theta = ln\,|csc\theta + cot\theta| + C$

$$\therefore U(\theta) = a\int \frac{d\theta}{2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$multiply\ with$: $\dfrac{cos\frac{\theta}{2}}{cos\frac{\theta}{2}}$

$$= a\int \frac{\frac{1}{2}sec^2(\frac{\theta}{2})d\theta}{tan(\frac{\theta}{2})}$$

But: $\frac{1}{2}sec^2(\frac{\theta}{2}) = \frac{d\left[\tan(\frac{\theta}{2})\right]}{d\theta}$

$$\boxed{\therefore U(\theta) = aln\left|tan\frac{\theta}{2}\right| + b \quad . \ . \ . \quad (\#)}$$

This equation represents the solution of Laplace equation in spherical coordinates, in on dimension which is θ.

$1. U(\theta_1) = a\ ln\left(tan\frac{\theta_1}{2}\right) + b = 0$

$$\therefore b = -a\ ln\left(tan\frac{\theta_1}{2}\right) \quad . \ . \ . \quad (1)$$

$2. U(\theta_2) = a\ ln\left(tan\frac{\theta_2}{2}\right) + b = V_\circ$

$or: V_\circ = a\left[ln\left(tan\frac{\theta_2}{2}\right) - ln\left(tan\frac{\theta_1}{2}\right)\right]$

$$\boxed{\therefore a = \frac{V_\circ}{ln\left[\frac{tan\frac{\theta_2}{2}}{tan\frac{\theta_1}{2}}\right]} \quad . \ . \ . \quad (2)}$$

Sub in eq.1

$$\boxed{\therefore b = -\frac{V_\circ ln\left(tan\frac{\theta_1}{2}\right)}{ln\left[\frac{tan\frac{\theta_2}{2}}{tan\frac{\theta_1}{2}}\right]} \quad . \ . \ . \quad (3)}$$

Sub. of eqs. 1 and 3 in eq.(#) yield:

$$U(\theta) = V_\circ ln\left[\frac{tan\frac{\theta}{2}}{tan\frac{\theta_1}{2}}\right] - V_\circ ln\left[\frac{tan\frac{\theta_2}{2}}{tan\frac{\theta_1}{2}}\right]$$

$$\boxed{\therefore\ U(\theta) = V_\circ \frac{ln\left[\dfrac{tan\frac{\theta}{2}}{tan\frac{\theta_1}{2}}\right]}{ln\left[\dfrac{tan\frac{\theta_2}{2}}{tan\frac{\theta_1}{2}}\right]}}$$

$$\vec{E} = -\vec{\nabla}U$$

$$= -\frac{1}{r}\frac{dU}{d\theta}\hat{\theta}$$

<u>Note from the Calculus (Math.):</u>

1. If: $y = lnu$, then: $\acute{y} = \frac{1}{u}.\acute{u}$
2. If $tan(a\theta)$, then: $\frac{d}{d\theta}tan(a\theta) = \mathrm{Sec}(a\theta)^2.a$
3. $\mathrm{Sec}^2\theta = \tan^2\theta + 1$

$$\therefore \vec{E} = -\frac{1}{r}\frac{d}{d\theta}\left\{a\ ln\left(tan\frac{\theta}{2}\right) + b\right\}\hat{\theta}$$

$$= -\frac{a}{r}.\frac{1}{\tan(\frac{\theta}{2})}.sec^2(\frac{\theta}{2}).\frac{1}{2}\hat{\theta}$$

$$= -\frac{a}{r}.\frac{1}{\dfrac{sin\frac{\theta}{2}}{cos\frac{\theta}{2}}}.\frac{1}{cos^2\frac{\theta}{2}}.\frac{1}{2}\hat{\theta}$$

$$= -\frac{a}{r}.\frac{1}{2\sin(\frac{\theta}{2}).\cos(\frac{\theta}{2})}\hat{\theta}$$

$$\vec{E} = -\frac{a}{r\sin\theta}\hat{\theta}$$

$$\boxed{\vec{E} = -\frac{V_\circ}{r\ sin\theta\ ln\left\{\dfrac{tan\frac{\theta_2}{2}}{tan\frac{\theta_1}{2}}\right\}}\hat{\theta}}$$

<u>H.W.</u>

For the example above, assume that $\theta_1 = \frac{\lambda}{10}$, $\theta_2 = \frac{\lambda}{6}$ and $U_\circ = 50V$, calculate $U(\theta)$ and $\vec{E}$ between the two cones.

Ans.

$$U(\theta) = 95.1\ ln\left\{\frac{tan\frac{\theta}{2}}{0.158}\right\}\ (V)$$

$$\vec{E}(\theta) = -\frac{95.1}{rsin\theta}\hat{\theta}\ \ (V/m)$$

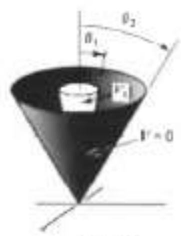


Fig. 8-10

and $$V = A\ln\left(\tan\frac{\theta}{2}\right) + B$$

The constants are found from

$$V_1 = A\ln\left(\tan\frac{\theta_1}{2}\right) + B \qquad 0 = A\ln\left(\tan\frac{\theta_2}{2}\right) + B$$

Hence $$V = V_1\frac{\ln\left(\tan\frac{\theta}{2}\right) - \ln\left(\tan\frac{\theta_2}{2}\right)}{\ln\left(\tan\frac{\theta_1}{2}\right) - \ln\left(\tan\frac{\theta_2}{2}\right)}$$

8.12. In Problem 8.11, let $\theta_1 = 10°$, $\theta_2 = 30°$, and $V_1 = 100$ V. Find the voltage at $\theta = 20°$. At what angle θ is the voltage 50 V?

Substituting the values in the general potential expression gives

$$V = -89.34\left[\ln\left(\tan\frac{\theta}{2}\right) - \ln 0.268\right] = -89.34\ln\left(\frac{\tan\frac{\theta}{2}}{0.268}\right)$$

Then, at $\theta = 20°$, $$V = -89.34\ln\left(\frac{\tan 10°}{0.268}\right) = 37.40\text{ V}$$

For $V = 50$ V, $$50 = -89.34\ln\left(\frac{\tan\theta/2}{0.268}\right)$$

Solving gives $\theta = 17.41°$.

8.13. With reference to Problems 8.11 and 8.12 and Fig. 8-11, find the charge distribution on the conducting plane at $\theta_2 = 90°$.

The potential is obtained by substituting $\theta_2 = 90°$, $\theta_1 = 10°$, and $V_1 = 100$ V in the expression of Problem 8.11. Thus

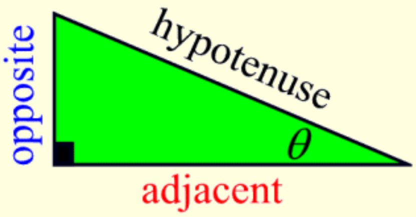
$$V = 100\frac{\ln\left(\tan\frac{\theta}{2}\right)}{\ln(\tan 5°)}$$

Then $$\mathbf{E} = -\frac{1}{r}\frac{dV}{d\theta}\mathbf{a}_\theta = \frac{-100}{(r\sin\theta)\ln(\tan 5°)}\mathbf{a}_\theta = \frac{41.05}{r\sin\theta}\mathbf{a}_\theta$$

$$\mathbf{D} = \epsilon_0\mathbf{E} = \frac{3.63\times10^{-10}}{r\sin\theta}\mathbf{a}_\theta\quad (\text{C/m}^2)$$

In a right triangle, there are actually six possible trigonometric ratios, or functions.

A Greek letter (such as theta θ or phi φ) will now be used to represent the angle.



sine of $\theta = \sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	cosecant of $\theta = \csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$
cosine of $\theta = \cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	secant of $\theta = \sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$
tangent of $\theta = \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	cotangent of $\theta = \cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

Notice that the three new ratios at the right are reciprocals of the ratios on the left.

Applying a little algebra shows the connection between these functions.

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\dfrac{\text{opposite}}{\text{hypotenuse}}} = \frac{1}{\sin\theta}$$

Reciprocal Functions	
$\sin\theta = \dfrac{1}{\csc\theta}$	$\csc\theta = \dfrac{1}{\sin\theta}$
$\cos\theta = \dfrac{1}{\sec\theta}$	$\sec\theta = \dfrac{1}{\cos\theta}$
$\tan\theta = \dfrac{1}{\cot\theta}$	$\cot\theta = \dfrac{1}{\tan\theta}$

Also Important
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$$\frac{\sin\theta}{\cos\theta} = \frac{opp/hyp}{adj/hyp} = \frac{opp}{adj} = \tan\theta$$

http://www.regentsprep.org/Regents/math/algtrig/ATT1/trigsix.htm