

Chapter Five

Electric Current

Up to this point we have been dealing with *charge at rest*, so in this chapter we shall consider *charges in motion*. Strictly speaking we will deal with *conductors of electricity*. In other word we will regard the material in which the *charges carriers are free* to move. It should keep in mind that conductors include not only the conventional conductors such as *metals* and alloys, but also *semiconductors, electrolytes, ionized gases, imperfect dielectrics* and even *vacuum* in the vicinity of a thermionic emitting cathode. In many conductors the charge carriers are *electrons*; in other cases the charge may be carried positive or negative ions.

Moving charge constitutes a **current**, and the process whereby charge is transported is called **electrical conduction** (σ or g) (the thermal conduction is another different concept). To be precise the current, I , is defined as *the rate at which charge is transported past a given point in conducting system*. Thus;

$$I = \frac{dQ}{dt} \quad \dots (6 - 1)$$

where $Q = Q(t)$ is the net charge transported in time t .

The unit of current in the MKS system is the Ampere. Evidently;

$$1(\text{Ampere}) = 1 \frac{\text{Coulomb}}{\text{Second}}$$

6-1: Nature of the current

- In a metal, current is carried entirely by electrons, while the heavy positive ions are *fixed* at regular positions in the crystal structure; see figure (6-1). Only the *valence* (outermost) atomic electrons are free to participate in the conduction process; the other electrons are tightly

bound to their positions. Under steady state conditions, electrons may be fed into the metal at one point and removed at another, producing a current, but the metal as a whole is electrostatically neutral.

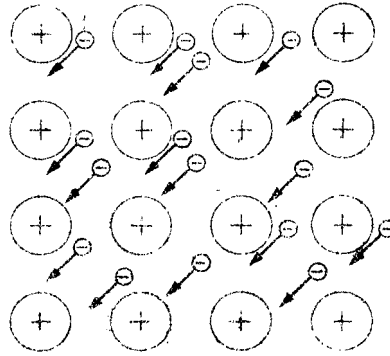


Fig. (6-1):

A schematic diagram of the motion of conduction electrons in a metal.

- In an *electrolyte*, the current is carried by both positive and negative ions, although, because some ions move faster than others, conduction by one type of ion usually predominates. It is important to note that positive and negative ions traveling in opposite directions, see figure (6-2) contribute to the current in the same direction. The basis for this fact is evident from equation (6-1), since the net charge transported past a given point depends on both the sign of the charge carrier and the direction in which it is moving. Thus, in figure (6-2), both the positive and negative carrier groups produce currents to the right; by convention, the direction in which the positive carrier moves (or, equivalently, the direction opposite to that in which the negative carrier moves) is taken as the direction, or sense, of the current. In general, an electric current arises in response to an electric field. If an electric field is imposed on a conductor, it will cause positive charge carriers to move in the general direction of the field and negative carriers in a direction opposite to the

field; hence all currents produced in the process have the same direction as the field.

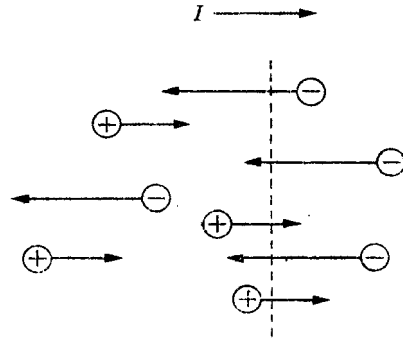


Fig. (6-2):

A schematic diagram of the motion of conduction electrons in an electrolyte.

The currents we have described thus far in this section are known as **conduction currents**. These currents represent the drift motion of charge carriers through the medium which usually be at rest.

- Liquids and gases may also undergo hydrodynamic motion, and if the medium has a charge density, this hydrodynamic motion will produce currents. Such currents, arising from mass transport called **convection currents**. Convection currents are important to the subject of *atmospheric electricity*. The motion of charged particles in vacuum (such as electrons in a vacuum diode) also constitutes convection currents.

6-2: Current Density: Equation of Continuity

Let us consider a conducting medium which has only one type of charge carrier, of charge q . The number of these carriers per unit volume will be denoted by N . However, the drift velocity for each carrier is v . We are now in a position to calculate the current through an element of area da such as shown in figure (6-3). During the time δt each carrier moves a distance $v\delta t$. From the figure it is evident that the charge amount δQ which crosses da during time δt is q times the sum of all charge carriers in the

volume ($v \cdot \hat{n} \delta t da$), where n is a unit vector normal to the area da , v is the drift velocity. From equation (6-1) we have;

$$dI = \frac{\delta Q}{\delta t} = \frac{qNv \cdot \hat{n} \delta t da}{\delta t} = Nqv \cdot nda \quad \dots (6-2)$$

If there is more than one kind of charge carrier present, there will be a contribution of the form (6-2) for each type of carrier. In general;

$$dI = \left[\sum_i N_i q_i v_i \right] \cdot nda \quad \dots (6-3)$$

represents the current through the area da . The summation is over the different carrier types. The quantity in bracket is a vector which has dimensions of current per unit area; this quantity is called the *current density*, and is given by the symbol J , where;

$$J = \sum_i N_i q_i v_i \quad \dots (6-4)$$

The current density J may be defined at each point in the conducting medium and is, therefore, a *vector point function*. The MKS unit of J is A/m². Equation (6-3) may be written as;

$$dI = J \cdot nda$$

and the current through the surface S , an arbitrary shaped surface area of macroscopic size, is given by:

$$I = \int_S J \cdot nda \quad \dots (6-5)$$

The current density J and the charge density ρ are not independent quantities, but are related at each point through a differential equation, the so-called ***equation of continuity*** (*the total amount (of the conserved quantity) inside any region can only change by the amount that passes in or out of the region through the boundary*), A conserved quantity cannot increase or decrease, it can only move from place to

place. This relationship has its origin in the fact that *charge can neither be created nor destroyed*. The electric current entering V , the volume enclosed by S , is given by;

$$I = - \oint_S \mathbf{J} \cdot \mathbf{n} \, da = - \int_V \text{div } \mathbf{J} \, dv \quad . . . \quad (6 - 6)$$

The minus sign in equation (6 – 6) comes about because n is the outward normal and we wish to call I positive when the net flow of charge is from the outside of V to within. From (6-1) I is equal to *the rate at which charge is transported* into V .

$$I = \frac{dQ}{dt} = \frac{d}{dt} \int_V \rho \, dV \quad . . . \quad (6 - 7a)$$

Since we are dealing with a fixed volume V , the time derivative operates only on the function ρ . However, ρ is a function of position as well as of time. So that the time derivative becomes the partial derivative with respect to time when it is moved inside the integral. Hence;

$$I = \int_V \frac{\partial \rho}{\partial t} \, dV \quad . . . \quad (6 - 7b)$$

Equation (6 – 6) and (6 – 7b) may now be equated,;

$$\int_V \left(\frac{\partial \rho}{\partial t} + \text{div } \mathbf{J} \right) dV = 0 \quad . . . \quad (6 - 8)$$

But V is completely arbitrary and the way that (6 – 8) can hold for an arbitrary volume segment of the medium is for the integrand to *vanish* at each point. Hence, the equation of continuity, as a *differential form* is:

$$\frac{\partial \rho}{\partial t} + \text{div } \mathbf{J} = 0 \quad . . . \quad (6 - 9)$$

Example: 6.10 @ Schaum p.88

6-3: Ohm's Law: Conductivity

It is found experimentally that in a *metal* at constant temperature the current density J is linearly proportional to the electric field (Ohm's law).

Thus:

$$J = gE \quad . . . \quad (6 - 10)$$

Where g is the proportionality constant and it is called **conductivity**. In order to make equation (6 – 10) valid for all conducting materials, it must be replaced by;

$$J = g(E)E$$

where $g(E)$ is a function of the electric field.

Materials for which equation (6 – 10) holds are called *linear media* or *Ohmic media*. The reciprocal of the conductivity is called the **resistivity** η , thus;

$$\eta = \frac{1}{g} \quad . . . \quad (6 - 11)$$

The unit of η in the *mks* system is (Volt-meters/Ampere) or Ohm-meters, where the Ohm is defined by;

$$1 \text{ Ohm} = \frac{\text{Volt}}{\text{Ampere}}$$

The unit of conductivity g is, therefore, $\text{Ohm}^{-1}\text{m}^{-1}$.

Consider a conducting specimen obeying Ohm's law, in the shape of a straight wire of uniform cross section whose ends are maintained at a constant potential difference ΔU . The wire is assumed to be homogeneous and characterized by the constant conductivity g . Under these conditions an electric field will exist in the wire, the field related to ΔU by the relation

$$\Delta U = \int E \cdot dl \quad . . . \quad (612a)$$

It is evident that the electric field is purely longitudinal, because of the geometry, the electric field must be the same at all points along the wire.

Therefore, equation (6-12a) reduces to;

$$\Delta U = E\ell \quad . . . \quad (6 - 13)$$

where ℓ is the length of the wire.

But the electric field implies a current of density $J = g E$. Thus the current through any cross section of the wire is;

$$I = \int_A J \cdot nda = JA \quad . . . \quad (6 - 14)$$

Where A is the cross section area of the wire.

Combining equations (6-14) with (6-10) and (6-13) we obtain;

$$I = \frac{gA}{\ell} \Delta U \quad . . . \quad (6 - 14)$$

which provides a linear relationship between I and ΔU . The quantity ℓ/gA is called the *resistance* of the wire resistance which will be denoted by the

symbol R . Therefore, equation (6 - 14) becomes;

$$\Delta U = RI \quad . . . \quad (6 - 15)$$

which is the familiar form of Ohm's law (R is evidently measured in units of Ohms).

Equation (6-15) may be considered to be a definition of the resistance of an object or device that is passing a constant current. In the general case, R will depend upon the value of this current.

6-4: Electromotive Force

$$\oint E_{eff} \cdot dl = \varepsilon \quad . . . \quad (6 - 16)$$

The quantity ε is called the *electromotive force* or simply the *emf*, represents the driving force for the current in a closed circuit. The unit of *emf* in the *mks* system is joules/coulomb, or Volt (the same as the unit for potential).

electromotive force, emf, (denoted ε and measured in volts) refers to voltage generated by a battery or by the magnetic force according to Faraday's law, which states that a time varying magnetic field will induce an electric current.

Electromotive "force" is not a force (measured in Newtons) but a *potential*, or *energy per unit of charge*, measured in volts. Formally, emf is the external work expended per unit of charge to produce an electric potential difference across two open-circuited terminals. The electric potential difference produced is created by separating positive and negative charges, thereby generating an electric field. The created electrical potential difference drives current flow if a circuit is attached to the source of emf. When current flows, however, the voltage across the terminals of the source of emf is no longer the open-circuit value, due to voltage drops inside the device due to its internal resistance.

Devices that can provide emf include electrochemical cells, thermoelectric devices, solar cells, electrical generators, transformers, and even Van de Graaff generators. In nature, emf is generated whenever magnetic field fluctuations occur through a surface. An example for this is the varying Earth magnetic field during a geomagnetic storm, acting on anything on the surface of the planet, like an extended electrical grid.

In chapter two it was shown that the integral of the tangential component of an electrostatic field around any closed path vanishes; i.e.

$$\oint E \cdot dl = 0$$

for an Ohmic material, $J = gE$.

In general case this is modified to $J = g(E)E$, but $g(E)$ is always a positive quantity. Thus it follows that a purely electrostatic force cannot cause a current to circulate in the same sense around an entire circuit. Or, in other words, steady current cannot be maintained by means of purely electrostatic forces.

A charged particle q may experience other forces (mechanical, chemical, *etc.*) in addition to the electrostatic force. If the total force per unit charge on a charged particle is called the effective electric field E_{eff} , then the above line integral will not necessarily vanish. *i.e.*

6-5: Steady currents in media without sources of *emf*

Consider a homogenous, Ohmic, conducting medium without internal sources of *emf*, under conditions of steady-state condition. Since the case is a steady state, the local charge density $\rho(x, y, z)$ is at equilibrium value, and $\partial\rho/\partial t = 0$ for each point in the medium. Hence, the equation of continuity (6 – 9) reduces to;

$$\text{div}J = 0 \quad . . . \quad (6 - 17)$$

Using ohm's law in combination with (6-17), we obtain;

$$\text{div} gE = 0$$

But with no sources of *emf*, E is derivable from a scalar potential. *i.e.*

$$E = -\nabla U$$

The combination of the last two equations yields;

$$\nabla^2 U = 0 \quad . . . \quad (6 - 18)$$

which is Laplace's equation.

Therefore, the steady-state conduction problem may be solved in the same way as electrostatic problems. Laplace's equation is solved by one of the techniques discussed in chapter three.

Under steady-state conduction the current which crosses an interfacial area between two conducting media may be computed in two ways: in terms of the current density in medium 1, or in terms of the current density in medium 2. Since the two procedures must yield the same result, the normal component of \mathbf{J} must be continuous across the interface. *i.e.*

$$J_{1n} = J_{2n} \dots (6 - 19a)$$

or: $g_1 E_{1n} = g_2 E_{2n} \dots (6 - 19b)$

So long as there are no sources of *emf* in either medium

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

For a closed path which links both media, and

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$$E_{1t} = E_{2t}$$

الصفحات:

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اعطيت للطلبة كمسائل محلولة

- 6.3.** A long copper conductor has a circular cross section of diameter 3.0 mm and carries a current of 10 A. Each second, what percent of the conduction electrons must leave (to be replaced by others) a 100 mm length?



Avogadro's number is $N = 6.02 \times 10^{26}$ atoms/kmol. The specific gravity of copper is 8.96 and the atomic weight is 63.54. Assuming one conduction electron per atom, the number of electrons per unit volume is

$$N_e = \left(6.02 \times 10^{26} \frac{\text{atoms}}{\text{kmol}}\right) \left(\frac{1 \text{ kmol}}{63.54 \text{ kg}}\right) \left(8.96 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right) \left(1 \frac{\text{electron}}{\text{atom}}\right)$$

$$= 8.49 \times 10^{28} \text{ electrons/m}^3$$

The number of electrons in a 100 mm length is

$$N = \pi \left(\frac{3 \times 10^{-3}}{2}\right)^2 (0.100) (8.49 \times 10^{28}) = 6.00 \times 10^{22}$$

A 10-A current requires that

$$\left(10 \frac{\text{C}}{\text{s}}\right) \left(\frac{1}{1.6 \times 10^{-19}} \frac{\text{electron}}{\text{C}}\right) = 6.25 \times 10^{19} \text{ electrons/s}$$

pass a fixed point. Then the percent leaving the 100 mm length per second is

$$\frac{6.25 \times 10^{19}}{6.00 \times 10^{22}} (100) = 0.104\% \text{ per s}$$

$$N_A = \frac{\text{number of atoms}}{\text{mole per gram}}$$

$$6.023 \times 10^{23} \text{ mol}$$

$$N_A = N/n \text{ عدد الذرات او الجزيئات الى كمية}$$

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