



Chapter 5

Multi Layer Feed-forward Network

4th Class

INTELLIGENT APPLICATIONS

التطبيقات الذكية

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Multi Layer Feed-forward Network

5.1 Introduction

5.2 Example



5.1 Introduction

Multi Layer Feed-forward Network, it consists of multiple layers. The architecture of this class of network, besides having the input and the output layers, also have one or more intermediary layers called *hidden layers*. The computational units of the hidden layer are known as *hidden neurons*.

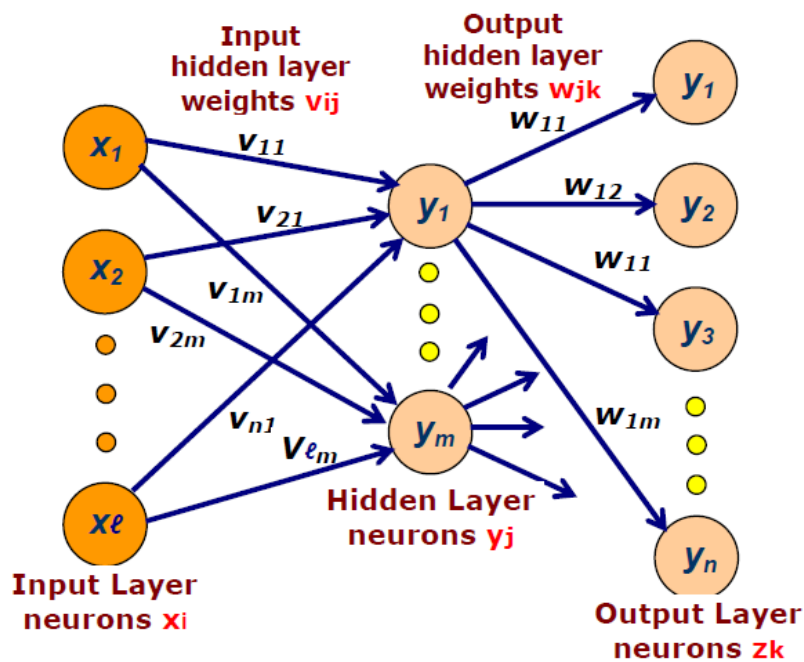


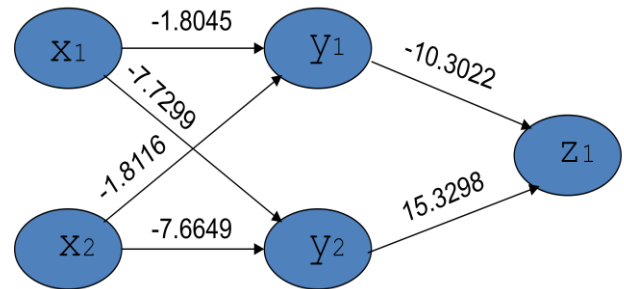
Fig. Multilayer feed-forward network in $(\ell - m - n)$ configuration.



5.2 Example

Consider Network Architecture in below figure, Find the value of (Z₁, Error) using Sigmoid function when inputs:

Inputs		Goal outputs (d)	Z ₁	Error
X ₁	X ₂	O _{desired}	O _{actual}	E
0	0	1	?	?
0	1	0	?	?
1	0	0	?	?
1	1	1	?	?



Sol:

INPUT #1:

When (X₁=0, X₂=0)

$$\begin{aligned}
 u_1 &= \sum W_{ij} * X_j \\
 &= (V_{11} * X_1 + V_{21} * X_2) \\
 &= ((-1.8045 * 0) + (-1.8116 * 0)) \\
 &= (0+0) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 u_2 &= \sum W_{ij} * X_j \\
 &= (V_{12} * X_1 + V_{22} * X_2) \\
 &= ((-7.7299 * 0) + (-7.6649 * 0)) \\
 &= (0+0) \\
 &= 0
 \end{aligned}$$

$$F(u) = 1 / (1 + e^{-x}) \quad (\text{Sigmoid function})$$

$$F(u_1) = 1 / (1 + e^{-u_1})$$

$$\begin{aligned}
 F(u_1) &= y_1 = 1 / (1 + e^0) && \text{note: } e^0 = 1, \\
 &= 1 / (1 + 1) \\
 &= 0.5
 \end{aligned}$$

Inputs		Goal outputs (d)	Z ₁	Error
X ₁	X ₂	O _{desired}	O _{actual}	E
0	0	1	0.925	0.075
0	1	0	0.1912	0.1912
1	0	0	0.1891	0.1891
1	1	1	0.4329	0.5671

NOTE¹ :

Goal output=desired output=Target
 الهدف وهي المخرجات التي يحددها المصمم
 او تسمى النتائج المرغوبة او النتائج المطلوبة

NOTE² :

Actual output= وهي المخرجات التي تظهر
 اثناء الحسابات على الشبكة او تسمى النتائج
 الحقيقية للشبكة



$$F(u_2) = 1 / (1 + e^{-u_2})$$

$$\begin{aligned} F(u_2) &= y_2 = 1 / (1 + e^0) \\ &= 1 / (1 + 1) \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} u_3 &= \sum W_{ij} * y_j \\ &= (W_{11} * y_1 + W_{21} * y_2) \\ &= ((-10.3022 * 0.5) + (15.3298 * 0.5)) \\ &= (-5.1511 + 7.6649) \\ &= 2.5138 \end{aligned}$$

$$F(u_3) = 1 / (1 + e^{-u_3})$$

$$\begin{aligned} F(u_3) &= Z_1 = 1 / (1 + e^{-2.5138}) \\ &= 0.925 \end{aligned}$$

note: $e^{-2.5138} = 0.0809$

$$\begin{aligned} \text{Error} = \Delta_i &= \mathbf{E} = / \mathbf{O}_{\text{desired}} - \mathbf{O}_{\text{actual}} / \\ E_1 &= / 1 - 0.925 / \\ &= 0.075 \end{aligned}$$

$$\begin{aligned} \text{Error signal} = \Delta_i &= \mathbf{E} = / \mathbf{O}_{\text{desired}} - \mathbf{O}_{\text{actual}} / \\ \mathbf{E} &= / \mathbf{d} - \mathbf{z}_1 / \end{aligned}$$

INPUT #2:

When ($X_1=0, X_2=1$)

$$\begin{aligned} u_1 &= \sum W_{ij} * X_j \\ &= (V_{11} * X_1 + V_{21} * X_2) \\ &= ((-1.8045 * 0) + (-1.8116 * 1)) \\ &= (0 + (-1.8116)) \\ &= -1.8116 \end{aligned}$$

$$\begin{aligned} u_2 &= \sum W_{ij} * X_j \\ &= (V_{12} * X_1 + V_{22} * X_2) \\ &= ((-7.7299 * 0) + (-7.6649 * 1)) \\ &= (0 + (-7.6649)) \\ &= -7.6649 \end{aligned}$$



$$F(u) = 1 / (1 + e^{-x}) \quad (\text{Sigmoid function})$$

$$F(u_1) = 1 / (1 + e^{-u_1})$$

$$\begin{aligned} F(u_1) = y_1 &= 1 / (1 + e^{-(-1.8116)}) \\ &= 1 / (1 + e^{1.8116}) \\ &= 1 / (1 + 6.1202) \\ &= 1 / 7.1202 \\ &= 0.140 \end{aligned}$$

note: $e^{1.8116} = 6.1202$

$$F(u_2) = 1 / (1 + e^{-u_2})$$

$$\begin{aligned} F(u_2) = y_2 &= 1 / (1 + e^{-(-7.6649)}) \\ &= 1 / (1 + e^{7.6649}) \\ &= 1 / (1 + 2132.179) \\ &= 1 / 2133.179 \\ &= 4.687839136^{-04} \\ &= 0.00004 \end{aligned}$$

note: $e^{7.6649} = 2132.179$

$$\begin{aligned} u_3 &= \sum W_{ij} * y_j \\ &= (W_{11} * y_1 + W_{21} * y_2) \\ &= ((-10.3022 * 0.140) + (15.3298 * 0.00004)) \\ &= (-1.442308 + 6.13192^{-04}) \\ &= (-1.442308 + 0.00006) \\ &= -1.442248 \end{aligned}$$

$$F(u_3) = 1 / (1 + e^{-u_3})$$

$$\begin{aligned} F(u_3) = Z_1 &= 1 / (1 + e^{-(-1.442248)}) \\ &= 1 / (1 + e^{1.442248}) \\ &= 1 / (1 + 4.2302) \\ &= 1 / 5.2302 \\ &= 0.1912 \end{aligned}$$

note: $e^{1.442248} = 4.2302$

$$\text{Error} = \Delta_i = \mathbf{E} = / \mathbf{O}_{\text{desired}} - \mathbf{O}_{\text{actual}} /$$

$$\begin{aligned} E_2 &= / 0 - 0.1912 / \\ &= / - 0.1912 / \\ &= 0.1912 \end{aligned}$$



INPUT #3:

When ($X_1=1, X_2=0$)

$$\begin{aligned}u_1 &= \sum W_{ij} * X_j \\&= (V_{11} * X_1 + V_{21} * X_2) \\&= ((-1.8045 * 1) + (-1.8116 * 0)) \\&= (-1.8045 + 0) \\&= -1.8045\end{aligned}$$

$$\begin{aligned}u_2 &= \sum W_{ij} * X_j \\&= (V_{12} * X_1 + V_{22} * X_2) \\&= ((-7.7299 * 1) + (-7.6649 * 0)) \\&= (-7.7299 + 0) \\&= -7.7299\end{aligned}$$

$$F(u) = 1 / (1 + e^{-x}) \quad (\text{Sigmoid function})$$

$$F(u_1) = 1 / (1 + e^{-u_1})$$

$$\begin{aligned}F(u_1) = y_1 &= 1 / (1 + e^{-(-1.8045)}) \\&= 1 / (1 + e^{1.8045}) \\&= 1 / (1 + 6.0769) \\&= 1 / 7.0769 \\&= 0.1413\end{aligned}$$

$$\text{note: } e^{1.8045} = 6.0769$$

$$F(u_2) = 1 / (1 + e^{-u_2})$$

$$\begin{aligned}F(u_2) = y_2 &= 1 / (1 + e^{-(-7.7299)}) \\&= 1 / (1 + e^{7.7299}) \\&= 1 / (1 + 2,275.375) \\&= 4.3948^{-04} \\&= 0.00004\end{aligned}$$

$$\text{note: } e^{7.7299} = 2,275.375$$

$$\begin{aligned}u_3 &= \sum W_{ij} * y_j \\&= (W_{11} * y_1 + W_{21} * y_2) \\&= ((-10.3022 * 0.1413) + (15.3298 * 0.00004)) \\&= (-1.4557 + 6.13192^{-04}) \\&= (-1.442308 + 0.00006) \\&= -1.4556\end{aligned}$$



$$F(u_3) = 1 / (1 + e^{-u_3})$$

$$\begin{aligned} F(u_3) = Z_1 &= 1 / (1 + e^{-(-1.4556)}) \\ &= 1 / (1 + e^{1.4556}) \\ &= 1 / (1 + 4.2870) \\ &= 1 / 5.2870 \\ &= 0.1891 \end{aligned}$$

note: $e^{1.4556} = 4.2870$

$$\begin{aligned} E_3 &= / 0 - 0.1891 / \\ &= / - 0.1891 / \\ &= 0.1891 \end{aligned}$$

INPUT #4:

When ($X_1=1, X_2=1$)

$$\begin{aligned} u_1 &= \sum W_{ij} * X_j \\ &= (V_{11} * X_1 + V_{21} * X_2) \\ &= ((-1.8045 * 1) + (-1.8116 * 1)) \\ &= -3.6161 \end{aligned}$$

$$\begin{aligned} u_2 &= \sum W_{ij} * X_j \\ &= (V_{12} * X_1 + V_{22} * X_2) \\ &= ((-7.7299 * 1) + (-7.6649 * 1)) \\ &= -15.3948 \end{aligned}$$

$$F(u) = 1 / (1 + e^{-x}) \quad (\text{Sigmoid function})$$

$$F(u_1) = 1 / (1 + e^{-u_1})$$

$$\begin{aligned} F(u_1) = y_1 &= 1 / 1 / (1 + e^{-(-3.6161)}) \\ &= 1 / 1 / (1 + e^{3.6161}) \\ &= 1 / (1 + 37.1922) \\ &= 1 / 38.1922 \\ &= 0.0262 \end{aligned}$$

note: $e^{3.6161} = 37.1922$



$$F(u_2) = 1 / (1 + e^{-u_2})$$

$$\begin{aligned} F(u_2) = y_2 &= 1 / (1 + e^{-(-15.3948)}) \\ &= 1 / (1 + e^{15.3948}) \\ &= 1 / (1 + 48515) \\ &= 1 / 48516 \\ &= 2.0611^{-05} \\ &= 0.000002 \end{aligned}$$

note: $e^{15.3948} = 48515$

$$u_3 = \sum W_{ij} * y_j$$

$$\begin{aligned} &= (W_{11} * y_1 + W_{21} * y_2) \\ &= ((-10.3022 * 0.0262) + (15.3298 * 0.000002)) \\ &= (-0.2699 + 3.0659^{-05}) \\ &= (-0.2699 + 0.000003) \\ &= -0.2699 \end{aligned}$$

$$F(u_3) = 1 / (1 + e^{-u_3})$$

$$\begin{aligned} F(u_3) = Z_1 &= 1 / (1 + e^{-(-0.2699)}) \\ &= 1 / (1 + e^{0.2699}) \\ &= 1 / (1 + 1.3098) \\ &= 1 / 2.3098 \\ &= 0.4329 \end{aligned}$$

note: $e^{0.2699} = 1.3098$

$$\text{Error} = \Delta_i = \mathbf{E} = \mathbf{O}_{\text{desired}} - \mathbf{O}_{\text{actual}} /$$

$$\begin{aligned} E_4 &= / 1 - 0.4329 / \\ &= 0.5671 \end{aligned}$$



Homework :

Q: Consider Network Architecture in below figure, Find the value of (Z_1 , Error) using Sigmoid function for hidden units (hidden layer) and linear function for output units (output layer) when inputs:

Inputs		Goal outputs (d)	Z_1	Error
X_1	X_2	$O_{desired}$	O_{actual}	E
0	0	1		
0	1	0		
1	0	0		
1	1	1		

