[محاضرة رقم (2)
$a_4 \rightarrow a_8$]

* Relation among the events:-

① If the event A has no elements, A is called the null event $\Rightarrow A = \phi$

② The Union of two events A_1, A_2 is the events of all elements in A_1 and A_2 and denoted by:-

$A_1 \cup A_2$ or $(A_1 \text{ or } A_2)$.

and the Union of several events $A_1, A_2, \ldots, A_K$ is the event of all elements in $A_1, A_2, A_3, \ldots$ and A_K and denoted by:- $A_1 \cup A_2 \cup A_3 \cdots \cup A_K$.

EX.s:- ① Let $A_1 = \{0, 1, \ldots, 10\}$, $A_2 = \{8, 9, 10, 11\}$

Then $A_1 \cup A_2 = \{0, 1, 2, \ldots, 10, 11\}$.

② Let $A_1 = \{0, 1, 2, \ldots, 10\}$, $A_2 = \phi$

Then $A_1 \cup A_2 = A_1$

③ The Intersection of two events A_1, A_2 is the event of is belong to each of the two events A_1 and A_2 and denoted by:-

$A_1 \cap A_2$ or $(A_1 \text{ and } A_2)$.

and the Intersection for the several events $A_1, A_2, A_3, \ldots, A_K$ the Intersection denoted by $A_1 \cap A_2 \cap A_3 \cdots \cap A_K$.

EX.s:- ① Let $A_1 = \{(0,0), (0,1), (1,1)\}$ and
$A_2 = \{(1,1), (1,2), (2,1)\}$ then the Intersection

$\Rightarrow A_1 \cap A_2 = \{(1,1)\}$

② $A \cap \phi = \phi$

④ The complement of an event A denoted by A^c is the set of all elements in sample space that are not in A.

H.W:- A die is thrown once. suppose:-
A_1: even numbers.
A_2: Numbers greater than 3.
A_3: odd numbers.
Find $A_1 \cap A_2$. $A_1 \cap A_3$. $A_1 \cup A_2$ - A_1^c.

** Probability Axioms:-
For A, B, C events:-

① $p(A \cup B) = p(B \cup A)$ and $p(A \cap B) = p(B \cap A)$.

② $p[A \cup (B \cup C)] = p[(A \cup B) \cup C]$.

$p[A \cap (B \cap C)] = p[(A \cap B) \cap C]$

③ $p[A \cap (B \cup C)] = p[(A \cap B) \cup (A \cap C)]$

$p[A \cup (B \cap C)] = p[(A \cup B) \cap (A \cup C)]$

④ $A \cup \phi = A$; $A \cap \phi = \phi$

$A \cup S = S$; $A \cap S = A$

⑤ $A \cup A^c = S$; $A \cap A^c = \phi$

$S^c = \phi$; $\phi^c = S$; $(A^c)^c = A$

⑥ $p[(A \cup B)^c] = p[A^c \cap B^c]$

$p[(A \cap B)^c] = p[A^c \cup B^c]$.

* Methods of Count:- طرق العد.

— There are two mathimatical methods which help us to know how many possible ways for any event.

[1] Combinations:- التوافيق

— Let n be denote collection of any thing and we selected $\underline{r}$ from them, then $\underline{r}$ occure as:-

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$ (نهتم بالتعويض دون الأخذ بنظر الاعتبار الترتيب).

In general or for taken all together. Then:.

$$C_n^n = \frac{n!}{n!\,(n-n)!} = 1$$ and for $r=1 \Rightarrow C_1^n = \frac{n!}{1!\,(n-1)!} = n$

where:- $n! = n(n-1)(n-2)(n-3)\cdots(n-r+1)$

$0! = 1$ and $1! = 1$

[2] Permutations:- التباديل

— Is the arrangment of n from the diffrent things in groups. each one contain $\underline{r}$ of different things too, Then:-

$$P_r^n = \frac{n!}{(n-r)!}$$; and if $r=1 \Rightarrow P_1^n = \frac{n!}{(n-1)!} = n$

(مراعاة الترتيب)

if $r=n \Rightarrow P_n^n = \frac{n!}{(n-n)!} = n!$

EX. 5:-

① A boy has five coins. How many different sums of mony occure head when he turn up.

sol/ $C_1^5 + C_2^5 + C_3^5 + C_4^5 + C_5^5 = 31$

② It is required to seat 5 mean and 4 women in a row. How many such arrangements are possible?

sol/ $P_5^5 \cdot P_4^4 = 2880$

③ A student is to answer 8 out of 10 quations in an exam:-

ⓐ How many choices has he?

ⓑ How many choises if he must answer the first three quations.

ⓒ How many choices if he must answer atleast four of the first five quations.

sol/:- ⓐ $C_8^{10} = 45$

ⓑ $C_3^3 \cdot C_5^7 = 21$

ⓒ $C_4^5 \cdot C_4^5 + C_5^5 \cdot C_3^5 = 35$

④ How many ways to divide 12 persons to 3 groups.

sol/ $P_3^{12} = 1320$

ملاحظة:- في بعض الاحيان يراد معرفة ترتيب مجموعة من العناصر تكون بعضها متماثلة اي ان عدد التباديل n في r_1 تختلف عن r_2 وتختلف عن r_3 --- وهكذا ففي هذه الحالة تحسب التباديل كما يلي:-

$$P^{n}_{r_1 \cdot r_2 \cdot r_3} = \frac{n!}{r_1! \cdot r_2! \cdot r_3! \cdots r_k!}.$$

$$\text{where } r_1 + r_2 + \cdots + r_k = n$$

Example:- ① If the student on the boat have 3 red 4 yellow and 2 blue flags to arrange. How many such arrangement are possible.

sol/:-
$$\frac{9!}{3! \cdot 4! \cdot 2!} = 1260$$

Example:- ② Find the number of Permutation for all Latters of the words. How many words can be made from:-

ⓐ queue ⓑ Comittee.

sol/

ⓐ $$\frac{5!}{1! \cdot 2! \cdot 2!} = 30$$

ⓑ $$\frac{8!}{1! \cdot 1! \cdot 1! \cdot 1! \cdot 2! \cdot 2!} = 10080.$$