* Law's of Probability

① $\sum P(A_i) = 1 \qquad i = 1, 2, \dots, k$

② $P(A^c) = 1 - P(A) \Rightarrow P(A) + P(A^c) = 1$

③ For every event A, $0 \leq P(A) \leq 1$

④ $P(\varphi) = 0$ for any sample space.

⑤ if $A \subset B \Rightarrow P(A) \leq P(B)$

⑥ For any two events A and B:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Note:

a if A, B are mutually exclusive events, then

$$P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$$

b if A, B are independent events, then

$$P(A \cap B) = P(A) \cdot P(B)$$

⑦ In a sample space, if we have the events A, B and C, then:

$$P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

and if they are independent, then

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

⑧ $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B)$

and $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B)$

Exs

1 A box contains 10 red and 8 white balls, we selected at random 5 balls. Find the Prob. that:

a) Three red and two white.

b) all five balls are red.

Sol./ $S = C_5^{18} = 8568$

a) $A = C_3^{10} \cdot C_2^{8} = 3360 \Rightarrow P(A) = \frac{3360}{8568} = 0.39$

b) $B = C_5^{10} = 252 \Rightarrow P(B) = \frac{252}{8268} = 0.029$

2 Find the Prob. of:

a) The number 3 or 4 occure in a die.

$S = P_1^6 = 6$ and $A = \{3,4\} \Rightarrow P(A) = \frac{2}{6}$

b) At least one head appears in two Coins.

$S = \{HH, HT, TH, TT\}$

$A = C_1^2 + C_2^2 = 3 \Rightarrow P(A) = \frac{3}{4}$

c) An even no. appear in a die.

$S = \{1,2,3,4,5,6\}$

$A = \{2,4,6\} \Rightarrow P(A) = \frac{3}{6}$

d) Selected 2 ball from box contain 7 red and 5 white if(the 2 are red).

$S = C_2^{12} = 66$

$A = C_2^{7} = 21 \Rightarrow P(A) = \frac{21}{66}$ (خواي)

3 A number chossen randomly from the numbers between 11-40, find the prob. that the no. is either 16 or an even number.

sol./ $S = C_1^{30} = 30 \Rightarrow A = \{16\} \Rightarrow P(A) = \frac{16}{30}$

$B = \{12, 14, 16, \dots, 40\} \Rightarrow P(B) = \frac{15}{30}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$A \cap B = \{16\}$

$\Rightarrow P(A \cap B) = \frac{1}{30}$

$\Rightarrow P(A \text{ or } B) = \frac{1}{30} + \frac{15}{30} - \frac{1}{30} = \frac{15}{30}$

4 Two dice are thrown once. what the prob. that.

a) The sum of both faces is 8.

b) The no. on the first face similer (آلاوز) on the other

c) The sum of both faces are 6 or 9

sol./ $S = \{1, 2, \dots, 6, 1, 2, \dots, 6\}$ $n = 36$

a) $A = \{(2,6), (3,5), (4,4), (6,2), (5,3)\} \Rightarrow P(A) = \frac{5}{36}$

b) $B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\} \Rightarrow P(B) = \frac{6}{36}$

c) $C = \{(1,5), (2,4), (3,3), (5,1), (4,2)\} \Rightarrow P(C) = \frac{5}{36}$

$D = \{(3,6), (6,3), (4,5), (5,4)\} \Rightarrow P(D) = \frac{4}{36}$

$$P(C \text{ or } D) = P(C) + P(D) - P(C \cap D)^{=0} \quad \text{متنافيه}$$
$$= \frac{5}{36} + \frac{4}{36} = \frac{9}{36}$$

5 A number is selected randomly from (1-9), at the same time a coin and die are tossed. Find the prob. of selecting an odd no., and the head appearing in a coin and getting the multiple of no. 3 in a die

Sol./ The three events are independent.

$$S = \{1,2,3,4,5,6,7,8,9\}$$
$$A = \{1,3,5,7,9\} \Rightarrow P(A) = \frac{5}{9}$$

$$S = \{H,T\} \text{ and } B = \{H\} \Rightarrow P(B) = \frac{1}{2}$$

$$S = \{1,2,3,4,5,6\} \text{ and } C = \{3,6\} \Rightarrow P(C) = \frac{2}{6}$$

$$\Rightarrow P(A \cap B \cap C) = \frac{5}{9} \cdot \frac{1}{2} \cdot \frac{2}{6} = \frac{5}{54}$$

6 A class contains 10 men and 20 women of which half of the men and half of the women have brown eyes. Find the prob. that a person chossen at random is a man or has brown eyes.

Sol./ $S = C_1^{30} = 30$

Let A is the person is a man $\Rightarrow A = C_1^{10} = 10$ and $P(A) = \frac{10}{30}$

and B is the person has brown eyes

$\Rightarrow B = C_1^{10} + C_1^{5} = 15$

then $P(B) = \frac{15}{30}$

$A \cap B$ is the Person is aman and has brown eyes.

then $P(A \cap B) = \frac{5}{30}$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{10}{30} + \frac{15}{30} - \frac{5}{30} = \frac{2}{3}$

H.W's:- واجبات طرق العد [تمارين $a_4 - a_{13}$]

① How many ways can distributed (9) playthings on (4) child inorder to the smallst children take 3 and each of another take two playthings?

② How many the parmutation for all latters of the words:

ⓐ sociological ⓑ unusual ⓒ them.

③ Find n from: ⓐ $P_2^n = 72$ ⓑ $C_4^n = 210$

④ How many ways can the teacher make group (4) person from class contain (9) boy and (3) girls.

H.W's واجبات موضوع الاحتمالية

[1] A class contains (9) boys and (3) girls, the teacher will chosse a group of 4 from them, the event that the grouwb contains:- ⓐ atleast one girl. ⓑ Exactly one boy.

Finds:- $p(A)$, $p(A^c)$, $p(B)$, and $p(B^c)$.

[2] A box containing (5) red, 25 white and 20 black balls. A ball is selected at randomm what the prob. of getting a red or black.

[3] If A, B are two independent events and $p(A) = 0.8$, $p(AB) = 0.4$, Find $p(B)$.

[4] A box (1) contains (8) units which (3) of them are defective and the box (2) contains (5) units which two of them are defective. one unit selected at random from each box. Find the prob. that

ⓐ The two units are non-defective.

ⓑ One of them is defective and the other is non-defective.