Conditioned probability: الاحتمال الشرطي

- For any two events A and B, the conditioned prob. of A given B is defined by:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad ; \text{where} \quad P(B) \neq 0$$

and

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad ; \text{where} \quad P(A) \neq 0$$

مجهول — تكون معلومة

وهي احتمالية حدوث حادثة معينة بوجود حادثة أخرى (معلومة).

Note:- If any of event not effect for the other, then they are Independent event, and:-

$$P(A \cap B) = P(A) \cdot P(B)$$

Then $P(A/B) = P(A)$ and $P(B/A) = P(B)$.

- and If they are dependent, then:-

$$P(A \cap B) = P(A) \cdot P(B/A) \text{ and } P(A \cap B) = P(B) \cdot P(A/B)$$

- For three events:-

$$P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B) \quad (\text{dep.})$$

حيث ذلك حدوث A,B,C يساوي احتمال حدوث A مضروب في احتمال حدوث B عندما تكون A معلومة مضروب باحتمال حدوث C عندما تكون A و B معلومة.

- and if they are Independent:-

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C).$$

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Examples:- 1 Three Coins are tossed. Find the probability that they are all head if:-

(a) The first Coin is heads.

(b) At Least one of the coins is heads.

Sol/ $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

$N = 8$

(a) $A = \{HHH, HHT, HTH, HTT\} \Rightarrow P(A) = \frac{n(A)}{N} = \frac{4}{8}$

$n(A) = 4 \Rightarrow B = \{HHH\} \Rightarrow P(B/A) = \frac{P(A \cap B)}{P(A)}$

$P(B) = \frac{1}{8}$ and $P(A \cap B) = \frac{1}{8}$

$\therefore P(B/A) = \frac{\frac{1}{8}}{\frac{4}{8}} = \frac{1}{4}$

(b) $A = \{HHH, HHT, HTH, THH, TTH, THT, HTT\} \Rightarrow n(A) = 7$

$B = \{HHH\} \Rightarrow n(B) = 1$

$\Rightarrow P(A) = \frac{n(A)}{N} = \frac{7}{8}$ and $P(B) = \frac{n(B)}{N} = \frac{1}{7}$

2 Two numbers are selected at random from (1–9) if the sum is even, fined the prob. that both number is odd.

Sol/ $S = C_2^9 = 36$

A = The sum is even
- 4 no.'s {2,4,6,8} even $C_2^4 = 6$
- 5 no.'s {1,3,5,7,9} odd $C_2^5 = 10$

$n(A) = 6 + 10 = 16$

$\Rightarrow P(A) = \frac{n(A)}{N} = \frac{16}{36}$

B = the two no.'s are odd $\Rightarrow C_2^5 = 10 \Rightarrow n(B) = 10$

$\Rightarrow p(B) = \frac{n(B)}{\sim} = \frac{10}{36}$

$\because p(B/A) = \frac{p(A \cap B)}{p(A)}$

$\because p(A \cap B) = p(B) = \frac{10}{36}$ (odd no.s)

$\Rightarrow p(B/A) = \frac{p(A/B)}{p(A)} = \frac{\frac{10}{36}}{\frac{16}{36}} = \frac{10}{16}$

[3] If $p(A) = 0.4$, $p(B/A) = 0.5$ and $p(A/B) = 0.4$:-

(a) Are A, B Independent or not?

(b) Find $p(B)$ (c) Find $p(A \cup B)$.

(d) Find $p(A^c \cap B^c)$ and $p(A^c \cup B^c)$

Sol:

(a) $\because p(A) = p(A/B) = 0.4 \Rightarrow$ A and B are Independent.

(b) $\because$ A, B Indep $\Rightarrow p(B) = p(A/B) = 0.5$

(c) $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$\because p(A \cap B) = p(A) \cdot p(B)$ (Indep)

$\Rightarrow p(A \cap B) = (0.4)(0.5) = 0.2$

$\therefore p(A \cup B) = 0.4 + 0.5 - 0.2 = 0.7$.

(d) $p(A^c \cap B^c) = p(A \cup B)^c = 1 - p(A \cup B) = 1 - 0.7 = 0.3$

and $p(A^c \cup B^c) = p(A \cap B)^c = 1 - p(A \cap B) = 1 - 0.2 = 0.8$.

[4] At a middle school (0.18) of all students play football and basket ball, and 0.32 of them play football. What the prob. that a student plays basket ball given that he plays foot ball?

Sol/ B = Basket ball and F = foot ball

$$\Rightarrow p(B/F) = \frac{p(B \cap F)}{p(F)} = \frac{0.18}{0.32} = 0.56$$

[5] A Coin is thrown three times. If A is the event that head occurs on each of the first two tosses, B is the event that a tail occurs on the third toss. Where C is the event that exactly two tails occur in the three tosses. Show that the events A and B are Independent; B and C are inedepent or not?

Sol/ $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\} = 8$

$\Rightarrow A = \{HHH, HHT\} = n(A) = 2 \Rightarrow p(A) = \frac{n(A)}{N} = \frac{2}{8} = \frac{1}{4}$

$B = \{HHT, THT, HTT, TTT\} \Rightarrow p(B) = \frac{n(B)}{N} = \frac{4}{8} = \frac{1}{2}$

$C = \{TTH, THT, HTT\} = n(C) = 3 \Rightarrow p(C) = \frac{n(C)}{N} = \frac{1}{8}$

then $A \cap B = \{HHT\} \Rightarrow n(A \cap B) = 1$

$\Rightarrow p(A \cap B) = \frac{n(A \cap B)}{N} = \frac{1}{8}$

and: $p(A \cap B) = p(A) \cdot p(B) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

and $p(C \cap B) = p(C) \cdot p(B) = \frac{3}{8} \cdot \frac{1}{2} = \frac{3}{16} \neq \frac{2}{8}$

then $p(A \cap B)$ and $p(C \cap B)$ are dependent.

H.W.5:-

*[1] A die is thrown once. If the number appearing is more than (3). what the prob. that :-

(a) The no. is even.

(b) The no. is 5.

*[2] A, B two events, if $p(A) = \frac{1}{2}$. $p(B) = \frac{1}{3}$ $p(A \cap B) = \frac{1}{4}$: Find:-

(a) $p(A/B)$

(b) $p(B/A)$

(c) $p(A \cup B)$.