Baye's Theorem: نظرية بيز

— Suppose the events $A_1, A_2, \ldots, A_n$ form a partition of sample space S; that is, the events A_i are mutually exclusive and their union is S.

— Let B be any other event, then:

$$B = S \cap B = (A_1 \cup A_2 \cup \ldots A_n) \cap B$$

$$= (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B) \ldots (A_n \cap B)$$

$$\Rightarrow p(B) = p(A \cap B) + p(A_2 \cap B) + \cdots + p(A_n \cap B).$$

$$\because p(B/A) = \frac{p(A \cap B)}{p(A)} \Rightarrow p(A \cap B) = p(A) \cdot p(A \cap B).$$

$$\Rightarrow p(B) = p(A_1) \cdot p(B/A_1) + p(A_2) \cdot p(B/A_2) + \cdots + p(A_n) \cdot p(B/A_n).$$

$$\Rightarrow p(B) = \sum p(A_i) \cdot p(B/A_i)$$

where $p(B/A_i) = \dfrac{p(B \cap A_i)}{p(A_i)}$

$$\Rightarrow p(A_i \cap B) = p(A_i) \cdot p(B/A_i)$$

and:-

$$p(A_i/B) = \frac{p(A_i \cap B)}{p(B)} = \frac{p(A_i) \cdot p(B/A_i)}{\sum p(A_i) \cdot p(B/A_i)}$$

$$\therefore p(A_i/B) = \frac{p(A_i) \cdot p(B/A_i)}{p(A_1) \cdot p(B/A_1) + p(A_2) \cdot p(B/A_2) + \cdots + p(A_n) p(B/A_n)}$$

Then: The $p(A_i)$ are called prior probability and the conditional probabilities $p(A_i/B)$ are called the posterior probability.

ملاحظة:-

prior :- هو الاحتمال القبلي الذي يعتمد على التخمينات المتوفرة قبل اجراء التجربة.

posterior :- " " البعدي " " " " المعلومات الميدانية بعد اجراء التجربة.

Example:- [1] In a factory, Three machines A, B, C are producing springs as 50%, 30%, 20% from the total production, and the ratio of defective spring as 3%, 4%, 5% respectively. One item selected at random. what:-

(a) The prob. that it is defective.

(b) If the item is defective what the prob. that it was product by machine B.

Sol/ Let X be the One wich is defective, then we find p(X) where:-

(a) $p(X) = p(A) \cdot p(X/A) + p(B) \cdot p(X/B) + p(C) \cdot p(X/C)$.

$$= \frac{50}{100} \cdot \frac{3}{100} + \frac{30}{100} \cdot \frac{4}{100} + \frac{20}{100} \cdot \frac{5}{100} = 0.037$$

(b) $$p(B/X) = \frac{p(B) \cdot p(X/B)}{p(A) \cdot p(X/A) + p(B) \cdot p(X/B) + p(C) \cdot p(X/C)} = \frac{12}{37} = 0.324$$

Examples - [2] In a College for which 60% of the student are women, 4% from the men and 1% from the women are taller than (1.8) m. One student selected at random and he (or she) taller than (1.8) m. Find the prob. that the student is women?

Sol/ Let A be the person taller than (1.8) m.

Then: p(W/A) is the prob. of the student is women and taller than (1.8).

$$p(W/A) = \frac{p(W) \cdot p(A/W)}{p(W) \cdot p(A/W) + p(m) \cdot p(A/m)}$$

$$= \frac{(0.60) \cdot (0.01)}{(0.60) \cdot (0.01) + (0.40)(0.04)} = \frac{0.006}{0.022} = \underline{\underline{0.272}}$$