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Chapter two :-

Random variables and Probability some important definitions.

① Random variable;

If we have a random experiment with a sample space S. A function X that assigns to each element in S one and only one real number is called a random variable.

It is depart by two types!

ⓐ Descrete r.v.

ⓑ Continuous r.v.

- A random variable is called discrete if it can assume only discrete values $X_1, X_2, \dots, X_n$.

for example/ The number of child, number of Passengers ..... etc.

- where the r.v. called continuous if it can assume any value over a continuous range of the real line

for example/ Heights of students in a class, age of People --- etc.

② Probability distribution.

If X is discrete r.v. - the function given by $f(x) = P(X = x)$. for each x within the range of x is called the Probability density function (P.d.f) and satisfy the following conditions;

① $f(x) \geq 0$ ② $\sum f(x) = 1$

where the cumulative dist. of X is (c.d.f) and denoted by:

$$F(x) = P(X \leq x) = \sum f(x) \text{ for all value of } X$$

and it is satisfy:

① For any real number a and b if $a < b$ then $F(a) \leq F(b)$

② $f(x_i) = F(x_i) - F(x_{i-1})$ for $i = 1, 2, \dots, n$.

dist. func. $\Rightarrow$ c.d.f

Prob. dist. fun. $\Rightarrow$ p.d.f.

Now, if X is continuous random variable and a, b are any real numbers, then:

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$ and it is called the Prob. density fun. It is satisfy:

① $f(x) \geq 0$ ② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

where the cumulative fun. is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt \quad -\infty < X < \infty$$

It is called dist. fun. and satisfy:

① If $a < b \Rightarrow F(a) \leq F(b)$

② $P(a \leq X \leq b) = F(b) - F(a) = \int_a^b f(x)\,dx$

③ $f(x) = \frac{\partial F(x)}{\partial x}$

Ex.'s :-

② Determine K from the p.d.f of the r.v. X given by:

$f(x) = KX \qquad x = 1, 2, 3, 4, 5$

Sol./ $\sum f(x) = 1 \Rightarrow K + 2K + 3K + 4K + 5K = 1$

$\Rightarrow K = \frac{1}{15}$

① Let $f(x) = \frac{x+2}{25} \qquad x = 1, 2, 3, 4, 5$

check whether the probability fun. is (p.d.f)

sol/ if $\sum f(x) = 1 \Rightarrow$ the fun. is p.d.f

$\sum f(x) = \frac{1+2}{25} + \frac{2+2}{25} + \cdots\cdots \frac{5+2}{25} = 1 \Rightarrow$ the fun. (p.d.f)

③ if X has the distribution fun.:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{28} & 0 \le x < 1 \\ \frac{18}{28} & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases} \qquad x = 0, 1, 2$$

Find the probability dist. fun. of X.