Sol/ $f(0) = F(0) = \frac{3}{28}$

$f(1) = F(1) - F(0) = \frac{18}{28} - \frac{3}{28} = \frac{15}{28}$

$f(2) = F(2) - F(1) = 1 - \frac{18}{28} = \frac{10}{28}$

④ Let $f(x) = \frac{x}{10}$ $x = 1, 2, 3, 4$

Find the dist. fun for X.

Sol/ $F(1) = f(1) = \frac{1}{10}$

$F(2) = f(1) + f(2) = \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$

$F(3) = f(1) + f(2) + f(3) = \frac{6}{10}$

$F(4) = f(1) + f(2) + f(3) + f(4) = 1$

$\Rightarrow$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{10} & 1 \le x < 2 \\ \frac{3}{10} & 2 \le x < 3 \\ \frac{6}{10} & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$

⑤ Let $f(x) = \frac{1}{4}$ $-2 < x < 2$. Find the cum(c.d.f) dist. fun.

Sol/

$$F(x) = \int_{-\infty}^{x} f(t)\,dt \Rightarrow \int_{-2}^{x} \frac{1}{4}\,dt \Rightarrow \frac{t}{4}\Big|_{-2}^{x} \Rightarrow \frac{x+2}{4}$$

⑥ Find the constant C and $P(X \geq 6)$ from the p.d.f $f(x) = C \quad 0 \leq X \leq 8$.

Sol/ $f(x) = C \Rightarrow \int_0^8 f(x)\,dx = 1 \Rightarrow$ p.d.f

$\Rightarrow C\int_0^8 dx = 1 \Rightarrow xC\Big|_0^8 = 1 \Rightarrow C = \frac{1}{8}$

$P(X \geq 6) = \int_6^8 f(x)\,dx \Rightarrow \int_6^8 \frac{1}{8}\,dx$

$\Rightarrow \frac{x}{8}\Big|_6^8 \Rightarrow \frac{1}{4}$

⑦ Find the Prob. fun from the dist. fun.

$F(x) = \frac{x^3}{2} \quad -1 \leq X \leq +1$ and $P(2X \leq 1)$.

Sol/ $f(x) = \frac{dF(x)}{dx} \Rightarrow \frac{3}{2}x^2 \quad -1 \leq X \leq 1$

$P(2X \leq 1) = P\left(X \leq \frac{1}{2}\right) \Rightarrow \frac{3}{2}\int_{-1}^{\frac{1}{2}} x^2\,dx$

$\Rightarrow \frac{3}{2}\cdot\frac{x^3}{3}\Big|_{-1}^{\frac{1}{2}} \Rightarrow \frac{1}{8}\cdot\frac{1}{2} + \frac{1}{2} = \frac{9}{16}$

⑧ Let a dist fun. is given by:

$F(x) = \frac{x+1}{2} \quad 0 \leq X < 1$, find $P\left(-1 \leq X \leq \frac{1}{2}\right)$ and $P(X = 0)$

Sol/

$P\left(-1 \leq X \leq \frac{1}{2}\right) = F\left(\frac{1}{2}\right) - F(-1) = \frac{3}{4} - 0 = \frac{3}{4}$

$P(X = 0) = \frac{1}{2}$

* H.W.8

① Let X be a r.v. have the p.d.f:

$f(x) = \frac{1}{2}$ $0 \leq X \leq 8$. find $P(2 \leq X \leq 5) \cdot P(3 \leq X \leq 7)$

② For the p.d.f below. $f(x) = kx$ $0 \leq X \leq 5$

Find: $P(1 \leq X \leq 3)$, $P(2 \leq X \leq 4)$, $P(X \leq 3)$

③ For the Prob. fun. below. find the dist. fun.

X	1	3	4	5
f(x)	0.4	0.1	0.2	0.3

④ For the Prob. dist. fun. below. Find the constant k and $(0.5 \leq X \leq 1)$.

$f(x) = ke^{-3x}$, $x > 0$

⑤ For the p.d.f. $f(x) = \frac{1}{3}$ $X = -1, 0, 1$

Find the dist. fun of X.

⑥ Let the dist fun. of X be:

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{6} & 1 \leq x < 2 \\ \frac{3}{6} & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

find the Prob. dist. fun

⑦ Let $f(x) = \frac{1}{100} e^{-\frac{x}{100}}$ $0 \leq X < \infty$:

(a) Is f(x) a. p.d.f ?

(b) Find the $P(X > 100)$?

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