المحاضرة السادسة
$a_{35} - 41$

Chapter Four

Probability function of two Random variables

* Joint. prob. fun. of two r.v.'s (dis. and con.)

Def :- if X and y are two r.v.'s (dis.), the fun. given by $f(x,y) = P(X = x, Y = y)$ for each pair of values withen the range of X and Y is called the joint prob. fun. of X and Y (J.P.d.f) and it is satisfy the conditions:

① $f(x,y) \geq 0$ for all values of x, y.

② $\sum\sum f(x,y) = 1$

where the Joint cumulative . dist. fun. of x, y is given by (J. C. d. f)

$$F(x,y) = \sum_{y}\sum_{x} f(s,t)$$, where $f(s,t)$ is the value of the joint prob. fun. of X and Y at (s,t)

Now - if the r.v.'s x and y are continuous r.v.'s, then the (J.P.d.f) is given by $f(x,y)$ which satisfy the conditions ① $f(x,y) \geq 0$

② $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$

and the (J.c.d.f) is :-

$$F(x,y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(s,t)\, ds\, dt \qquad \begin{matrix} -\infty < x < \infty \\ -\infty < y < \infty \end{matrix}$$

and if we want to find the (J.p.d.f)

$$f(x,y) = \frac{\partial^2}{\partial x\, \partial y} F(x,y)$$

Ex.'s / ① Let $f(x,y) = x + y \qquad 0 \le x \le 1 \quad 0 \le y \le 1$

1- Is this fun. is (J.P.d.f) or not ?
2- Find the Joint. c. dist. fun. of x, y. (J.C.d.f)
3- Find $P(\frac{1}{2} \le X \le 1) \quad 0 \le y \le 1$

Sol/ ① if $\int_0^1 \int_0^1 f(x,y)\,dx\,dy = 1 \Rightarrow$ is J.P.d.f

$$\int_0^1 \int_0^1 x + y \;dx\,dy = \int_0^1 \frac{x^2}{2} + xy \Big|_0^1 dy$$

$$\Rightarrow \int_0^1 \frac{1}{2} + y\,dy \Rightarrow \frac{y}{2} + \frac{y^2}{2} \Big|_0^1 = 1 \Rightarrow \text{J.P.d.f}$$

② $F(x,y) = \int_0^y \int_0^x f(s,t)\,ds\,dt$

$$= \int_0^y \frac{s^2}{2} + st \Big|_0^x dt \Rightarrow \int_0^y \frac{x^2}{2} + xt\,dt$$

$$= \frac{x^2 t}{2} + \frac{x t^2}{2} \Big|_0^y \Rightarrow \frac{x^2 y}{2} + \frac{x y^2}{2}$$

③ $P(\frac{1}{2} \le X \le 1 \,,\; 0 \le y \le 1) = \int_0^1 \int_{\frac{1}{2}}^1 (x+y)\,dx\,dy$

$$= \int_0^1 \frac{x^2}{2} + xy \Big|_{\frac{1}{2}}^1 dy \Rightarrow \int_0^1 \left(\frac{1}{2} + y - \frac{1}{8} - \frac{y}{2}\right) dy$$

$$= \int_0^1 \frac{3}{8} + \frac{y}{2}\,dy \Rightarrow \frac{3}{8}y + \frac{y^2}{4} \Big|_0^1 \Rightarrow \frac{5}{8}$$

② Find the constant C from the J.P.d.f below then find $P(X=y=2)$

$f(x,y) = Cxy \qquad x = 1,2 \qquad y = 1,2$

sol/ $\sum_y \sum_x f(x,y) = 1$

$\Rightarrow f(1,1) = C \qquad f(1,2) = 2C \qquad f(2,1) = 2C \qquad f(2,2) = 4C$

$\Rightarrow C + 2C + 2C + 4C = 1 \Rightarrow C = \frac{1}{9}$

$P(X=y=2) = f(2,2) = 4C \Rightarrow \frac{4}{9}$

③ Given J.c.d.f $F(x,y) = (x-1)(y-1)$

$1 \le x \le 3 \qquad 1 \le y \le 3$

find the joint Prob. dist. fun. of x,y

sol/ $f(x,y) = \frac{\partial^2}{\partial x \partial y} F(x,y)$

$\frac{\partial}{\partial x} = y-1 \Rightarrow \frac{\partial^2}{\partial x \partial y} = 1 \Rightarrow f(x,y) = 1$