* Marginal Prob. function

Def/ If X and Y are discrete random variables and $f(x,y)$ is the value of their joint prob. dist. at (x,y), the fun. given by:

$g(x) = \sum_y f(x,y)$ is called the marginal dist. of X

where
$h(y) = \sum_x f(x,y)$ is called the mar. dist. of y

And if X,Y are continuous r.v.'s then:

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad -\infty < y < \infty$$

is called the marginal distribution of X.

where
$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx \qquad -\infty < x < \infty$$

is called the marginal dis. of y.

Ex.'s ① For the following J.P.d.f find the marginal prob. fun. of x and of y.

$$f(x,y) = \frac{x+y}{21} \qquad x = 1,2,3 \qquad y = 1,2$$

Sol/ $g(x) = \sum_y f(x,y)$

$$= \frac{x+1}{21} + \frac{x+2}{21} \Rightarrow \frac{2x+3}{21} \qquad x = 1,2,3$$

$h(y) = \sum_x f(x,y)$

$$\frac{1+y}{21} + \frac{2+y}{21} + \frac{3+y}{21} \Rightarrow \frac{3y+6}{21} \qquad y = 1,2$$

(2) Given the J.P.d.f:

$f(x,y) = \frac{2}{3}(x+2y)$ $\quad 0<x<1 \quad 0<y<1$

Find $g(x)$ and $h(y)$?

Sol/ $g(x) = \int_0^1 f(x,y)\,dy$

$= \frac{2}{3}\int_0^1 (x+2y)\,dy$

$= \frac{2}{3}\left(xy + \frac{2y^2}{2}\right)\Big|_0^1 \Rightarrow \frac{2}{3}(x+1) \quad 0<x<1$

$h(y) = \int_0^1 f(x,y)\,dx$

$= \frac{2}{3}\int_0^1 (x+2y)\,dx \Rightarrow \frac{2}{3}\left(\frac{x^2}{2}+2xy\right)\Big|_0^1$

$= \frac{2}{3}\left(\frac{1}{2}+2y\right) \quad 0<y<1$

* Independent Random variables *

Def: Discrete r.v.'s X and Y are said to be independent if and only if:

$P(X=x, Y=y) = P(X=x)\cdot P(Y=y)$ for all real numbers x and y

Continuous r.v.'s X and Y are said to be ind. if and only if:

$f(x,y) = g(x)\cdot h(y)$ for all real numbers x and y.

Ex.'s

① Let $f(x,y) = 4xy \quad 0<x<1 \quad 0<y<1$
are x,y independent or not?

Sol/ $g(x) = \int_0^1 f(x,y)\,dy \Rightarrow \int_0^1 4xy\,dy$

$= 4x\frac{y^2}{2}\Big|_0^1 \Rightarrow 2x \quad 0<x<1$

$h(y) = \int_0^1 f(x,y)\,dx \Rightarrow \int_0^1 4xy\,dx$

$= 4y\frac{x^2}{2}\Big|_0^1 \Rightarrow 2y \quad 0<y<1$

if $f(x,y) = f(x)\cdot f(y) \Rightarrow$ ind.

$f(x,y) = 4xy = f(x)\cdot f(y) \Rightarrow$ x,y ind.

② Let $f(x,y) = \frac{xy^2}{13}$ $(x,y) = (1,1)(1,2)(2,2)$
Find the marginal fun. of x and y then check wither they are ind. or not?

Sol/ $g(x) = \sum_y f(x,y) \Rightarrow \frac{5x}{13} \quad x = 1,2$

$h(y) = \sum_x f(x,y) \Rightarrow \frac{3y^2}{13} \quad y = 1,2$

$g(x)\cdot h(y) = \frac{5x}{13}\cdot\frac{3y^2}{13} \Rightarrow \frac{15xy^2}{169} \neq \frac{xy^2}{13}$

then x,y not independent.