* Expectation of sum and product of two Random variables.

- If the random variables X and y are two discret r.v.'s then:

$$E(x+y) = E(x) + E(y) \Rightarrow \sum_y \sum_x x+y \; f(x,y)$$

$$= \int_y \int_x (x+y) f(x,y) \, dx \, dy \quad \text{if they are con.}$$

and $E(x,y) = E(x) \cdot E(y)$ if they are independent

if they are not independent r.v.'s, then

$$E(xy) = E(x) \cdot E(y) - E(x-\bar{x})(y-\bar{y})$$

where $\bar{x}, \bar{y}$ are the mathematical mean of x, y and

$E(x-\bar{x})(y-\bar{y})$ is the covariance between x and y

if x, y are independent, then:

$$Cov(x,y) = E(x-\bar{x})(y-\bar{y}) = 0$$ ممكن ان يكون قيمة سالبة وموجبة

Since:

$$E(x-\bar{x})(y-\bar{y}) = E(xy) - E(x) \cdot E(y) \quad [dep.]$$

the ratio defined as:

$$\rho_{xy} = \frac{\operatorname{cov}(x,y)}{\sqrt{\operatorname{var}(x)\operatorname{var}(y)}} \Longrightarrow \frac{E(x-\bar{x})(y-\bar{y})}{\sqrt{\underbrace{E\cancel{(x-\bar{x})}^2}_{V(x)}}\sqrt{\underbrace{E\cancel{(y-\bar{y})}^2}_{V(y)}}}$$

$$E(x^2)-(E(x))^2 \qquad E(y^2)-(E(y))^2$$

[المحاضرة السابقة
$a_{42} \rightarrow a_{49}$
[illegible]]

is called the correlation coefficient between X and Y

$\underline{Ex}$ /:- Let X, Y be r.v.'s have the J.P.d.f:

$$f(x,y) = x+y \qquad 0 \le x \le 1 \qquad 0 \le y \le 1$$

find the correlation coefficient between X and Y.

$\underline{\underline{Sol}}$ /

$$\rho_{x,y} = \frac{E(xy) - E(x)E(y)}{\sqrt{V(x)V(y)}} \qquad -1 \le \rho \le +1$$

$$\mu_{xy} = E(xy) = \int_0^1\int_0^1 x(x+y)\,dx\,dy$$

$\underline{\underline{or}}$ $E(x) = \int_0^1 x f(x)\,dx$

$$g(x) = \int_0^1 (x+y)\,dy \Longrightarrow x+\frac{1}{2} \qquad 0 \le x \le 1$$

$$E(x) = \int_0^1 x\left(x+\frac{1}{2}\right)dx \longrightarrow \frac{x^3}{3}+\frac{x^2}{4}\Big|_0^1 \Rightarrow \frac{7}{12}$$

$$\mu_y = E(y) = \int_0^1 y f(y)\,dy$$

$$h(y) = \int_0^1 (x+y)\,dx \Longrightarrow \frac{1}{2}+y \qquad 0 \le y \le 1$$

$$E(y) = \int_0^1 y\left(\frac{1}{2}+y\right)dy = \frac{y^2}{4}+\frac{y^3}{3}\Big|_0^1 = \frac{7}{12}$$

$$V(X) = E(X^2) - (EX)^2$$

$$EX^2 = \int_0^1 x^2 f(x)\,dx \Rightarrow \int_0^1 x^2\left(x+\tfrac{1}{2}\right)dx \Rightarrow \frac{x^4}{4}+\frac{x^3}{6}\Big|_0^1$$

$$EX^2 = \frac{10}{24} \Rightarrow V(X) = \frac{10}{24} - \left(\frac{7}{12}\right)^2 \Rightarrow \frac{11}{144}$$

$$V(y) = E(y^2) - (E(y)^2)$$

$$E(y^2) = \int_0^1 y^2 f(y)\,dy \Rightarrow \int_0^1 y^2\left(\tfrac{1}{2}+y\right)dy \Rightarrow \frac{y^3}{6}+\frac{y^4}{4}\Big|_0^1$$

$$E(y^2) = \frac{10}{24} \Rightarrow V(y) = \frac{10}{24} - \left(\frac{7}{12}\right)^2 \Rightarrow \frac{11}{144}$$

$$\operatorname{cov}(x,y) = E(xy) - E(x)\cdot E(y)$$

$$E(xy) = \int_0^1\int_0^1 xy(x+y)\,dx\,dy$$

$$= \int_0^1 \frac{x^3 y}{3} + \frac{x^2y^2}{2}\Big|_0^1 dy \Rightarrow \int_0^1 \frac{y}{3}+\frac{y^2}{2}\,dy$$

$$\Rightarrow \frac{y_2}{6} - \frac{y^3}{6}\Big|_0^1 = \frac{1}{6}+\frac{1}{6}$$

$$E(xy) = \frac{2}{6} \Rightarrow \operatorname{cov}(x,y) = \frac{2}{\cancel{6}\,24} - \frac{49}{144} \Rightarrow -\frac{1}{144}$$

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$$\rho_{x,y} = \frac{-\frac{1}{144}}{\sqrt{\left(\frac{11}{144}\right)\left(\frac{11}{144}\right)}} = -\frac{1}{11}$$