* Conditional function, Expection and Variance of two random variables

Def :— Let $f(x,y)$ be the J.p.d.f of two r.v.'s x and y and let $g(x)$ denote the marginal p.d.f. of x, then the conditional p.d.f of y given x is given by;

$$f(y|x) = \frac{f(x,y)}{g(x)}, \qquad g(x) > 0$$

and for x given y is given by:

$$f(x|y) = \frac{f(x,y)}{h(y)}$$, where $h(y)$ is the marginal of y. $h(y) > 0$.

then the conditional mean (expectation) of y.

given x as;

$$E(y|x) = \int_{-\infty}^{\infty} y\, f(y|x)\, dy$$

دائماً y مجهول و x معلوم

continuous

and for x given y is:

$$E(x|y) = \int_{-\infty}^{\infty} x\, f(x|y)\, dx$$

x مجهول و y معلوم
لانه شرطي يجب ان يكون
احدهما مجهول ويستخرج عن
طريق المعلوم الاخر.

and

$$E(y|x) = \sum y f(y|x)$$
$$E(x|y) = \sum x f(x|y)$$

dis

واذا لم يعطى رقم معين
للمتغير المعلوم يظهر التوقع
او التباين بدلالة ذلك المتغير

where the conditional variance of y given X defined by:

$$V(y|x) = E(y^2|x) - [E(y|x)]^2 \Rightarrow E[(y - E(y|x))^2 | x]$$

and

$$V(x|y) = E(x^2|y) - [E(x|y)]^2 \Rightarrow E[(x - E(x|y))^2 | y]$$

Ex/:- Let $f(x,y) = \frac{x+y}{21}$ $\quad x = 1, 2, 3 \quad y = 1, 2$

Find the conditional variance of $y|x$ in $X = 3$

Sol - $V(y|x) = E[(y - E(y|x))^2 | x = 3]$

$$E(y|x) = \sum_y y f(y|x=3)$$

$$f(y|x) = \frac{f(x,y)}{g(x)}$$

$$g(x) = \sum_y f(x,y) \Rightarrow \frac{x+1}{21} + \frac{x+2}{21} \Rightarrow \frac{3+2x}{21}$$

$$f(y|x=3) = \frac{\frac{x+y}{21}}{\frac{3+2x}{21}} \Rightarrow \frac{x+y}{3+2x} \Big|_{x=3} \Rightarrow \frac{3+y}{9}$$

$$E(y|x) = \sum_y y\left(\frac{3+y}{9}\right) \Rightarrow \frac{4}{9} + \frac{10}{9} \Rightarrow \frac{14}{9}$$

$$V(y|x=3) = E\left[\left(y - \frac{14}{9}\right)^2 \Big| x = 3\right]$$

$$= \sum_y \left(y - \frac{14}{9}\right)^2 f(y|x=3) \quad \swarrow E(y|x)$$

$$= \sum \left(y - \frac{14}{9}\right)^2 \left(\frac{3+y}{9}\right) \Rightarrow \left(1 - \frac{14}{9}\right)^2\left(\frac{4}{9}\right) + \left(2 - \frac{14}{9}\right)^2\left(\frac{5}{9}\right)$$

$$V(y|x=3) = \boxed{\frac{20}{81}}$$

$\underline{Ex}$:- Let $f(x,y) = \frac{xy}{2}$ $\quad 0 \leq x \leq 2 \quad 0 \leq y \leq x$

Find the conditional Expectation of y given x.

$\underline{Sol}$ - $V(y|x) = E\left[(y - E(y|x))^2 \mid x\right]$

$$E(y|x) = \int_0^x y\, f(y|x)\, dy$$

$$f(y|x) = \frac{f(x,y)}{g(x)}$$

$$g(x) = \int_0^x f(x,y)\, dy \Rightarrow \int_0^x \frac{xy}{2}\, dy \Rightarrow \frac{1}{2} \cdot \frac{xy^2}{2} \Big|_0^x$$

$$\underline{\left|g(x) = \frac{x^3}{4}\right|} \quad 0 \leq x \leq 2$$

$$f(y|x) = \frac{xy}{\cancel{2}} \cdot \frac{\cancel{4}}{x^{\cancel{3}}} \Rightarrow \underline{\left|\frac{2y}{x^2}\right|}$$

$$E(y|x) = \int_0^x y \cdot f(y|x)\, dy \Rightarrow \int_0^x y \cdot \frac{2y}{x^2}\, dy$$

$$= 2\left[\frac{y^3}{3x^2}\right]_0^x = \underline{\left|\frac{2}{3} x\right|}$$