Linear Transformations of two Random variables

Def: is a change to avariable by one or more of the following operations by adding a constant to the variable, subtracting multiplying or dividing the variable by a constant.

Then a new random variable is appearing. The relation between any two random variables is alinear transformation

$\Rightarrow$ Let X and y be arandom variables, and let b, m be any constants. then:

① $y = X + b$ ② $y = X - b$ ③ $y = mX$ ④ $y = \frac{X}{m}$

⑤ $y = mX + b$ ⑥ $y = \frac{X}{m} - b$

Note: If X and Z are variables, and the corr. between them is ρ. Anew variable y is created by applying alinear transformation to X, then the correlation between X, y is ρ too.

Then if $y = mX + b \Rightarrow E(y) = mEX + b$

and $V(y) = m^2 V(x)$

Ex / If X is a r.v. having p.d.f:

$$f(x) = 4x(1-x^2) \qquad 0 \le x \le 1$$

and $y = 15x + 9$ find the mean and variance of y

sol / $E(y) = 15\,E(x) + 9$

$V(y) = 225\,V(x)$

$$E(x) = \int_0^1 x f(x)\,dx \Rightarrow 4\int_0^1 x^2(1-x^2)\,dx$$

$$= 4\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \boxed{\frac{8}{15}}$$

$$V(x) = E x^2 - (E x)^2$$

$$E x^2 = \int_0^1 x^2 f(x)\,dx \Rightarrow 4\int_0^1 x^2(x-x^3)\,dx$$

$$= 4\left[\frac{x^4}{4} - \frac{x^6}{6}\right]_0^1 \Rightarrow \boxed{\frac{1}{3}}$$

$$V(x) = \frac{1}{\underset{75}{\cancel{3}}} - \frac{64}{225} \Rightarrow \frac{11}{225}$$

$$E(y) = 15\left(\frac{8}{15}\right) + 9 \Rightarrow 17$$

$$V(y) = 225\left(\frac{11}{225}\right) \Rightarrow 11$$

$\underline{Ex}$: Let $f(x) = \frac{1}{3}$, $X = 1, 2, 3$ be p.d.f of r.v. and y is another v. such that:

$y = 2X+1$, Find the mean and variance of r.v. y i.e find $E(y) = 2E(X)+1$

$V(Y) = 4V(X)$.

sol/ $\because y = 2X+1$

$\Rightarrow E(y) = 2E(X)+1$

and $V(Y) = 4V(X)$.

$\Rightarrow E(x) = \sum x f(x) = \frac{1}{3} + \frac{2}{3} + \frac{3}{3} = \frac{6}{3}$

and $V(x) = EX^2 - (EX)^2$

$\Rightarrow E(x^2) = \sum x^2 f(x) = \frac{1}{3} + \frac{4}{3} + \frac{9}{3} = \frac{14}{3}$

$\therefore V(x) = \frac{14}{3} + \frac{12}{3} = \frac{26}{3}$

and $E(y) = 2\left(\frac{6}{3}\right) + 1 = 4+1 = 5$

$V(y) = 4\left(\frac{26}{3}\right) = \frac{104}{3} = 34.6$