[950 → 956 (8) محاضرة

Chapter Five :- Some special Probability dist.

بعض التوزيعات الخاصة .

—— The random variables classification due to the probability density function into two types (distret and continuous). Then there is two types of the prob. distributions :-

* Discrete distributions :- التوزيعات المتقطعة

The important distributions in this type are:-

[1] Bernoulli distribution :- توزيع برنولي

Defn:- If an experiment has two possible outcomes (success and failure) and their probabilities are respectively, p and (1-p) then the number of success X = 1 or X = 0 has a Bernoulli dist. and denoted by ⇒ $X \sim Ber(p)$.

The p.d.f is given by:-

$$f(x) = p^x \cdot (1-p)^{1-x} \quad ; \; x = 0, 1$$

* The moments :-

① the mean $\mu_x = E(x) = p$

② The variance $\sigma^2_x = V(x) = p \cdot q \quad ; (q = 1-p)$

③ The M.g.f $M_x(t) = q + pe^t$

① $\mu = E(x) = \sum x \cdot f(x) = 0 \cdot p^0 \cdot (1-p)^{1-0} + 1 \cdot p^x (1-p)^{1-1}$

$= 0 \cdot (1-p) + 1 \cdot p = \underline{\underline{p}}$

② $\sigma^2_x = V(x) = E(x^2) - (E(x))^2 = \sum x^2 \cdot f(x) - p^2$

$= 0 \cdot (1-p) + 1 \cdot p - p^2$

$= p(1-p) = \underline{\underline{p \cdot q}}$

③ $M_x(t) = E(e^{tx}) \Rightarrow \sum e^{tx} \cdot f(x) = \sum e^{tx} \cdot p^x \cdot (1-p)^{1-x}$

$$= e^{0t} \cdot p^0 (1-p)^{1-0} + e^{1t} \cdot p^1 (1-p)^{1-1}$$

$$= (1-p) + e^t p \Rightarrow \underline{q + pe^t}$$

Example:- Let X be a r.v of getting a tail in a single toss of a coin. Find the distribution the mean the variance and M.g.f of X.

Sol/ $X \sim Ber(p = \frac{1}{2})$

$\Rightarrow f(x) = p^x \cdot (1-p)^{1-x} \quad ; x = 0, 1$

* $\mu_x = E(x) = \frac{1}{2}$
* $\sigma_x^2 = V(x) = (\frac{1}{2}) \cdot (\frac{1}{2}) = \frac{1}{4}$
* $M_x(t) = \frac{1}{2} + \frac{1}{2} e^t$

[2] Binomial distribution:-

Def:- The repeating of Bernoulli case in (n) trials will give a r.v. having Binomial dist. with parameters n, p, and the p.d.f. is:-

$$f(x) = C_x^n \cdot p^x \cdot (1-p)^{n-x} \quad ; x = 0, 1, 2, 3, \ldots, n$$

then $X \sim Bin(n, p)$

where: n is the number of trials, p is the prob. of success, X is the number of successes among n trials which is independent.

The moments:-

① The mean $\mu_x = E(x) = np$

② The Variance $\sigma_x^2 = V(x) = n \cdot p \cdot q$

③ The M.g.f $M_x(t) = [q + pe^t]^n$

where: ① $\mu_x = E(x) = E\sum(x_i) = \sum E(x_i) = \sum p = np$

② $\sigma_x^2 = V(x) = \sum V(x_i) = \sum p(1-p) = np \cdot q$

Notes:- If we have the m.g.f of r.v. $X \sim Bin(n, p)$, then the mean an the variance of X are:-

$M_X(t) = [q+pe^t]^n$

$E(X) = M'_X(t)\big|_{t=0} = n(q+pe^t)^{n-1}\cdot(pe^t)\big|_{t=0} = np$

$V(X) = E(X^2) - (E(X))^2$.

$E(X^2) = M''_X(t)\big|_{t=0} \Rightarrow$

$= n(q+pe^t)^{n-1}\cdot(pe^t) + n(n-1)(q+pe^t)^{n-2}\cdot(pe^t)^2$

$= np + n(n-1)p^2$

$\therefore V(X) = np + n^2p^2 - np^2 - n^2p^2 = \underline{npq}$

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Examples:- ① Let the r.v. X denoted the head appearing in a coin tossing 3 times, find the distribution of X if appears:-

ⓐ 3 heads ⓑ 2 head and 1 tail ⓒ 2 tails and 1 head ⓓ 3 tails.

sol/ $X \sim Bin(3, \frac{1}{2})$

$f(x) = C^3_x \cdot (\frac{1}{2})^x \cdot (1-\frac{1}{2})^{3-x}$; $x = 0, 1, 2, 3$

ⓐ $f(3) = C^3_3 \cdot (\frac{1}{2})^3 \cdot (1-\frac{1}{2})^{3-3} = \frac{1}{8}$

ⓑ $f(2) = C^3_2 \cdot (\frac{1}{2})^2 \cdot (1-\frac{1}{2})^{3-2} = \frac{3}{8}$

ⓒ $f(1) = C^3_1 \cdot (\frac{1}{2})^1 \cdot (1-\frac{1}{2})^{3-1} = \frac{3}{8}$

ⓓ $f(0) = C^3_0 \cdot (\frac{1}{2})^0 \cdot (1-\frac{1}{2})^{3-0} = \frac{1}{8}$.