Examples-②: Two dice are thrown 8 times, find the prob. that the two dice show odd number.
Then find the expected number of times that the two dice show odd. number. [variance]

Sol/:-

$X \sim Bin(8, p)$, which is denoted odd no.

i.e $X = 1, 3, 5$

$f(x) = C_x^8 \cdot p^x \cdot (1-p)^{8-x}$, $x = 1, 3, 5$, $(p = \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{4})$

$f(x) = C_3^x \cdot (\frac{1}{4})^3 \cdot (\frac{3}{4})^5 = 0.208$

$\Rightarrow E(X) = n \cdot p = 8 \cdot (\frac{1}{4}) = 2$

$V(X) = ?$

3 Poisson Distribution:-

Defn:- A random variable is said to have a poisson distibution if the probability mass function p.d.f of X is:-

$f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$; $x = 0, 1, 2, \ldots$, $\lambda > 0$

then $X \sim P(\lambda)$

* **The moments:-**

① The mean $\mu_x = E(X) = \lambda$

② the variance $\sigma_x^2 = V(X) = \lambda$

③ the M.g.f $M_x(t) = e^{\lambda(e^t - 1)}$

Note:- If the M.g.f of X is exist then:-

$M_x(t) = e^{\lambda(e^t - 1)}$

$E(X) = M_x'(t)\Big|_{t=0} = \lambda e^t \cdot e^{\lambda(e^t-1)}\Big|_{t=0} = \lambda$

$E(X^2) = M_x''(t)\Big|_{t=0} = (\lambda e^t)^2 \cdot e^{\lambda(e^t-1)} + \lambda e^t \cdot e^{\lambda(e^t-1)}\Big|_{t=0}$

$= \lambda^2 + \lambda$

$\therefore V(X) = \lambda^2 + \lambda - \lambda^2 = \lambda$.

Note:- From Binomial dist. if n is very Larg and p is very small then the r.v. gone to have a poisson dist. Such that the following situations:-

① the percentage of telephone calls arriving per unit time.

② The typing errors per page in a big book.

... ect.

— Where $\underline{\underline{n}}$ is the number of calling and errors typing and p is the percentage of appearing these no.'s.

Then the parameter $\lambda = np$.

Example ① Let X be a r.v with m.g.f $= e^{2(e^t-1)}$

Find $p(X=3)$ and $p(X>2)$.

sol/ $M_X(t) = e^{2(e^t-1)} \Rightarrow \lambda = 2 \Rightarrow X \sim p(2)$.

$$P(X=3) = \frac{e^{-2} \cdot 2^3}{3!} = 0.180$$

$$p(X>2) = 1 - P(X \leq 2)$$

$$= 1 - [f(0) + f(1) + f(2)]$$

$$f(0) = \frac{e^{-2} \cdot 2^0}{0!} = e^{-2}$$

$$f(1) = \frac{e^{-2} \cdot 2^1}{1!} = 2 \cdot e^{-2}$$

$$f(2) = \frac{e^{-2} \cdot 2^2}{2!} = 2 \cdot e^{-2}$$

$$\therefore p(X>2) = 1 - [e^{-2}(1+2+2)]$$

$$= 1 - 5e^{-2}$$

$$= 0.323$$

Example ②:- In a factory 2% of the product ar defective. Determind the prob. that 100 boxes of that product contains at most 3 defective items.

Sol/ $n=100$, $p=0.02$, $\lambda = np \Rightarrow \lambda = 2$

$\therefore X \sim p(2)$

$$f(x) = \frac{e^{-2} \cdot 2^x}{x!}$$

$$P(X \leq 3) = f(0) + f(1) + f(2) + f(3)$$

$$= e^{-2}\left(1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!}\right) = 0.857$$

4 Uniform distribution:-

Defn:- If the r. v. X having the p.d.f.:-

$$f(x) = \frac{1}{N} \quad , x = 1, 2, \dots, N$$

then we say the dist. of X is uniform dist. and N is the parameter of the dist.

* The moments:-

① The mean $\mu_x = E(x) = \frac{N+1}{2}$

② The Variance $\sigma^2_x = V(x) = \frac{N^2-1}{12}$

③ The M.g.f $M_x(t) = \sum_{j=1}^{N} \frac{e^{jt}}{N}$