5 Geometric distribution:-

Defⁿ:- A r.v. x is defined to have gemetric dist. if the density of x is given by:-

$$f(x) = p \cdot q^{x} \quad ; \quad x = 0, 1, --$$

$$x \sim G(p)$$

where:- p is the prob. of success.
q is the prob. of failuer.
x is the number of failure before the first success.

نحسب عدد مرات الفشل قبل أول نجاح.

* The moments:-

① The mean $\mu_x = E(x) = \frac{q}{p}$

② The variance $\sigma^2_x = V(x) = \frac{q}{p^2}$

③ The m.g.f $M_x(t) = \frac{p}{1 - qe^t}$

- Example:- A dic is rolling. Calculate the prob. of getting 1 on the 5 th roll, Then Find the expected number of rolls before 1 comein. ماهي عدد الرميات المتوقعة قبل ظهور الرقم واحد.

Sol:- $p = \frac{1}{6}$, $q = \frac{5}{6}$

في السؤال إيجاد احتمال أن يظهر الرقم واحد في الرمية الخامسة يعني ذلك الرمية الخامسة هي نجاح وما قبلها فشل أي أن $x = 4$

$$f(x) = \left(\frac{1}{6}\right) \cdot \left(\frac{5}{6}\right)^4$$

$$= \frac{625}{7776} = 0.08$$ ← احتمال ظهور الرقم واحد من الرمية الخامسة هو

$$E(x) = \frac{q}{p} = \frac{5/6}{1/6} = 5$$ ← عدد الرميات المتوقعة قبل ظهور رقم ١

$V(x) = ?$

المحاضرة التاسعة :
957 → 962

⊛ **Continuos Distributions:-** التوزيعات المستمرة

— The Important distributions in this type are:-

1. Gamma distribution:-

— The prob. dist. function for Gamma dist. is:-

$$f(x) = \frac{1}{\Gamma\alpha \cdot \beta^{\alpha}} \cdot x^{\alpha - 1} \cdot e^{-\frac{x}{\beta}} \quad ; x > 0, \; \alpha, \beta > 0$$

Then $X \sim Gamma(\alpha, \beta)$ $[\, Ga(\alpha, \beta) \,]$

Where the moments are:-

1. The mean $\mu_x = E(x) = \alpha\beta$
2. The variance $\sigma_x^2 = V(x) = \alpha\beta^2$
3. The m.g.f $M_x(t) = (1 - \beta t)^{-\alpha}$

and $\Gamma\alpha = (\alpha - 1)!$

Example:- For the p.d.f below. Find the distribution, the mean and variance for r.v. X.

$$f(x) = \frac{1}{\Gamma\alpha \cdot 50^{\alpha}} \cdot x^{30} \cdot e^{-\frac{x}{50}} \quad ; x > 0.$$

sol/ $X \sim Ga(31, 50)$

$\Rightarrow \alpha = 31$ and $\beta = 50$

$\therefore E(X) = \alpha \cdot \beta = 31 * 50 = 1550$

$V(X) = \alpha \cdot \beta^2 = 31 * (50)^2 = 77500$

$M_x(t) = (1 - 50t)^{-31}$

2 Exponential distribution :-

- In Gamma dist. when $\alpha = 1$, the r.v. X gone to have exonential dist. with p.d.f. :-

$$f(x) = \frac{1}{\beta} \cdot e^{\frac{-x}{\beta}} \quad ; \quad x > 0 \quad ; \quad \frac{1}{\beta} = \lambda > 0$$

* The moments :-

1 The mean $\mu_x = E(x) = \beta$

2 The variance $\sigma_x^2 = V(x) = \beta^2$

3 The m.g.f $M_x(t) = (1 - \beta t)^{-1}$

3 Chi-square distribution :-

- In Gamma dist. when $\alpha = \frac{K}{2}$ and $\beta = 2$, the r.v.x gone to have chi-square dist. with parameter K which is called degree of freedom; then :-

$$X \sim \chi^2_{(K)}$$

the density function of the r.v.x is :.

$$f(x) = \frac{1}{\Gamma\frac{K}{2} \cdot 2^{K/2}} \cdot x^{\frac{K}{2} - 1} \cdot e^{-\frac{1}{2} \cdot x} \quad , \quad \begin{matrix} x > 0 \\ K > 0 \end{matrix}$$

* the moments :-

1 the mean $\mu_x = E(x) = K$

2 The variance $\sigma_x^2 = V(x) = 2K$

3 the m.g.f $M_x(t) = (1 - 2t)^{-\frac{K}{2}}$