Example: ① If X has the m.g.f:

$$M_X(t) = (1-2t)^{-8}$$; find the dist. the mean and the variance of the r.v. X, then write p.d.f ?

Solution:-

$$X \sim \chi^2_{(16)}$$

$$\Rightarrow E(X) = K = 16$$

$$V(X) = 2K = 32$$

Then the p.d.f is:-

$$f(x) = \frac{1}{\Gamma 8 \ 2^8} \cdot x^7 \cdot e^{-x/2} \quad ; \quad x > 0$$

Example: ② For $X \sim \chi^2_{(8)}$; find $E(3X^2 - 6X + 1)$?

Solution:-

$$E(3X^2 - 6X + 1) = 3E(X^2) - 6E(X) + 1$$

$$\Rightarrow E(X) = K = 8$$

$$V(X) = 2K = 16$$

$$\because V(X) = E(X^2) - (E(X))^2 \Rightarrow 16 = E(X^2) - (8)^2$$

$$\Rightarrow E(X^2) = 8^2 + 16 = 80$$

$$\Rightarrow 3E(X^2) - 6E(X) + 1 = 3(80) - 6(8) + 1 = 289$$

4 Normal distribution:- التوزيع الطبيعي

— One of the importants cont. dist.'s is the normal dist

— the r.v.X has a normal dist. if it's p.d.f is defined by:-

$$f(x)=\frac{1}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad ; -\infty<x<\infty \;,\; \mu \text{ and } \sigma>0.$$

where μ, σ are the parameters of the dist.

* The moments are:-

1 The mean $\mu=E(x)=\mu$

2 The variance $\sigma_x^2=V(x)=\sigma^2$

3 The m.g.f $M_x(t)=e^{\mu t+\frac{1}{2}\sigma^2 t^2}$

Example: 1 :- If the p.d.f of r.v.X is:-

$$f(x)=\frac{1}{4\sqrt{2\pi}}\cdot e^{-\frac{(x+7)^2}{32}} \quad ; -\infty<x<\infty.$$

Find the dist of X, the mean, the variance and the m.g.f.

Sol:- $X\sim N(-7,16)$

$E(X)=\mu=7$

$\sigma^2(x)=V(x)=16$

$M_x(t)=e^{-7t+8t^2}$

Example:- 2 :- If the moment generating function of the r.v. X is:-

$M_x(t)=\exp\{5t+12t^2\}$; Find the dist. of X, write the p.d.f, then find the mean and the variance of X.

Soll:- $X\sim N(5,24)$

$$\Rightarrow f(x)=\frac{1}{4.9\sqrt{2\pi}}\cdot e^{-\frac{(x-5)^2}{48}} \quad ; -\infty<x<\infty$$

$\Rightarrow E(X)=\mu=5$ and $V(x)=\sigma^2=24$

Notes:- ① The mean, median and mode are identical. and

② The Curve of normal dist is symmetric about the mean.

③ The Curves of this dist difference due to the values of the mean and standard deviation.

- لصعوبة إيجاد قيمة التكامل تحت المنحنى للدالة، أي المساحة تحت المنحنى الطبيعي جاءت فكرة التوزيع الطبيعي القياسي بجعل قيم المعالم كالتالي :-

$\mu = 0$ and $\sigma^2 = 1$

معنى ذلك أننا سنأخذ شكل ثابت لمنحنى التوزيع مهما اختلفت قيم المتغير العشوائي .

*** Standard Normal dist:-** التوزيع الطبيعي القياسي

— Any normally distributed random variable can by transformed to the standard normal: such that:-

$$P(X \leq x) = P\left(Z \leq \frac{x-\mu}{\sigma}\right) = \phi(z)$$

- where the value of Z between 0 and 3.09.

$\Rightarrow Z \sim N(0,1)$

where:-
$$Z = \frac{X-\mu}{\sigma}$$

$$\Rightarrow E(Z) = \frac{1}{\sigma} E(X-\mu) = 0$$

$$V(Z) = \frac{1}{\sigma^2} V(X-\mu) = 1$$

and the p.d.f of Z is:-

$$f(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}} \quad ; \quad -\infty < z < \infty$$