

Notes - There is table of the values of  $Z$  called (the normal dist. table). We can find any area under the normal curve in one or two sides about the mean and for these cases -

Then -

$$\textcircled{1} P(Z \geq 0) = P(Z \leq 0) = 0.5$$

$$\textcircled{2} P(0 \leq Z \leq t) = \phi(t) \text{ and } P(-t \leq Z \leq 0) = \phi(-t)$$

$$\textcircled{3} P(Z \geq +t) = P(Z \geq 0) - \phi(t)$$

$$P(Z \geq -t) = \phi(-t) + P(Z \geq 0).$$

where  $\phi(+t) = \phi(-t)$  ;  $t$  any value.

$$\textcircled{4} P(a \leq X \leq b) = P\left(\frac{a-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{b-\mu}{\sigma}\right)$$

$$= P(\bar{a} \leq Z \leq \bar{b}) = \phi(\bar{b}) - \phi(\bar{a})$$

where  $a < b$  and:-

$$P(-a \leq X \leq b) = \phi(\bar{a}) + \phi(\bar{b}).$$

Examples - Find the area under the normal dis. curve in each of the cases.

$$\textcircled{1} P(0 \leq Z \leq 1.2) = \phi(1.2) = 0.3849$$

$$\textcircled{2} P(-0.68 \leq Z \leq 0) = \phi(0.68) = 0.2517$$

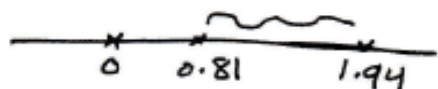
$$\textcircled{3} P(-0.46 \leq Z \leq 2.21) = \phi(0.46) + \phi(2.21)$$

$$= 0.1772 + 0.4864$$

$$= 0.6636.$$

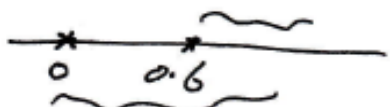
④  $P(0.81 \leq Z \leq 1.94) = \Phi(1.94) - \Phi(0.81)$

$= 0.9738 - 0.7910 = 0.1828$



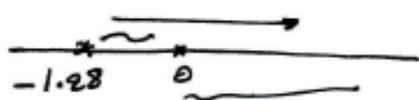
⑤  $P(Z \geq 0.6) = P(Z \geq 0) - \Phi(0.6)$

$= 0.5 - 0.2257 = 0.2743$



⑥  $P(Z \geq -1.28) = \Phi(-1.28) + P(Z \geq 0)$

$= 0.3997 + 0.5 = 0.8997$



⑦  $P(0 \leq Z \leq Z_0) = 0.9901$  Find  $Z_0$ ?

سؤال /  $0.9901$  احتمال عن أي مجال  
 فنجد أنه يشير إلى أن  $Z = 2.33$

⑧ If  $X \sim N(3, 16)$ ; Find  $P(4 \leq X \leq 8)$  and  $P(0 \leq X \leq 5)$ .

Sol  $P(4 \leq X \leq 8) = P\left(\frac{4-3}{4} \leq \frac{X-3}{4} \leq \frac{8-3}{4}\right)$   
 $= P\left(\frac{1}{4} \leq Z \leq \frac{5}{4}\right) \Rightarrow P(0.25 \leq Z \leq 1.25)$   
 $= \Phi(1.25) - \Phi(0.25)$   
 $= 0.8944 - 0.5987$   
 $= 0.2957$

$P(0 \leq X \leq 5) = P\left(\frac{0-3}{4} \leq Z \leq \frac{5-3}{4}\right)$   
 $= P(-0.75 \leq Z \leq 0.5)$   
 $= \Phi(0.5) + \Phi(0.75)$   
 $= 0.6915 + 0.2734$   
 $= 0.4649$

⑨ If  $X \sim N(16, 1)$ , Find  $P(X \geq 17)$

$$\begin{aligned}\text{Sol/ } P(X \geq 17) &= P\left(\frac{X - \mu}{\sigma} \geq \frac{17 - 16}{1}\right) \\ &= P(Z \geq 1) = P(Z \geq 0) - \phi(1) \\ &= 0.5 - 0.3413 = 0.1587\end{aligned}$$

⑩ If  $X \sim N(70, 100)$ , Find  $P(60 \leq X \leq 95)$ ?

$$\begin{aligned}\text{Sol/ } P(60 \leq X \leq 95) &= P\left(\frac{60 - 70}{10} \leq Z \leq \frac{95 - 70}{10}\right) \\ &= P(-1 \leq Z \leq 2.5) \\ &= \phi(-1) + \phi(2.5) \\ &= 0.3413 + 0.4938 \\ &= 0.8351\end{aligned}$$

⑪ The life time of a battery is normally distributed with mean 42 hours and standard deviation of 1.2 hours. Find the prob. that a randomly selected battery with life equal or longer than 40 hours.

Sol/ ( $\sum p = 1 - q$ )

$$\begin{aligned}P(X \geq 40) &= 1 - P(X < 40) \\ &= 1 - P\left(\frac{X - \mu}{\sigma} < \frac{40 - 42}{1.2}\right) \\ &= 1 - P\left(\frac{X - 42}{1.2} < \frac{40 - 42}{1.2}\right) \\ &= 1 - P(Z < 1.67) \\ &= 1 - 0.4525 \\ &= 0.5475\end{aligned}$$