Ex.: If $X \sim Beta(a,b)$, show that

$$E(X) = \frac{a}{a+b}$$

Sol.

$$E(X) = \int_0^1 x \, \frac{1}{\beta(a,b)} \, x^{a-1} (1-x)^{b-1} \, dx$$

$$= \int_0^1 x^1 \, \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} \, x^{a-1} (1-x)^{b-1} \, dx$$

$$= \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} \int_0^1 x^{\cancel{1}+a-\cancel{1}} (1-x)^{b-1} \, dx$$

$$\boxed{\therefore \int_0^1 x^{a-1} (1-x)^{b-1} \, dx} = \beta(a,b)$$

وبالمقارنة نستنتج

مضاف واحد

$$\frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} \left[\beta(\underline{a+1}, b) \right]$$

$$\Rightarrow \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} \times \frac{\Gamma(a+1)\,\Gamma(b)}{\Gamma(a+1+b)}$$

$$= \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} \left[\frac{(a+\cancel{1-1})! \times (b-1)!}{(a+b+\cancel{1-1})!} \right]$$

$$\Rightarrow \frac{(a+b-1)!}{(a-1)!\,\cancel{(b-1)!}} \left[\frac{a!\,\cancel{(b-1)!}}{(a+b)!} \right]$$

$$\Rightarrow \frac{\cancel{(a+b-1)!}}{\cancel{(a-1)!}} \times \frac{a\,\cancel{(a-1)!}}{(a+b)\,\cancel{(a+b-1)!}}$$

$$\Rightarrow E(X) = \frac{a}{a+b}$$

Ex.: On a final examination in math the mean was (72) and the SD was (15). Determine the standared scores of marks

a) 60 b) 93 c) 72

Sol.

$$Z = \frac{X - M}{\sigma}$$

تحول الدرجات الاعتيادية
الى درجات معيارية.

a) $Z = \frac{60 - 72}{15} = -0.8$

b) $Z = \frac{93 - 72}{15} = 1.4$

c) $Z = \frac{72 - 72}{15} = 0$

and then find the marks corresponding to the S. scores

a) -1 b) 1.6

Sol.

$$Z = \frac{X - M}{\sigma}$$

a) $X = \sigma (Z) + M$

$= 15 (-1) + 72 = 57$

b) $X = \sigma (Z) + M$

$= 15 (1.6) + 72 = 96$

~~Ex 1~~ e) $p(Z \geq 0.6)$

Sol.

$p(Z \geq 0) = 0.5$ ملاحظة / دائماً

$p(Z \geq 0) - p(0 \leq Z \leq 0.6)$

$= 0.5 - \phi(0.6)$

$= 0.5 - 0.2257$

$= 0.2743$

f) $p(Z \geq -1.28)$

Sol.

$p(Z \geq 0) + p(0 \leq Z \leq 1.28)$

$= 0.5 + 0.3997$

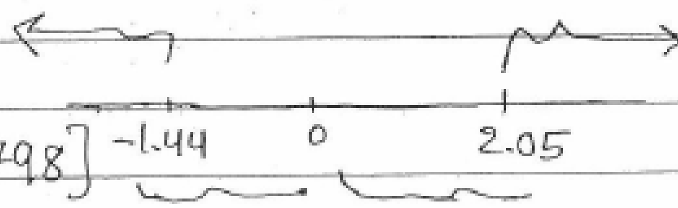
$= 0.8997$

g) $p(2.05 \leq Z \leq -1.44)$

Sol.

$1 - [\phi(2.05)$

$+ \phi(1.44)]$



$= 1 - [0.4251 + 0.4798]$

$= 0.0951$

h) $p(|Z| \leq 0.5) = p(-0.5 \leq Z \leq 0.5)$

$= p(-0.5 \leq Z \leq 0) + p(0 \leq Z \leq 0.5)$

$= p(0 \leq Z \leq 0.5) + p(0 \leq Z \leq 0.5)$

$= 2\,\phi(0.5)$

$= 2\,(0.1915)$

$= 0.3830$