** Relation between two variables (x, y) :-

Correlation and Regression (الارتباط والانحدار)

In this section study the relation between two variable by Correlation and Regression :-

1. **Correlation** :- They are three types of Corr.

a- Simple Corr. [two variables, one dependent, and one independent.].

b- Partial Corr. [one depe. and two indep.]

c- Multiple Corr. [one depe. and more than two var. indep.]

a :- Simple Corr. :-

Defn :- It is a linear relation between two variables. it is denoted by (r), where $-1 \leq r \leq 1$, and it is Calculate by two Coefficients :-

1. **Pearson's Coeff. as** :- معامل بيرسون

$$r_{x,y} = \frac{n \cdot \sum x_i \cdot y_i - [(\sum x_i)(\sum y_i)]}{\sqrt{[n \cdot \sum x_i^2 - (\sum x_i)^2][n \cdot \sum y_i^2 - (\sum y_i)^2]}}$$

or

$$r_{x,y} = \frac{\sum x_i \cdot y_i - n \bar{x} \bar{y}}{\sqrt{[\sum x_i^2 - n \bar{x}^2][\sum y_i^2 - n \bar{y}^2]}}$$

Notes :-
* $r = -1$ (عكسي تام)
* $r = +1$ (طردي تام)
* $r = 0$ لا يوجد ارتباط

* $-\frac{1}{2} < r < \frac{1}{2}$ ضعيف

B1

Example:- Find the Pearson's Coefficient between X and y if:- $n = 8$, $\sum x_i = 56$, $\sum y_i = 44$

$\sum x_i y_i = 359$, $\sum x_i^2 = 524$, $\sum y_i^2 = 256$?

Sol/:-

$$r_{xy} = \frac{n.\sum x_i y_i - [(\sum x_i).(\sum y_i)]}{\sqrt{[n.\sum x_i^2 - (\sum x_i)^2][n.\sum y_i^2 - (\sum y_i)^2]}}$$

$\therefore \bar{X} = \frac{\sum x_i}{n} \Rightarrow \bar{X} = \frac{56}{8} = 7$

and $\bar{y} = \frac{\sum y_i}{n} \Rightarrow \bar{y} = \frac{44}{8} = 5.5$

$$\therefore r_{x,y} = \frac{8.(359) - [(56)(44)]}{\sqrt{[8.(524) - (56)^2].[8.(256) - (44)^2]}}$$

$\therefore r_{x,y} = 0.97$ (طردي تام)

② Sperman's Coefficient :-

The sperman's coefficient of rank correlation it is define as:- $r_k = 1 - \frac{6.\sum d_i^2}{n(n^2-1)}$

where:- ① d_i (تمثل الفرق بين رتب المتغيرين)

② n (عدد المشاهدات)

طريقة الحل:-

① أعطاء رتب لقيم المتغيرات [وفي حالة تكرار القيم يؤخذ الوسط الحسابي]

② ايجاد قيم (d_i) وتطبيق القانون .

Example:- Find the rank Correlation Coefficient between x, y from the following data

x	y	R_x	R_y	d_i^2
85	93	4	5	1
60	75	2	3	1
73	65	3	2	1
40	50	1	1	0
90	80	5	4	1

$$r_k = 1 - \frac{6 \cdot \sum d_i^2}{n(n^2-1)}$$

$$r_k = 1 - \frac{6 \cdot (4)}{5(25-1)} = 1 - \frac{24}{120} = 0.8$$

علاقة طردية قوية .

Example:- Find the rank Corr.

x	y	R_x	R_y	d_i^2
0	3	1	1	0
2	5	2	2	0
3	7	3.5	3	0.25
3	9	3.5	4	0.25
4	11	5	5	0

$$r_k = 1 - \frac{6 \cdot \sum d_i^2}{n(n^2-1)}$$

$$= 1 - \frac{6(0.5)}{5(24)} = 0.97$$

علاقة طردية قوية

(b):- Partial Corr. Coefficient :-

— For X_1, X_2, X_3 three r.v.'s, then the partial Corr. between X_1, X_2, where X_3 is not exist define as:- غير موجود (هنعد).

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}} \quad ; \text{where } -1 \le r_{12.3} \le +1$$

مثال:- X_1 - انفاق شهري (متغير معتمد).

X_2 - دخل الافراد في الاسرة .

X_3 - عدد افراد الاسرة .