Example:- Find the Estimation regression for X/y from the following data, then find $\hat{x}$ if y = 1, 5 and 9

$\sum x_i y_i = 337$, $\sum y_i^2 = 286$, $n = 12$, $\sum x_i = 60$, $\sum y_i = 48$?

Sol/:- $\because X_i = \beta_0 + \beta_1 y_i + e_i$

$$\hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum y_i^2 - n\bar{y}^2} = \frac{337 - n(\bar{x})(\bar{y})}{286 - n(\bar{y})^2}$$

$\because \bar{x} = \frac{\sum x_i}{n} = \frac{60}{12} = 5$ and $\bar{y} = \frac{\sum y_i}{n} = \frac{48}{12} = 4$

then $\hat{\beta}_1 = \frac{337 - [(12)(5)(4)]}{286 - [12(4)^2]} = \frac{337 - 240}{286 - 192} = \frac{97}{94} = \boxed{1.03}$

$\because \hat{\beta}_0 = \bar{x} - \hat{\beta}_1 \bar{y} = 5 - (1.03)(4) = 0.88$

then $\hat{x}_i = \beta_0 + \beta_1 y_i = 0.88 + 1.03 y_i$

$\hat{x}_1 = (0.88) + (1.03)(1) = 1.91$

$\hat{x}_2 = (0.88) + (1.03)(5) = 6.03$

$\hat{x}_3 = (0.88) + (1.03)(9) = 10.15$

Note:- there is arelation between Corr. and Reg. by:-

* For Reg. Equation y/x

$$\hat{\beta}_{y/x} = \frac{S_y}{S_x} * r_{x,y}$$

* For Reg. Equation x/y

$$\hat{\beta}_{x/y} = \frac{S_x}{S_y} \cdot r_{x,y}$$, where

S_x, S_y the standard divaiation for x, y respectively.

$r_{x,y}$ the Corr. Coefficient.

② Coefficient of determination :-

— It is the squar of Corr. Coeff. as :-

$$R^2 = (r_{x,y})^2 \quad ; \quad 0 \leq R^2 \leq 1$$

or

$$R^2 = \hat{\beta}^2_{y/x} * \frac{S_x^2}{S_y^2} \quad ; \text{ For equation } y/x .$$

and

$$R^2 = \hat{\beta}^2_{x/y} + \frac{S_y^2}{S_x^2} \quad ; \text{ For equation } x/y .$$

— For Large value of R^2, that is mean the indep. Varible, and for $R^2 = 0$, There is no effect from independed variable.

Example :- From the data below Find :-

① The simple Corr. coefficied -

② The coefficied of determination.

③ The regression Coefficient of y/x from ② .

Where $\sum x_i = 56$, $\sum y_i = 44$, $\sum x_i y_i = 359$, $\sum x_i^2 = 524$

$\sum y_i^2 = 256$, $n = 8$, $S_y = 2.83$, $S_x = 4.34$

Sol/ ① $r_{xy} = 0.98$ (اسلوب بيرسون)

② $R^2 = (r_{x,y})^2 = 0.96$

③ $\hat{\beta}^2_{y/x} = \frac{S_y^2}{S_x^2} * R^2$

$\Rightarrow \hat{\beta}^2_{y/x} = \frac{(2.83)^2}{(4.34)^2} . (0.96) = 0.41$

$\therefore \hat{\beta}_{y/x} = 0.6$.

$\frac{B}{8}$

Example:- If the Estimated Reg. equ. of y/x given by:- $\hat{y} = 12.65 - 0.95 X_i$, Find:-

① $\hat{\beta}_0$ and reg. coefficient.

② $\hat{y}$ where $X = 4, 8, 12$

③ e_q where $X_q = 4$ and $y_q = 9$?

Sol/:- ① $\hat{\beta}_0 = 12.65$ and the Reg. Coeff. $\hat{\beta}_1 = -0.95$

② $\hat{y} = 12.65 - 0.95(4) = 8.84$

$\hat{y} = 12.65 - 0.95(8) = 5.05$

$\hat{y} = 12.65 - 0.95(12) = 1.25$

③ $e_q = y_q - \hat{y}_q$

$\Rightarrow \hat{y}_q = 12.65 - 0.95(4) = 8.85$

$\Rightarrow e_q = 9 - 8.85 = \underline{0.15}$

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* **Multiple Linear Regression:-**

— It is Estimation the Reg. Equ. between many variables, one of them is (dependent) define as :-

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + e_i$$

Where:- y is depe. var.

X_1, X_2 are indep. var.

β_0: Constant of Equation

β_1, β_2: Reg. Coefficients.