Conv

ومن خلال ما سبق بالإمكان إيجاد التباين المشترك بين المتغيرين x, y (Covergance) الذي صيغته ، (Covariance) ←

$$Cov(x,y) = E(x\,y) - E(x)E(y)$$

أما في حالة x, y مستقلين فإن :-

$$f(x,y) = f(x)\,f(y) \quad , x,y \text{ indep}$$

and $cov(x,y) = 0$

Example: Let we have the following joint probability table.

x \ y	2	3	4	جمع افقي وعمودي يعطي (1)
1	0.06	0.15	0.09	0.30
2	0.14	0.35	0.21	0.70
	0.20	0.50	0.30	

Find if $f(x,y)$ a J pdf

① $p(x=y)$, $p(x=2, y \leq 3)$

② $f(x)$, $f(y)$, $E(x,y)$

Sol

$$p(x=y) = p(x=2, y=2) = 0.14$$

$$p(x=2, y \leq 3) = p(x=2, y=2) + p(x=2, y=3)$$

$$= 0.14 + 0.35 = .49$$

x	1	2
$f(x)$	0.30	0.70

y	2	3	4
$f(y)$	0.20	0.50	0.30

$$E(xy) = \sum_{x=1}^{2} \sum_{y=2}^{4} xy\, f(x,y)$$

$$= (1)(2)(0.06) + (1)(3)(0.15) + (1)(4)(0.09) + (2)(2)(0.14) + (2)(3)(0.35) + (2)(4)(0.21)$$

$$= 5.27$$

Example:- Let x and y be two r.v's with JPdf given as

$$f(x,y) = \begin{cases} \frac{3}{5} x(y+x) & 0<x<1, \\ & 0<y<2 \\ 0 & \text{o.w.} \end{cases}$$

Show it is realy JPdf and, then find

$$P(0<x<\tfrac{1}{2}, 1<y<2)$$

Sol.

$$\int_x \int_y f(x,y)\, dy\, dx = 1$$

$$\int_0^1 \int_0^2 \frac{3}{5} x(y+x)\, dy\, dx$$

$$\int_0^1 \frac{3}{5} x\left(\frac{y^2}{2} + xy\right)\Big|_0^2 dx$$

$$\int_0^1 \frac{3}{5} x\left(\frac{4}{2} + 2x\right) dx$$

$$\int_0^1 \frac{3}{5} x(2+2x)\, dx$$

$$\frac{3}{5}\left(\frac{2x^2}{2} + \frac{2x^3}{3}\right)\Big|_0^1$$

$$= \frac{3}{5}\left(1 + \frac{2}{3}\right) = \frac{3}{5} \cdot \frac{5}{3} = 1$$