Review Some Informations

[1] $\sum_{i=1}^{n} X_i = X_1 + X_2 + X_3 + \cdots + X_n$ مجموع n من المتغيرات

$$\sum_{i=1}^{n} i = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{6} i = 1+2+3+4+5+6 = \frac{6(7)}{2} = 21$$

[2] $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{n}$ (مجموع مربعات الأعداد)

$$\sum_{i=1}^{6} i^2 = 1^2+2^2+3^2+4^2+5^2+6^2 = \frac{6(7)(13)}{6} = 91$$

$$\Rightarrow \sum_{i=1}^{n} i^2 \neq \left(\sum_{i=1}^{n} i\right)^2$$

$$91 \neq (21)^2$$

[3] $\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Notes:

ⓐ $\sum_{i=1}^{n} K = K+K+K+\cdots+K = nK$: K is Constant

$$\sum_{a}^{n} K = (n-a+1)K \Rightarrow \sum_{i=3}^{10} -2 = (10-3+1)(-2) = -16$$

ⓑ $\sum_{i=1}^{n} KX_i = K\sum_{i=1}^{n} X_i \Rightarrow \sum_{i=1}^{n} -2X_i = -2\sum_{i=1}^{n} X_i$

ⓒ $\sum_{i=1}^{n} (aX_i \mp b) = a\sum_{i=1}^{n} X_i \mp bn$

$$= \sum_{i=1}^{n} aX_i \mp \sum_{i=1}^{n} b$$

Permutations:- التباديل

إن عملية التباديل هي عملية اختيار r من الأشياء من أصل n من الأشياء بحيث أن الترتيب يؤثر على النتيجة $n \geq r$

i.e $P_r^n = \frac{n!}{(n-r)!}$

*For examples:-

1) $6! = 6(5)(4)(3)(2)(1) = 720$

$P_3^5 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5(4)(3)2!}{2!} = 5 \times 4 \times 3 = 60$

2) $P_2^n = 30$ Find the value of n?

Sol/

$$\frac{n!}{(n-r)!} = \frac{n!}{(n-2)!} = 30$$

$$\Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 30 \quad \Rightarrow n(n-1) = 30$$

$$n^2 - n - 30 = 0 \quad \Rightarrow (n-6)(n+5) = 0$$

$$n - 6 = 0 \Rightarrow \boxed{n = 6}$$

or $n + 5 = 0 \Rightarrow \boxed{n = -5}$ — does

Ex5:- $4n^3 - 2n^2 - 3n = 18$

Notes:

1. $P_1^n = n$

   $P_1^{10} = 10$

2. $P_n^n = n!$ and $0! = 1$

   $P_5^5 = 5! = 5(4)(3)(2)(1) = 120$

3. number of ways can be seated n pearson a row.

   $P_n^n = n!$

Examples- In how many ways can (9) person seted in a row?

Sol:- $P_9^9 = 9! =$

4. number of ways can be seated n person in a circale ? عدد الطرق الممكنة لجلوس n شخص على شكل دائرة.

   $P_{n-1}^{n-1} = (n-1)!$

Example: In how many ways can (9) person seater in around table? حول الطاولة.

Sol $P_{n-1}^{n-1} = P_{9-1}^{9-1} = P_8^8 = 8!$

5. عدد الطرق الممكنة لاختيار n من الأشياء في حالة وجود تكرارات في القيم:-

$$P_{n_1, n_2, \dots, n_k}^{n} = \frac{n!}{r_1! \cdot r_2! \cdot r_3! \cdots r_k!}$$

Examples- In how may ways you can arrange the word "MESSISSIPE"

Sol/ $P_{r_1, r_2, \dots, r_k}^{n} = P_{1,2,4,2,1}^{10} = \frac{10!}{1!\,2!\,4!\,2!\,1!} =$