Combinations and Permutations

Permutation: **A set of objects in which position (or order) is important.**
Combination: **A set of objects in which position (or order) is NOT important.**

A permutation **is the choice of r things from a set of nthings without replacement and where the order matters.**

A combination **is the choice of r things from a set of nthings without replacement and where order does notmatter.**

Example 1: What is the total number of possible 4-letter arrangements of the letters m, a, t, h if each letter is used only once in each arrangement?

Example 2: There are 12 boys and 14 girls in Mrs. Schultzkie's math class. Find the number of ways Mrs. Schultzkie can select a team of 3 students from the class to work on a group project. The team is to consist of 1 girl and 2 boys.

Example 3: From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

A. 564 **B. 645**

C. 735 **D. 756**

E. None of these

Example 4: In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

A. 360 **B. 480**

C. 720 **D. 5040**

E. None of these

Example 5: In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

A. 810
B. 1440
C. 2880
D. 50400
E. 5760

Example 6: In how many ways can the letters of the word 'LEADER' be arranged?

A. 72
B. 144
C. 360
D. 720
E. None of these

Example 7: A die is rolled, find the probability that an even number is obtained.

Solution:

- Let us first write the sample space S of the experiment.

  S = {1,2,3,4,5,6}

- Let E be the event "an even number is obtained" and write it down.

  E = {2,4,6}

- We now use the formula of the classical probability.

  ∴P(E) = m(E) / n(S) = 3 / 6 = 1 / 2

Example 8: Two coins are tossed, find the probability that two heads are obtained. **Note:** Each coin has two possible outcomes H (heads) and T (Tails).

Solution:

- The sample space S is given by.

  S = {(H,T),(H,H),(T,H),(T,T)}

- Let E be the event "two heads are obtained".

  E = {(H,H)}

- We use the formula of the classical probability.

  ∴P(E) = m(E) / n(S) = 1 / 4

Example 9: Which of these numbers cannot be a probability?

a) -0.00001
b) 0.5
c) 1.001
d) 0
e) 1
f) 20%

Solution:

- A probability is always greater than or equal to 0 and less than or equal to 1, hence only **a)** and **c)** above cannot represent probabilities: -0.00010 is less than 0 and 1.001 is greater than 1.

Example 9: Two dice are rolled, find the probability that the sum is

a) equal to 1

b) equal to 4

c) less than 13

Solution:

- a) The sample space S of two dice is shown below.

  S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
  (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
  (3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
  (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
  (5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
  (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) }

- Let E be the event "sum equal to 1". There are no outcomes which correspond to a sum equal to 1, hence

  $\therefore P(E) = m(E) / n(S) = 0 / 36 = 0$

- b) Three possible outcomes give a sum equal to 4: E = {(1,3),(2,2),(3,1)}, hence.

  $\therefore P(E) = m(E) / n(S) = 3 / 36 = 1 / 12$

- c) All possible outcomes, E = S, give a sum less than 13, hence.

  $\therefore P(E) = m(E) / n(S) = 36 / 36 = 1$

Example 10: A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.

Solution:

- The sample space S of the experiment described in question 5 is as follows
  S = { (1,H),(2,H),(3,H),(4,H),(5,H),(6,H)
  (1,T),(2,T),(3,T),(4,T),(5,T),(6,T)}

- Let E be the event "the die shows an odd number and the coin shows a head". Event E may be described as follows

  E={(1,H),(3,H),(5,H)}

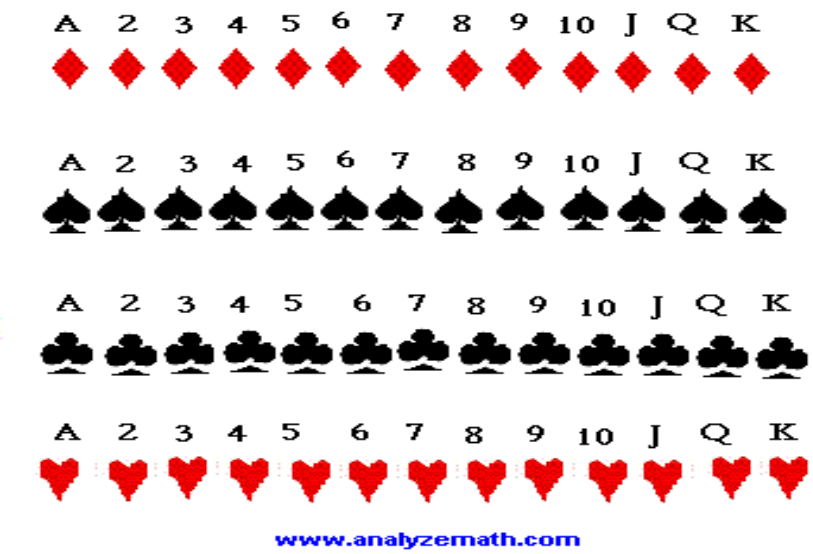
- The probability P(E) is given by

  $\therefore P(E) = m(E) / n(S) = 3 / 12 = 1 / 4$

Example 11: A card is drawn at random from a deck of cards. Find the probability of getting the 3 of diamond.

Solution:

- The sample space S of the experiment in question 6 is shwon below



- Let E be the event "getting the 3 of diamond". An examination of the sample space shows that there is one "3 of diamond" so that m(E) = 1 and n(S) = 52. Hence the probability of event E occurring is given by

  $\therefore P(E) = 1 / 52$

Example 12: A card is drawn at random from a deck of cards. Find the probability of getting a queen.

Solution:

- The sample space S of the experiment in question 7 is shwon above (see question 6)
- Let E be the event "getting a Queen". An examination of the sample space shows that there are 4 "Queens" so that m(E) = 4 and n(S) = 52. Hence the probability of event E occurring is given by

∴P(E) = 4 / 52 = 1 / 13

Example 13: A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white?

Solution: We first construct a table of frequencies that gives the marbles color distributions as follows

color	frequency
red	3
green	7
white	10

- We now use the empirical formula of the probability

$$P(E) = \frac{\text{Frequency for white color}}{\text{Total frequencies in the above table}}$$

∴P(E) = 10 / 20 = 1 / 2

Example 14: The blood groups of 200 people is distributed as follows: 50 have type **A** blood, 65 have **B** blood type, 70 have **O** blood type and 15 have type **AB** blood. If a person from this group is selected at random, what is the probability that this person has O blood type?

Solution: We construct a table of frequencies for the the blood groups as follows

group	frequency
a	50
B	65
O	70
AB	15

- We use the empirical formula of the probability

$$P(E) = \frac{\text{Frequency for O blood}}{\text{}}$$

Total frequencies

$\therefore P(E) = \mathbf{70 / 200 = 0.35}$

Exercises:

a) A die is rolled, find the probability that the number obtained is greater than 4.

b) Two coins are tossed, find the probability that one head only is obtained.

c) Two dice are rolled, find the probability that the sum is equal to 5.

d) A card is drawn at random from a deck of cards. Find the probability of getting the King of heart.

MORE COMBINATION AND PERMUTATION PRACTICE PROBLEMS:

1.Suppose that 7 people enter a swim meet. Assuming that there are no ties, in how many ways could the gold, silver, and bronze medals be awarded?

2.How many different committees of 3 people can be chosen to work on a special project from a group of 9 people?

3. A coach must choose how to line up his five starters from a team of 12 players. How many different ways can the coach choose the starters?

4. John bought a machine to make fresh juice. He has five different fruits: strawberries, oranges, apples, pineapples, and lemons. If he only uses two fruits, how many different juice drinks can John make?

5. How many different four-letter passwords can be created for a software access if no letter can be used more than once?

6.How many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from.

7. There are 25 people who work in an office together. Five of these people are selected to go together to the same conference in Orlando, Florida. How many ways can they choose this team of five people to go to the conference?

8. There are 25 people who work in an office together. Five of these people are selected to attend five different conferences. The first person selected will go to a conference in Hawaii, the second will go to New York, the third will go to San Diego, the fourth will go to Atlanta, and the fifth will go to Nashville. How many such selections are possible?

9. John couldn't recall the Serial number on his expensive bicycle. He remembered that there were 6 different digits, none used more than once, but couldn't remember what digits were used. He decided to write down all of the possible 6 digit numbers . How many different possibilities will he have to create?

10. How many different 7-card hands can be chosen from a standard 52-card deck?

11. One hundred twelve people bought raffle tickets to enter a random drawing for three prizes. How many ways can three names be drawn for first prize, second prize, and third prize?

12. A disc jockey has to choose three songs for the last few minutes of his evening show. If there are nine songs that he feels are appropriate for that time slot, then how many ways can he choose and arrange to play three of those nine songs?