**Lecture 1**

**CH1: Functions**

**S1.1 : Functions and Their Graphs**

**Definition : A function *f* ( or a mapping *f* ) from a set  *A* to a set *B* is a rule that assigns to each element a of *A* exactly one element b of *B* . The set *A* is called the domain of *f* and the set *B* is called the codomain of *f* . If *f* assigns b to a , then b is called the image of a under *f* . The subset of *B* comprised of all the images of elements of *A* under *f* ( which is denoted by  ) is called the image of *A* under *f* ( or the range of *f* ) .**

**We use** ** to mean that *f*  is a function from *A* to *B* . We will write *f* ( a ) = b to indicate that b is the image of a under *f .***

**Example 1.1.1:**

**Let *A* = { 2, 4, 5 } ,  *B* = { 1, 2, 3, 6 } , and  be the function defined by  *f* ( 2 ) = 1 , *f* ( 4 ) = 3 , *f* ( 5 ) = 6 . Then the domain of *f* is**

 ***A* = { 2, 4, 5 } , the codomain of *f*  is *B* = { 1, 2, 3, 6 }, and the range**

**of  *f* = { 1, 3, 6 } .**

**Counter example :**

**Let *C* = { 1, 2, 3, 4 } and *D* = { 2, 3, 4, 5 } , and let  *h* be the rule defined by  *h* ( 1 ) = 2 , *h* ( 1 ) = 4 , *h* ( 2 ) = 3 , *h* ( 3 ) = 5 , *h* ( 4 ) = 4 , then *h* is not a function from  *C* to *D* since there are two different elements ( 2 and 4 ) belong to *D* are assigned to the same element 1 of  *C* .**

**Example 1.1.2: Find the domain and the range of the function *f* defined by  .**

**Solution : For  to be real , *x* + 10 must be greater than or equal to 0 . That is ,  which means that  .**

**Thus the domain is  and the range is  .**

**Exercises:**

**1) Let *A* = { 2, 4, 5,7 } ,  *B* = { 1, 2, 3, 6,9 } , and  be the**

 **function defined by  *f* ( 2 ) = 9 , *f* ( 4 ) = 3 , *f* ( 5 ) = 6 , *f* ( 7 ) = 2. Find**

 **the domain of *f*  , the codomain of *f*  , and the range of *f* .**

**2) Find the domain and the range of the function *f*  defined by**

 **.**

**3) Find the domain and the range of the function *f*  defined by**

 ** .**

**Definition: The graph of a function *f* is the line passing through all**

**the points ( *x* , ) on the  *x y* - plane .**

**Definition: The *y* - coordinate of the point where a graph of a function**

**intersect the *y* - axis is called the *y* - intercept of the function .**

**Definition: The *x* - coordinate of a point where a graph of a function**

**intersects the *x* - axis is called an *x* - intercept of the function .**

**Remarks :**

**1) The graph of any function *f* has at most one *y* - intercept . The**

 **graph of the function *f* has exactly one *y* - intercept if 0 is in the**

 **domain of the function *f* and the *y* - intercept is *f* (** 0 **) .**

**2) The graph of any function *f* has no *x* - intercept if there is no *x***

 **in the domain of the function *f* such that *f* ( *x* ) =** 0 **.**

 **The graph of a function *f* has one or more than one *x* - intercepts**

 **if *f* ( *x* ) =** 0 **for some *x* in the domain of *f* , and the number of**

 ***x* - intercepts is the number of the distinct solutions of the**

 **equation *f* ( *x* ) =** 0 **.**

**Properties of Functions :**

1. **A function** $y=f(x)$ **is called an even function of** $x$ **if**

$ f\left(-x\right)=f\left(x\right) , ∀ x $ **.**

1. **A function** $y=f(x)$ **is called an odd function of** $x$ **if**

$ f\left(-x\right)=-f\left(x\right) , ∀ x $ **.**