**Lecture 5**

**Algebra of Functions**

**Definition: The sum , difference , product , and quotient of the functions *f* and *g* are the functions defined by**

**( *f* + *g* ) ( *x* ) = *f* ( *x* ) + *g* ( *x* ) sum function**

**( *f* – *g* ) ( *x* ) = *f* ( *x* ) – *g* ( *x* ) difference function**

**( *f* . *g* ) ( *x* ) = *f* ( *x* ) . *g* ( *x* ) product function**

** quotient function**

**The domain of each function is the intersection of the domains of *f* and *g* , with the exception that the values of *x* where *g*( *x* ) = 0 must be excluded from the domain of the quotient function .**

**Definition: Let *f* and *g*  be functions , then *f***  o ***g*  is called the composite of *g* and *f* and is defined by the equation**

**( *f***  o ***g* )( *x* ) = *f* ( *g* ( *x* ) ) .**

**The domain of *f***  o ***g* is the set  .**

**Example 1.3.13 : Let *f* and *g* be the functions defined by**

** and  . Find the functions *f* + *g* , *f* – *g***

 **, *f* . *g* , , *f***  o ***g*  , *g*** o ***f* and find their domains .**

**Solution :**

**( *f* + *g* ) ( *x* ) = *f* ( *x* ) + *g* ( *x* ) =  +  **

**( *f* – *g* ) ( *x* ) = *f* ( *x* ) –  *g* ( *x* ) =   **

**( *f* . *g* ) ( *x* ) = *f* ( *x* ) . *g* ( *x* ) =  .  **

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**( *f***  o ***g* )( *x* ) = *f* ( *g* ( *x* ) ) = *f*  =  =**

**( *g***  o ***f* )( *x* ) = *g* ( *f* ( *x* ) ) = *g*  = **

 **= =**

**The domain of *f* = R**

**The domain of *g* = R**

**The intersection of the domains of *f* and *g* is R**

**Thus the domain of each of the functions  *f* + *g* , *f* – *g* , *f* . *g* , *f***  o ***g***

**, and *g*** o ***f* is R .**

**The domain of** **** = **** .

**Remark : The domain of any polynomial function is R .**

**Example 1.3.14 : Let *f* and *g* be the functions defined by**

***f* ( *x* ) = *x* + 5 and *g* ( *x* )** $=x^{2} - 3$ **, Find *f* o *g* ( *x* ) , *g* o *f* ( *x* ) ,**

***f* o *g* ( 3) and *g* o *f* ( 3) .**

**Solution: *f* o *g* ( *x* ) = *f* ( *g* ( *x* ) )** $=$$ f \left( x^{2} - 3 \right)$

$= x^{2} - 3+5 $

$= x^{2}+ 2 $

 ***g* o *f* ( *x* )** $=$ ***g* ( *f* ( *x* ) )** $ =$ ***g* ( *x* + 5 )**

$ =$$(x+5)^{2}-3$

$ = x^{2} +10 x+25-3 $

$= x^{2} +10 x+22 $

***f* o *g* ( 3)** $= (3)^{2}+2=9 +2=11 $

***g* o *f* ( 3)** $=$$ (3)^{2} +10 (3)+22$$ = 9+30+22 = 61$

**Exersice 1.3.15 : Let *f* and *g* be the functions defined by**

 ***f* ( *x* ) = *x* – 4 and  . Find the functions *f* + *g* , *f* – *g***

 **, *f* . *g* ,  and find their domains .**