# Chapter Four

## The Electrostatic Field In Dielectric Media

An ideal dielectric material is one which has no free charges. All material media are composed of molecules, and the molecules of the dielectric are certainly affected by the presence of an electric field. The electric field causes a force to be exerted on each charged particle, positive particles pushed in the direction of the field, negative particles oppositely, so that the positive and negative parts of each molecule are displaced from their equilibrium positions in opposite direction. The dielectric is said to be polarized. The polarization of the dielectric depends on the total electric field in the medium, but a part of the electric field is produced by the dielectric itself.

### 1. Polarization

Consider a small volume element $\Delta v$ of a dielectric medium which, as a whole, is electrically neutral. If the medium is polarized, then a separation of positive and negative charge has been effected, and the volume element is characterized by an electric dipole moment

$$\Delta \vec{p} = \int_{\Delta v} \vec{r}\, dq \qquad (1)$$

Since $\Delta\vec{p}$ depends on the size of the volume element, it is more convenient to work with $\vec{P}$, the electric dipole moment per unit volume is:

$$\vec{P} = \frac{\Delta\vec{p}}{\Delta v} \qquad (2)$$

$\vec{P}$ must be defined as the limit of this quantity as $\Delta v$ becomes very small from the macroscopic point of view. In this way $\vec{P}$ becomes a point function, $\vec{P}(x,y,z)$, $\vec{P}$ is called the electric polarization, or the polarization of the medium. Its dimensions are charge per unit area; in mks units, $C/m^2$.

It is apparent that $\vec{P}(x,y,z)$ is a vector quantity which, in each volume element, has the direction of $\Delta\vec{p}$. This, in turn, has the direction of displacement of positive charge relative to the negative charge (see fig.(1)).

$\Delta v$ contains many molecules. It is sometimes desirable to speak about the electric dipole moment of a single molecule, that is,

$$\vec{p}_m = \int_{molecule} \vec{r}\, dq \qquad (3)$$

The dipole moment associated with $\Delta v$ is given by $\Delta\vec{\mathrm{p}} = \sum \vec{\mathrm{p}}_m$, where the summation extends over all molecules inside the element $\Delta v$. Hence,

$$\vec{P} = \frac{1}{\Delta v}\sum_m \vec{\mathrm{p}}_m \qquad (4)$$

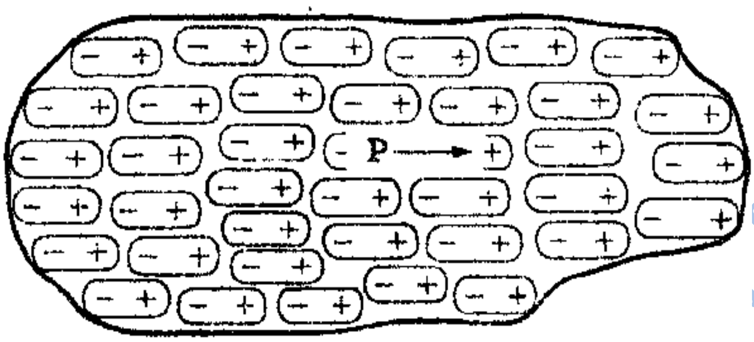


*Fig.(1): A piece of polarized dielectric material. Each volume element is represented as a dipole $\Delta\vec{p}$.*

Although fig.(1) represents each volume element of the polarized dielectric as a small dipole, it may be more instructive to visualize the dielectric in terms of its molecules, and to imagine that each dipole of Fig.1 represents a single molecule.

## 2.External Field of a Dielectric Medium

Consider a finite piece of dielectric material which is *polarized*, i.e., which is characterized at each point $\vec{r}'$ by a polarization $\vec{P}(\vec{r}')$. The polarization gives rise to an electric field, and our problem is to calculate the field at point $\vec{r}$ which is outside of the dielectric body (see Fig.2). Firstly, calculate the potential $U(\vec{r})$, and obtain the electric field as minus the gradient of $U$.

Each volume element $\Delta v'$ of the dielectric medium is characterized by a dipole moment $\Delta\vec{p} = \vec{P}\Delta v'$, and since the distance between $(x, y, z)$ and $\Delta v'$ is large compared with the dimensions of $\Delta v'$, thus, the dipole moment completely determines $\Delta v'$'s contribution to the potential:

$$\Delta U(\vec{r}) = \frac{\Delta\vec{\boldsymbol{p}}.(\vec{r}-\vec{r}')}{4\pi\epsilon_0|\vec{r}-\vec{r}'|^3} = \frac{\vec{\mathrm{P}}(\vec{r}').(\vec{r}-\vec{r}')\Delta\boldsymbol{v}'}{4\pi\epsilon_0|\vec{r}-\vec{r}'|^3} \qquad (5)$$

Here $\vec{r}-\vec{r}'$ is the vector directed out from $\boldsymbol{\Delta v'}$**,** whose magnitude is given by

$$|\vec{r}-\vec{r}'| = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2} \qquad (6)$$

The entire potential at point $\vec{r}$ is obtained by summing the contributions from all parts of the dielectric:

$$U(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{V_0} \frac{\vec{P}(\vec{r}').(\vec{r}-\vec{r}')\boldsymbol{dv'}}{4\pi\epsilon_0|\vec{r}-\vec{r}'|^3} \qquad (7)$$

If the functional form $\vec{P}$ is known, the function $U$ can be evaluated directly from equation (7)

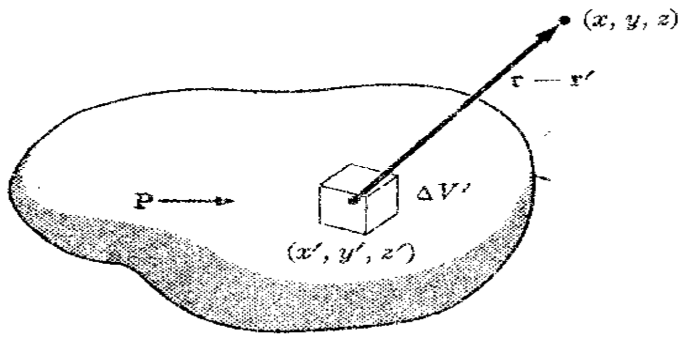


*Fig.(2): The electric field at $(x, y, z)$ may be calculated by summing up the contributions due to the various volume elements $\Delta v'$ in $V_0$. The surface $V_0$ is denoted by $S_0$.*

If $|\vec{r}-\vec{r}'|$ is given by equation(6), then

$$\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = +\frac{(\vec{r}-\vec{r}')}{||\vec{r}-\vec{r}'||^3} \qquad (8)$$

Where $\vec{\nabla}'$ operator involves derivatives with respect to the primed coordinates.

$$\vec{\nabla}' = \frac{\partial}{\partial x'}\hat{a}_x + \frac{\partial}{\partial y'}\hat{a}_y + \frac{\partial}{\partial z'}\hat{a}_z$$

The integrand of (7) may be transformed by means of (8)

$$\frac{\vec{P}.(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \vec{P}.\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) \qquad (9)$$

Equation (9) may be transformed by means of the vector following quantity

$$div\varphi\vec{F} = \varphi\, div\vec{F} + \vec{F}.\vec{\nabla}\varphi$$

$$div' f\vec{A} = f\ div'\ \vec{A}\ + \vec{A}.\vec{\nabla}' f \qquad (10)$$

where $f$ is any scalar point function and $\vec{A}$ is an arbitrary vector point function.

Letting f=(1/$|\vec{r} - \vec{r}'|$) and $\vec{A} = \vec{P}$, the integrand (9), becomes

$$\frac{\vec{P}.(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = div'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right) - \frac{1}{|\vec{r}-\vec{r}'|} div'\vec{P} \qquad (11)$$

Finally, the potential in (7) may be written as

$$U(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{V_0} [div'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right) - \frac{1}{|\vec{r}-\vec{r}'|} div'\vec{P}]dv'$$

Using the divergence theorem on the first term of the last equation, then,

$\int_{V_0} div'(\frac{\vec{P}}{|\vec{r}-\vec{r}'|})\, dv' = \oint_{S_0} \frac{\vec{P}.\hat{n}da'}{|\vec{r}-\vec{r}'|}$ , *thus, the potential in equation (7) becomes*

$$U(\vec{r}) = \frac{1}{4\pi\epsilon_0}\oint_{S_0} \frac{\vec{P}.\hat{n}da'}{|\vec{r}-\vec{r}'|} + \frac{1}{4\pi\epsilon_0}\int_{V_0} \frac{(-div'\vec{P})}{|\vec{r}-\vec{r}'|} dv' \qquad (12)$$

Where $\hat{n}$ is the outward normal to the surface element $da'$

The quantities $\vec{P}.\hat{n}$ and $-div\vec{P}$ which appear in the integral of equation (12) are two scalar functions obtained from the polarization $\vec{P}$. Since they have dimensions of charge per unit area and charge per unit volume, respectively, we write

$$\sigma_P = \vec{P}.\hat{n} \qquad (13)$$

and $\rho_P = -div\vec{P}$ (14)

$\sigma_P$ and $\rho_P$ are called polarization charge densities or bound charge densities. The term "bound charge" is used to emphasize that the charges are not free to move around or be extracted from the dielectric material.

The surface density of bound charge is given by the component of the polarization normal to the surface, and the volume density of bound charge is a measure of the nonuniformity of the polarization inside the material.

The potential due to the dielectric material is,

$$U(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\oint_{S_0}\frac{\sigma_P\, da'}{|\vec{r}-\vec{r}'|} + \int_{V_0}\frac{\rho_P}{|\vec{r}-\vec{r}'|}dv'\right] = \frac{1}{4\pi\epsilon_0}\int\frac{dq_P'}{|\vec{r}-\vec{r}'|} \qquad (15)$$

Therefore, the dielectric material has been replaced by an appropriate distribution of bound charge.

The total polarization charge of a dielectric body,

$$Q_p = \int_{V_0}\left(-div'\vec{P}\right)dv' + \oint_{S_0}\vec{P}.\hat{n}\, da') \qquad (16)$$

must equal zero, since it was our premise that the dielectric as a whole is electrically neutral. This result is immediately obvious from the form of equation (16), which clearly vanishes as a consequence of the divergence theorem.

We now have two distinct expressions for the electrostatic potential $U(\vec{r})$ due to a polarized dielectric specimen, namely, equations (7) and (15). Both are correct, but we shall find the latter expression (equation (15)) more convenient in most cases.

The electric field $\vec{E}$ may be obtained as minus the gradient of equation (15). Since $U(\vec{r})$ is a function of the coordinates$(x, y, z)$, the appropriate gradient is $-\vec{\nabla}$.

$$\vec{E}(\vec{r}) = -\vec{\nabla}U = -\frac{1}{4\pi\epsilon_0}\vec{\nabla}\left[\oint_{S_0}\frac{\sigma_P\, da'}{|\vec{r}-\vec{r}'|} + \int_{V_0}\frac{\rho_P}{|\vec{r}-\vec{r}'|}dv'\right]$$

Since $\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = -\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$

$$\vec{E}(\vec{r}) = -\vec{\nabla}U = \frac{1}{4\pi\epsilon_0}\vec{\nabla}'\left[\oint_{S_0}\frac{\sigma_P\, da'}{|\vec{r}-\vec{r}'|} + \int_{V_0}\frac{\rho_P}{|\vec{r}-\vec{r}'|}dv'\right]$$

Since $\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\int_{S_0}\frac{\sigma_P\,(\vec{r}-\vec{r}')\, da'}{|\vec{r}-\vec{r}'|^3} + \int_{V_0}\frac{\rho_P\,(\vec{r}-\vec{r}')\, dv'}{|\vec{r}-\vec{r}'|^3}\right] \qquad (17)$$

***Problem.1:*** *A thin dielectric rod with cross section area A and length L located on the x-axis at the range x=0 and x=L, and the polarization through the rod is* $\vec{P} = (ax^2 + b)\hat{a}_x$. *Find* $\sigma_p$ *and* $\rho_b$ *and show that* $Q_p=0$, *see Fig.3.*

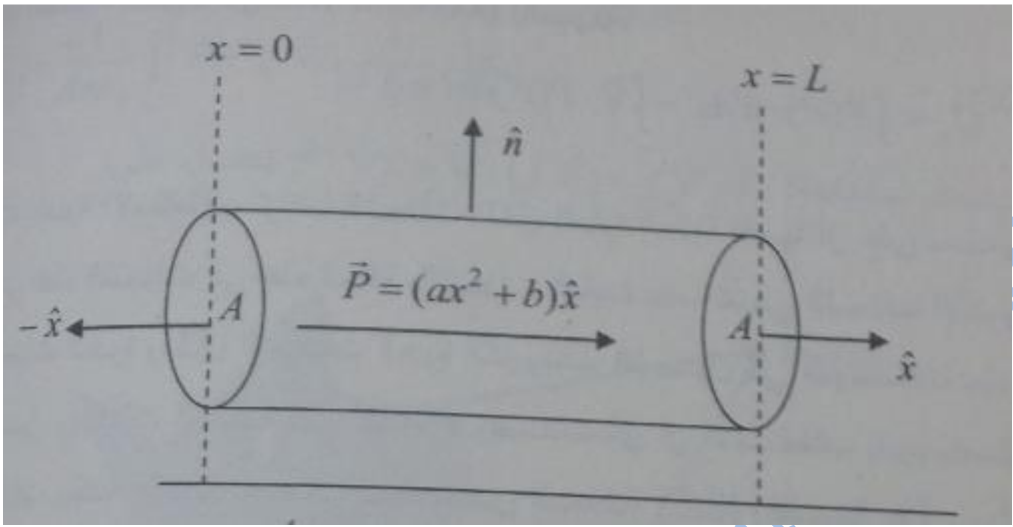


*Fig.(3): A thin dielectric rod with polarization in the x-direction.*

***Solution:***

It is noted that the surface of the rod is formed of three parts are: The circular part and the two bases. On the circular part $\sigma_p = \vec{P}\,.\hat{n} = 0$, while on the two bases the surface charge density is

$$\sigma_p = \hat{a}_x.\vec{P}\big|_{x=L} + (-\hat{a}_x).\vec{P}\big|_{x=0} = \hat{a}_x.(aL^2+b)\hat{a}_x + (-\hat{a}_x).b\hat{a}_x$$

$$\sigma_p = aL^2 + b - b = aL^2$$

While the volume charge density becomes

$$\rho_p = -\vec{\nabla}.\vec{P} = -\frac{\partial}{\partial x}(ax^2+b) = -2ax$$

and the total polarized charge is

$$Q_p = \int \rho_p dv + \int \sigma_p dS = \int_0^L (-2ax)Adx + aL^2A = -aL^2A + aL^2A = 0$$

***Problem.2:*** *A dielectric cub of side L and center at the origin has a radial polarization given* $\vec{P} = a\vec{r}$, *where a is a constant and* $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$. *Find all bound charge densities* $\sigma_p$ , $\rho_p$, *and show that the total bound charge vanishes* $Q_p=0$, *see Fig.(4).*

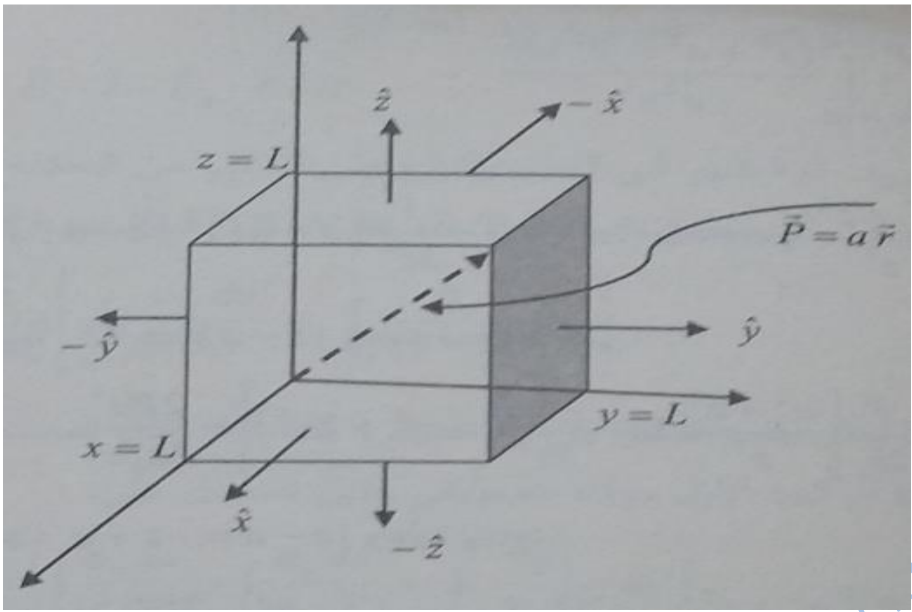


*Fig.(4): A dielectric cub of side L centered at the origin with radial polarization.*

**<u>Solution:</u>** *The surface charge density is*

$$\sigma_p = \hat{a}_x.\vec{P}\big|_{x=L} + (-\hat{a}_x).\vec{P}\big|_{x=0} + \hat{a}_y.\vec{P}\big|_{x=L} + (-\hat{a}_y).\vec{P}\big|_{y=0} + \hat{a}_z.\vec{P}\big|_{z=L} + (-\hat{a}_z).\vec{P}\big|_{z=0}$$

$$= aL + aL + aL = 3aL$$

The volume charge density is

$$\rho_p = -\vec{\nabla}.\vec{P} = -a\left(\frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}z\right) = -3a$$

and $Q_p = \int \rho_p dv + \int \sigma_p dS = -3aAL + 3aAL = 0$

## 3. <u>The Electric Field Inside a Dielectric</u>

The electrostatic field in a dielectric must have the same basis properties which we found applied to $\vec{E}$ in vacuum; in particular, $\vec{E}$ is a conservative field, and hence derivable from a scalar potential. Thus,

$Curl\vec{E} = 0$ Or, equivalently, $\oint \vec{E}.d\vec{l} = 0$

Apply the last equation to the path ABCD shown in the Fig.5, where the segment AB lies in a needle-shaped cavity cut out of the dielectric, and the segment CD lies in the dielectric proper. Since the segments AD and BC may be small, the line integral reduces to,

$\vec{E}_v.\vec{l} - \vec{E}_d.\vec{l} = 0$ or equivalently, $E_{vt} = E_{dt}$ (18)

Where the subscript $v$ and $d$ refer to vacuum and dielectric respectively, and the subscript t stands for tangential component.

***The important conclusion***

*The electric field in a dielectric is equal to the electric field inside a needle-shaped cavity in the dielectric provided the cavity axis is oriented parallel to the direction of the electric field.*

Evidently, the problem of calculating the electric field inside a dielectric reduces to calculating the electric field inside a needle-shaped cavity in the dielectric. But the electric field in the cavity is *an external field*, and hence may be determined by means of the results of section-2. Just as in section-2, we assume here that the polarization of the dielectric is a given function $\vec{P}(x', y', z')$, and we calculate the potential and electric field arising from the polarization. Taking the field point at the center of the cavity and using equation (15), we obtain for the potential

$$U(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{V_0-V_1}\frac{\rho_P(x',y',z')\,dv'}{|\vec{r}-\vec{r}'|} + \frac{1}{4\pi\epsilon_0}\int_{S_0-S'}\frac{\sigma_P(x',y',z')\,da'}{|\vec{r}-\vec{r}'|} \qquad (19)$$

Where $V_0 - V_1$ is the volume of the dielectric excluding the ''needle'', $S_0$ is the exterior surface of the dielectric, and $S' = S_1 + S_2 + S_c$ are the needle surfaces. But from Fig.5 it is seen that $\sigma_P = 0$ on the cylindrical surface $S_c$ of the needle; furthermore, may be made arbitrary thin so that the surfaces $S_1$ and $S_2$ have negligible area. Thus, only the exterior surfaces of the dielectric contribute, and the surface integral of equation (19) becomes identical in form of the surface integral of equation (15). The volume integral of equation (19) excludes the cavity; however, the contribution of the cavity to this integral is negligible.

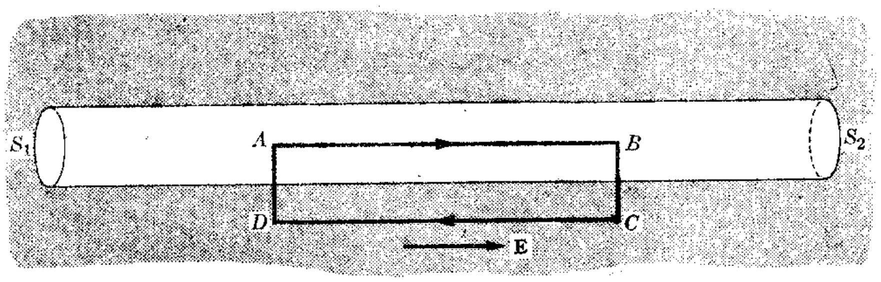


*Fig.(5): The path ABCD lies partly in the needle-shaped cavity, and partly in the dielectric. In an isotropic dielectric(see Fig.5) the polarization $\vec{P}$ has the direction of $\vec{E}$, so that, for the orientation of the needle shown, $\sigma_P = 0$ on the cylindrical walls. In an anisotropic dielectric, $\sigma_P$ is not necessary zero, but its value does not affect the longitudinal component of electric field in the cavity.*

## 4. Gauss's Law in a Dielectric: The Electric Displacement

In chapter two we derived an important relationship between electric flux and charges, namely, Gauss; law. *This law states that the electric flux across an arbitrary closed surface is proportional to the total charge enclosed by the surface.*

In applying Gauss's law to a region containing free charges embedded in a dielectric, we must be careful to include all of the charge inside the gaussian surface, bound charge as well as free charge.

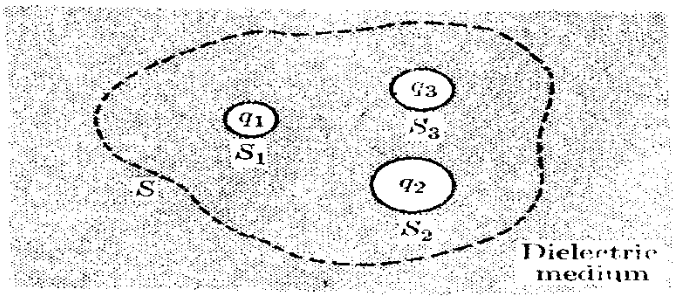


*Fig.(6): Construction of a gaussian surface S in a dielectric medium.*

In Fig.(6) the dashed surface S is an imaginary closed surface located inside a dielectric medium. There is a certain amount of free charge, Q, in the volume bounded by S, and we shall assume that this free charge exists on the surfaces of three conductors in amounts $q_1$, $q_2$, and $q_3$. By Gauss's law,

$$\oint_S \vec{E}.\hat{n}da = \frac{1}{\varepsilon_0}(Q + Q_p) \qquad (20)$$

Where $Q$ is the total free charge, i.e., $Q = q_1 + q_2 + q_3$

and $Q_p$ is the polarization charge:

$$Q_p = \int_{S_1+S_2+S_3} \vec{P}.\hat{n}da + \int_V (-div\vec{P})dv \qquad (21a)$$

Here $V$ is volume of the dielectric enclosed by S. There is no boundary of the dielectric at S, so that the surface integral in equation (21a) does not contain a contribution from S.

If we transform the volume integral in equation(21a) to a surface integral by means of the divergence theorem, we must be careful to include contributions from all surfaces bounding $V$, namely, S, $S_1$, $S_2$, and $S_3$.

$\int_V \; div\,\vec{F}\,dv = \oint_S \; \vec{F}.\hat{n}\,da$ (The divergence theorem)

By applying this theorem to the volume integral part of equation (21a), obtaining;

$$\int_V \left(-div\,\vec{P}\right)dv = -\oint_{S+S_1+S_2+S_3} \vec{P}.\hat{n}\,da$$

It is evident that the last three contributions will be cancel the first term of equation (21a), so that

$$Q_p = -\oint_S \; \vec{P}.\hat{n}\,da \qquad (21b)$$

Combining this result with equation (20) as follows;

$$\oint_S \; \vec{E}.\hat{n}da = \frac{1}{\varepsilon_0}\left(Q - \oint_S \; \vec{P}.\hat{n}\,da\right)$$

$\oint_S \; \varepsilon_0\vec{E}.\hat{n}da + \oint_S \; \vec{P}.\hat{n}\,da = Q$, Thus, we obtain

$$\oint_S \; \left(\varepsilon_0\vec{E} + \vec{P}\right).\hat{n}\,da = Q \qquad (22)$$

Equation (22) states that the flux of the vector $\varepsilon_0\vec{E} + \vec{P}$ through a closed surface is equal to the total free charge enclosed by the surface. We define, a new macroscopic field vector $\vec{D}$, The electric displacement:

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \qquad (23)$$

Which evidently has the same unit as $\vec{P}$, charge per unit area. In terms of $\vec{D}$ equation (22) becomes

$$\oint_S \; \vec{D}.\hat{n}\,da = Q \qquad (24)$$

And this is usually referred to as Gauss's law for the electric displacement. Equation (24) is applicable to a region of space bounded by any closed surface S; if we apply it to a small region in which all of the free charge enclosed is distributed as a charge density $\rho$, the Gauss' law becomes

$$\oint_S \; \vec{D}.\hat{n}\,da = \rho\Delta v$$

By replacing the charge in equation (24) by the integral of the volume charge density and using the divergence theorem, we obtain the differential form of Gauss' law, as follows

$$\oint_S \vec{D}.\hat{n}\, da = \int_v \rho\, dv$$

$$\int_v div\, D\, dv = \int_v \rho\, dv$$

Thus, the differential form of Gauss' law is

$$div\, D = \rho \qquad (25)$$

The advantage of the procedure just followed is that the total electrostatic field at each point in the dielectric medium is expressed as the sum of two parts.

$$\vec{E}(x,y,z) = \frac{1}{\varepsilon_0}\vec{D}(x,y,z) - \frac{1}{\varepsilon_0}\vec{P}(x,y,z) \qquad (26)$$

Where the first term, $\frac{1}{\varepsilon_0}\vec{D}$, is related to the free charge density through its divergence, and the second term, $-\frac{1}{\varepsilon_0}\vec{P}$, is proportional to the polarization of the medium. In a vacuum the electric field is given entirely by the first term in equation (26).

## 5. <u>Electric Susceptibility and Dielectric Constant</u>

In the introduction of this chapter it was stated that the polarization of a dielectric medium occurs in response to the electric field in the medium. The degree of polarization depends not only on the electric field, but also on the properties of molecules which make up the dielectric material. From the macroscopic point of view, the behavior of the material is completely specified by an experimentally determined relationship, $\vec{P} = \vec{P}(\vec{E})$, where $\vec{E}$ is the macroscopic electric field, if $\vec{E}$ varies from point to point in the material, then $\vec{P}$ will vary accordingly.

For most material $\vec{P}$ vanishes when $\vec{E}$ vanishes. Since this is the usual behavior, we shall limit our discussion here to materials of this type. (dielectric with a permanent polarization will be discussed briefly in chapter five.

If the material is isotropic, then the polarization should have the same direction as the electric field which causing it. These results are summarized by the equation:

$$\vec{P} = \chi(E)\vec{E} \qquad \textbf{(27)}$$

Where the scalar quantity $\chi(E)$ is called the *electric susceptibility* of the material, which is more or less a measure of how susceptible (or sensitive) a given dielectric is to electric fields. A great many materials are electrically isotropic; this category includes fluids, polycrystalline and amorphous solid, and some crystals.

Note: A treatment of the electrical properties of anisotropic materials is beyond the scope of the $4^{th}$ stage study.

Combining (27) with (23), we obtain an expression for $\vec{D}$ in isotropic media:

$$\vec{D} = \varepsilon_0\vec{E} + \chi(E)\vec{E}$$

$$\vec{D} = \varepsilon(E)\vec{E} \qquad (28)$$

$$\varepsilon(E) = \varepsilon_0 + \chi(E) \qquad (29)$$

Where $\varepsilon(E)$ is the permittivity of the material. It is evident that $\varepsilon$, $\varepsilon_0$, and $\chi$ all have the same units.
If $\chi$ and $\varepsilon$ are constants characteristic of the material, materials of this type will be called *linear dielectrics*, and they obey the relations

$$\vec{P} = \chi\vec{E} \qquad (30a)$$
$$\vec{D} = \varepsilon\vec{E} \qquad (30b)$$

The electrical behavior of a material is now completely specified by either the permittivity $\varepsilon$ or the susceptibility $\chi$.

It is more convenient, to work with a dimensionless quantity $k$ defined by:

$$\varepsilon = k\varepsilon_0 \qquad (31)$$

$K$ is called the dielectric coefficient (relative permittivity), or simply the dielectric constant. From (29) it is evident that

$$k = \frac{\varepsilon}{\varepsilon_0} = 1 + \frac{\chi}{\varepsilon_0} \qquad (32)$$

However, the dielectric constant (or relative permittivity) $k$ is the ratio of the permittivity of the dielectric to that of free space.

It should also be noticed that $k$ is a dimensionless quantity whereas $\varepsilon$, $\varepsilon_0$ and $\chi$ are in farads/meter.

The dielectric constants for a few commonly encountered materials are given in Table-1. Except for a few examples in which the polarization $\vec{P}$ of the material is specified, the problems in this book deal with linear dielectrics.
If the electric field in a dielectric is made very intense, it will begin to pull electrons completely out of the molecules, and the material, will become conducting. 'The maximum electric field which a dielectric can withstand without breakdown is called its dielectric strength. The dielectric strengths $E_{max}$, of a few substances are also given in Table-1.

*Table-1: Properties of dielectric materials, (dielectric constant K and dielectric strength $E_{max}$.*

| Material | $K$ | $E_{max}$, volts/m |
|---|---|---|
| Glass† | 5–10 | $9 \times 10^6$ |
| Mica | 6.0 | $5\text{–}20 \times 10^6$ |
| Nylon | 3.5 | $16 \times 10^6$ |
| Rubber† | 2–3.5 | $16\text{–}40 \times 10^6$ |
| Sulfur | 4.0 | |
| Wood† | 2.5–8.0 | |
| Alcohol, ethyl (0°C) | 28.4 | |
| Benzene (0°C) | 2.3 | |
| Petroleum oil | 2.1 | $12 \times 10^6$ |
| Water (distilled, 0°C) | 88.0 | |
| Water (distilled, 20°C) | 80.0 | |
| Air (1 atm) | 1.00059 | $3 \times 10^6$ |
| Air (100 atm) | 1.0548 | |
| $CO_2$ (1 atm) | 1.000985 | |

## 6. Point Charge in a Dielectric Fluid

One of the simplest problems involving a dielectric is that of a point charge $q$ in a homogeneous isotropic medium of infinite extent. The dielectric medium will be assumed to be linear and characterized by a dielectric constant *k*.
If the point charge $q$ were situated in a vacuum, the electric field would be a pure *radial field*. But since $\vec{E}$, $\vec{D}$, and $\vec{P}$ are all parallel to one another at each point, the radial nature of the field is not having by the presence of the medium. Furthermore, from the symmetry of the problem, $\vec{E}$, $\vec{D}$, and $\vec{P}$ can depend only on the distance from the point charge, not on any angular coordinate.
Apply Gauss's law (equation (24)) to spherical surface of radius $r$ which is located concentrically about $q$, for convenience, $q$ will be located at the origin. Then

$\oint_S \vec{D}.\hat{n}\, da = Q$ , $4\pi r^2 D = q$, or

$$\vec{D} = \frac{q}{4\pi r^3}\vec{r} \qquad (33)$$

The electric field and polarization may now be evaluated quite easily:

$$\vec{E} = \frac{q}{4\pi k\varepsilon_0 r^3}\vec{r} \qquad (34)$$

$$\vec{P} = \frac{(k-1)q}{4\pi K r^3}\vec{r} \qquad (35)$$

Thus the electric field is smaller by the factor K than would be the case if the medium were absent.

Now, we try to see why the dielectric has weakened the electric field. The electric field has its origin in all of the charge, bound and free. The free charge is just the point charge $q$. The bound charge, however, is made up from two contributions, a volume density $\rho_P = -\boldsymbol{div}\vec{P}$, and a surface density $\sigma_P = \vec{P}.\hat{n}$ on the surface of the dielectric in contact with point charge. Using equation (35), we find that $\boldsymbol{div}\vec{P}$ vanishes, so there is no volume density of the bound charge in this case.

Our point charge $q$ is a point in the macroscopic sense. Actually, it is large on a molecular scale, and we can assign to it a radius b which eventually will be made to approach zero. The total surface bound charge is then given by

$$Q_P = \lim_{b\to 0} 4\pi b^2 \left(\vec{P}.\hat{n}\right)_{r=b} = -\frac{(k-1)q}{k} \qquad (36)$$

The total charge, $\quad Q_P + q = \frac{1}{k}q \qquad (37)$

Appears as a point charge from the macroscopic point of view, and it is now clear why the electric field is a factor K smaller than it would be if the medium were absent. A schematic diagram of the point charge $q$ in a dielectric medium is shown in Fig.7.

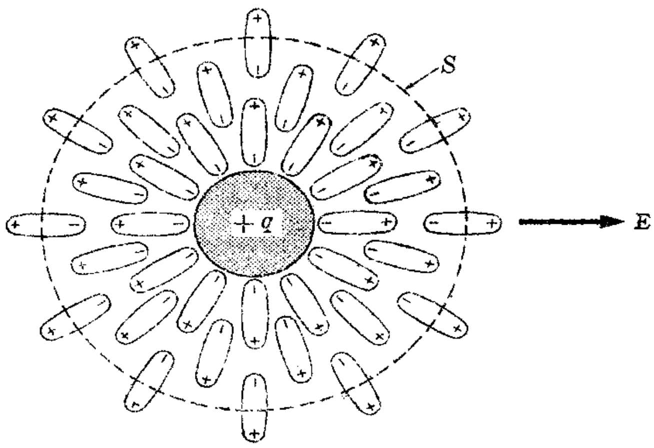

Fig.(7): Schematic diagram showing the orientation of polarized molecules in a dielectric medium surrounding a ‘’point charge’’ $q$.

## 7.<u>Boundary Conditions on the Field Vectors</u>

Before we can solve more complicated problems, we must know that how the field vectors, $\vec{E}$ and $\vec{D}$ change in passing an interface between two media. The two media may be two dielectrics with different properties, or a dielectric and a conductor, vacuum may be treated as a dielectric with permittivity $\varepsilon_0$.
Consider two media, 1 and 2 in contact as shown in Fig.8, if there is a surface density of free charge, σ, which may vary from point to point on the interface. If a small pillbox-shaped surface S which intersects the interface and encloses an area ΔS of the interface, the height of the pillbox being negligibly small in comparison with the diameter of the bases. The free charge enclosed by S is

$$\sigma\Delta S + \frac{1}{2}(\rho_1 + \rho_2) \times volume$$

but the volume of the pillbox is negligibly small, so that the last term may be neglected. Applying Gauss’s law to S, we find

$$\vec{D}_2.\hat{n}_2\Delta S + \vec{D}_1.\hat{n}_1\Delta S = \sigma\Delta S, \text{ or}$$

$$(\vec{D}_2 + \vec{D}_1).\hat{n}_2 = \sigma \qquad (38a)$$

Since $\hat{n}_2$ may serve as the normal to the interface,

$$D_{2n} - D_{1n} = \sigma \qquad (38b)$$

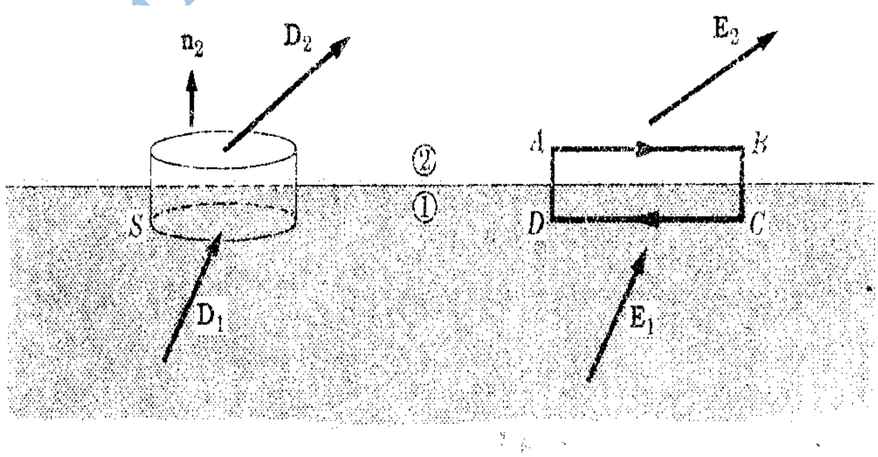

*Fig.(8): Boundary conditions on the field vectors at the interface between two media may be obtained by applying Gauss' law to S and integrating $\vec{E}.d\vec{l}$ around the path ABCD.*

Thus, the discontinuity in the normal component of $\vec{D}$ is given the surface density of free charge on the interface. In another way, if there is no free charge on the interface between two media, the normal component of $\vec{D}$ is continuous, this means that $D_{2n} = D_{1n}$. Thus, the normal component of $\vec{D}$ is continuous across the interface, that is, $D_n$ undergoes no change at the boundary. Since, $\vec{D} = \boldsymbol{\varepsilon}\vec{E}$, $\varepsilon_2 E_{2n} = \varepsilon_1 E_{1n}$ showing that the normal component of $\vec{E}$ is discontinuous at the boundary.

Because the electrostatic field $\vec{E}$ may be obtained as minus the gradient of a potential, the line integral of $\vec{E}.d\vec{l}$ around any closed path vanishes. Applying this result to the rectangular path ***ABCD*** of the figure. On this path the lengths ***AB*** and ***CD*** will be taken equal to $\Delta l$ and the segments *AD* and *BC* will be assumed to be negligibly small. Therefore,

$$\vec{E}_{\mathbf{2}}.\Delta\vec{l} + \vec{E}_{\mathbf{1}}.(-\Delta\vec{l}) = 0\ , \text{ or}$$

$$(\vec{E}_{\mathbf{2}} - \vec{E}_{\mathbf{1}}).\Delta\vec{l} = 0 \qquad (39a)$$

Hence, the desired result:

$$E_{2t} = E_{1t} \qquad (39b)$$

That is, the tangential component of the electric field is continuous across an interface. Since, $\vec{D} = \boldsymbol{\varepsilon}\vec{E}$**,** thus, equation (39) can be written as

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

That is, $D_t$ undergoes some change across the interface. Hence, $D_t$ is said to discontinuous across the interface.

The above results have been obtained for two arbitrary media. If medium 1 is taken as the conductor, then $E_1 = 0$. Since $E_1$ vanishes, there is no polarization, and by equation (23) $D_1$ also vanishes. Thus equations (38b) and (39b) become

$D_{2n} = \sigma$ (40)

$E_{2t} = 0$ (41)

for the displacement and electric field in a dielectric just outside of a conductor. From the discussion above and in preceding sections, it may be inferred that the electric displacement $\vec{D}$ is closely related to free charge.

We should now like to prove an important property of $\vec{D}$, namely, that the flux of $\vec{D}$ is continuous in regions containing no free charge. We imagine a tube of displacement, a volume bounded on the sides by lines of $\vec{D}$ (see Fig.9). The tube is terminated at its ends by the surfaces $S_1$ and $S_2$. Applying Gauss's law, we obtain,

$$\int_{S_2} \vec{D}.\hat{n}\, da - \int_{S_1} \vec{D}.\hat{n}'\, da = Q \quad (42)$$

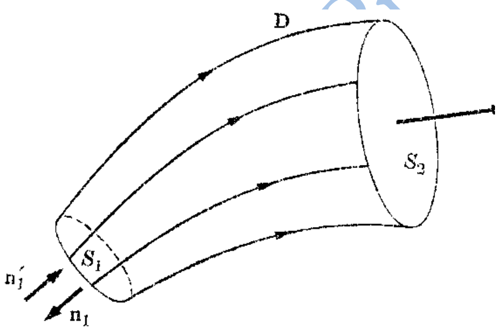


*Fig.(9): A tube of displacement flux.*

If there is no free charge in the region, then $Q = 0$, and the same amount of flux enters the tube through $S_1$ as leaves through $S_2$. When free charge is present, it determines the discontinuous in the displacement flux, thus lines of displacement terminate on free charges. The lines of force, on the other hand, may terminate on free or bounded charges.

As mentioned earlier, the boundary conditions are usually applied in finding the electric field on one side of the boundary given the field on the other side. Besides this, we can use the boundary conditions to determine the "refraction" of the electric field across the interface. Consider $\vec{D}_1$ or $\vec{E}_1$, and $\vec{D}_2$ or $\vec{E}_2$ making angles $\theta_1$ and $\theta_2$ with the *normal* to the interface as illustrated in Fig.10. Using equation (39b), we have

$$E_1 \sin\theta_1 = E_{1t} = E_{2t} = E_2 \sin\theta_2 \text{ , or } \quad E_1 \sin\theta_1 = E_2 \sin\theta_2$$

Similarly, by applying the equation $D_{2n} = D_{1n}$ or $\varepsilon_2 E_{2n} = \varepsilon_1 E_{1n}$

$$\varepsilon_1 E_1 \cos\theta_1 = D_{1n} = D_{2n} = \varepsilon_2 E_2 \cos\theta_2 \quad \text{or} \quad \varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$$

Dividing the latter two equations, gives

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

Since $\varepsilon_1 = \varepsilon_0 k_1$ and $\varepsilon_2 = \varepsilon_0 k_2$, the last equation becomes

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{k_1}{k_2}$$

This is the *law of refraction* of the electric field at a boundary free of charge (since $\sigma = 0$ is assumed at the interface). Thus, in general, an interface between two dielectrics produces bending of the flux lines as a result of unequal polarization charges that accumulate on the sides of the interface.

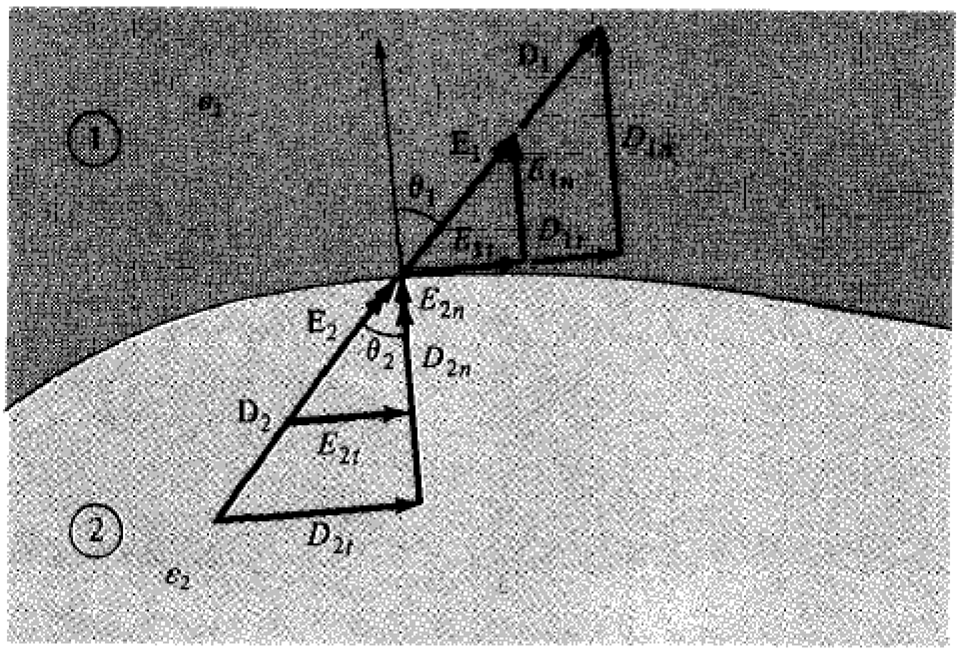

Fig.(10): Refraction of $\vec{D}$ or $\vec{E}$ at a dielectric-dielectric boundary.

## Special Cases of boundary conditions

The boundary conditions on the field vectors above may be considered a general case which is called ***dielectric-dielectric boundary conditions*** in the present we take special two cases:

### I. Conductor-Dielectric Boundary Conditions

This is the case shown in Fig.11. The conductor is assumed to be perfect (i.e.,$\sigma \to \infty$ $or$ $\rho_c \to 0$ ). Although such a conductor is not practically realizable, we may regard conductors such as copper and silver as though they were perfect conductors.

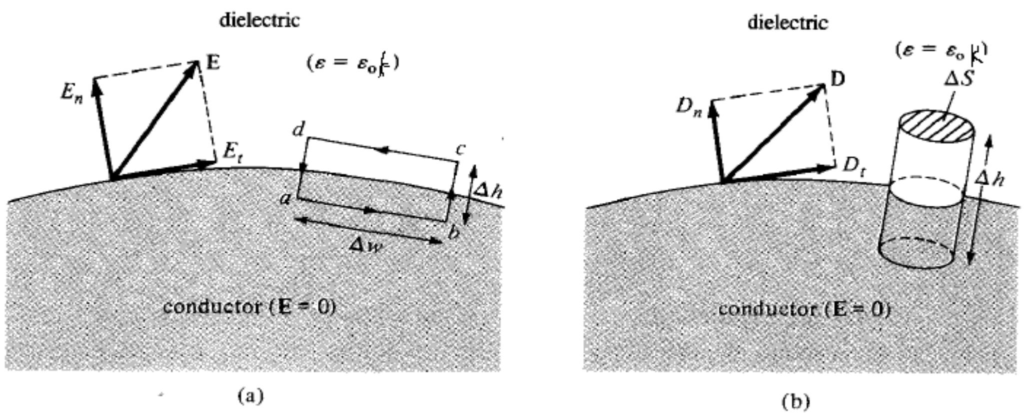


Fig.(11): Conductor-dielectric boundary.

To determine the boundary conditions for a conductor-dielectric interface, we follow the same procedure used for dielectric-dielectric interface except that we incorporate the fact that E = 0 inside the conductor. Applying the equation $\oint \vec{E}.d\vec{l} = 0$ to the closed path *abcda* of Fig.(11a), gives

$$0 = 0.\Delta w + 0.\frac{\Delta h}{2} + E_n.\frac{\Delta h}{2} - E_t.\Delta w - E_n.\frac{\Delta h}{2} - 0.\frac{\Delta h}{2}$$

as $\Delta h \rightarrow 0$

$E_t = 0$

Similarly, by applying the equation $\oint \vec{D}.d\vec{S} = Q$ to the pillbox of Fig.(11b) and letting $\Delta h \rightarrow 0$, we get

$\Delta Q = D_n.\Delta S - 0.\Delta S$ ,

because $\vec{D} = \varepsilon\vec{E} = 0$ inside the conductor, the latter equation may be written as

$D_n = \frac{\Delta Q}{\Delta S} = \sigma$ or $D_n = \sigma$

Thus, under static conditions, the following conclusions can be made about a perfect conductor:

1- No electric field may exist *within* a conductor; that is, $\rho = 0,\ \vec{E} = 0$
2. Since $\vec{E} = -\vec{\nabla}V = 0$, there can be no potential difference between any two points in the conductor; that is, a conductor is an equipotential body.
3. The electric field $\vec{E}$ can be external to the conductor and *normal* to its surface; that is

$$D_t = \varepsilon_0 k E_t = 0,\ D_n = \varepsilon_0 k E_n = \sigma \qquad (*)$$

II. **<u>Conductor-Free Space Boundary Conditions</u>**

This is a special case of the conductor-dielectric conditions and is illustrated in Fig.12. The boundary conditions at the interface between a conductor and free space can be obtained from equation (*) by replacing $k$ by 1 (because free space may be regarded as a special dielectric for which $k = 1$ ). We expect the electric field E to be external to the conductor and normal to its surface. Thus the boundary conditions are

$$D_t = \varepsilon_0 E_t = 0,\ D_n = \varepsilon_0 E_n = \sigma \qquad (**)$$

It should be noted again that equation(**) implies that E field must approach a conducting surface normally.

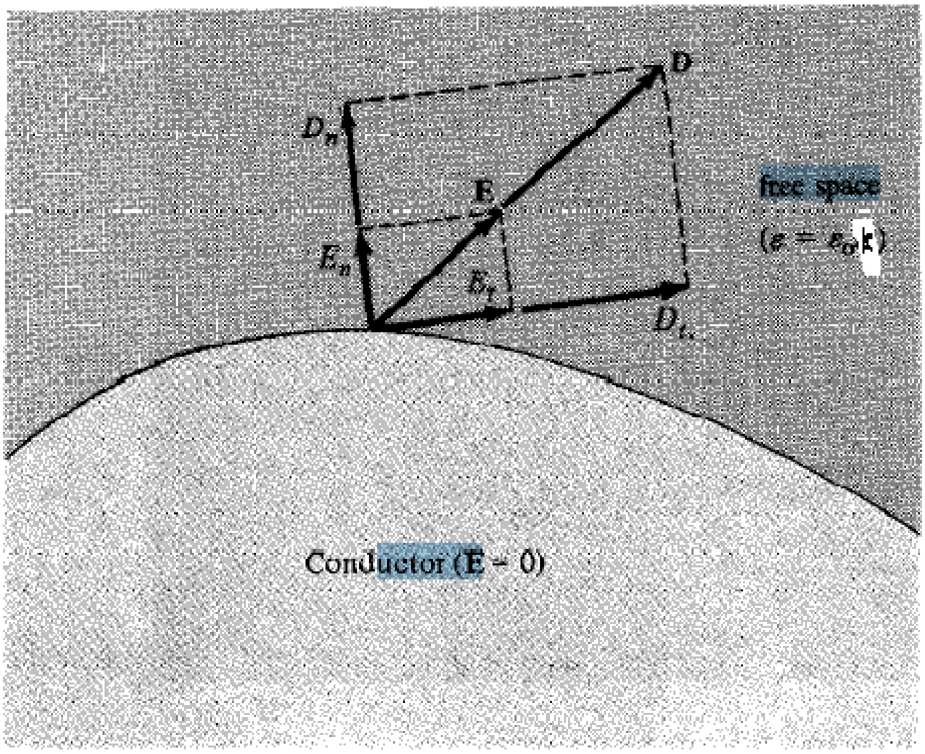


*Fig.(12): Conductor-free space boundary.*



***Problem.3:*** *Two extensive homogeneous isotropic dielectrics meet on plane z = 0. For* $z \geq 0$, $k_1 = 4$ *and for* $z \leq 0$, $k_2 = 3$. *A uniform electric field* $\vec{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ *kV/m exists for z > 0. Find*

*(a)* $\vec{E}_2$ *for* $z \leq 0$

*(b) The angles* $E_1$ *and* $E_2$ *make with the interface*

***Solution:*** Let the problem be as illustrated in Fig.13.

(a) Since $\hat{a}_z$ is normal to the boundary plane, we obtain the normal components as

$$E_{1n} = \vec{E}_1.\hat{a}_n = \vec{E}_1.\hat{a}_z = 3, \qquad E_{1n} = 3\hat{a}_z$$

Also, $E_{2n} = (\vec{E}_2.\hat{a}_z)\hat{a}_z$

$$\vec{E} = \vec{E}_n + \vec{E}_t$$

Hence, $\vec{E}_{1t} = \vec{E}_1 - \vec{E}_{1n} = 5\hat{a}_x - 2\hat{a}_y$

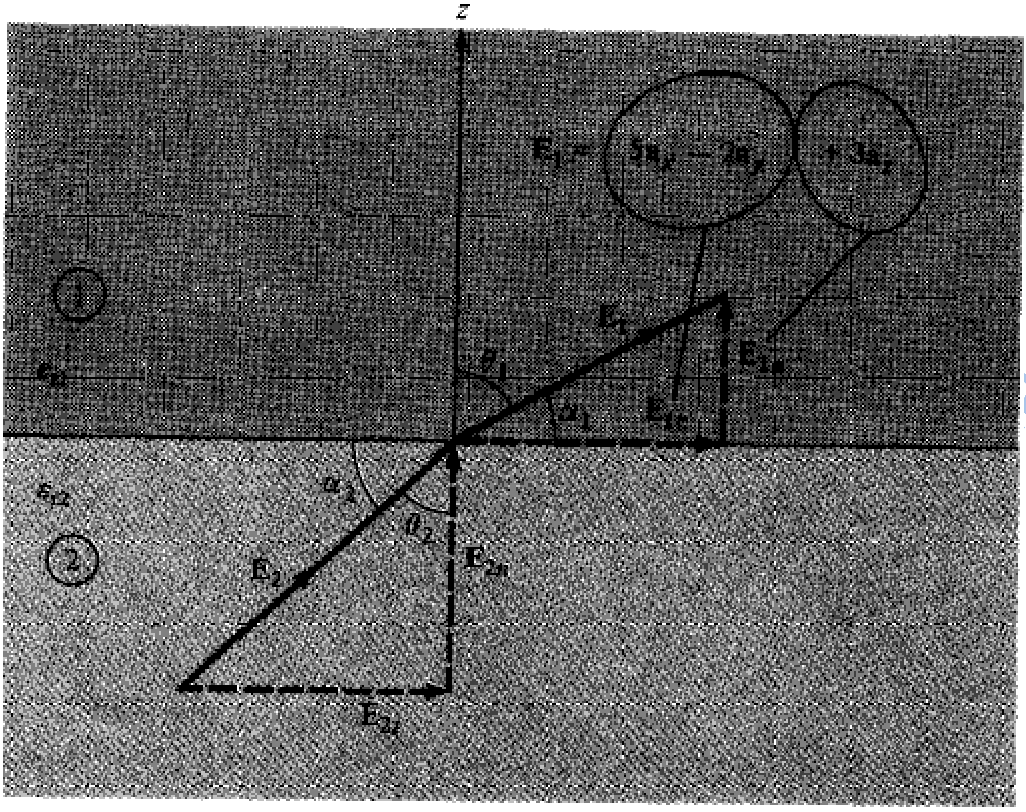

Fig.(13): *Two extensive homogeneous isotropic dielectrics meet on plane z = 0.*

$\vec{E}_{2t} = \vec{E}_{1t} = 5\hat{a}_x - 2\hat{a}_y$

Similarly, $\vec{D}_{2n} = \vec{D}_{1n} \rightarrow k_2\vec{E}_{2n} = k_1\vec{E}_{1n}$, or $\vec{E}_{2n} = \frac{k_1}{k_2}\vec{E}_{1n} = \frac{4}{3}(3\hat{a}_z) = 4\hat{a}_z$

Thus, $\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n} = 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z$ KV/m

(b) Let $\alpha_1$ and $\alpha_2$ be the angles $E_1$ and $E_2$ make with the interface while $\theta_1$ and $\theta_2$ are the angles they make with the normal to the interface as shown in Fig.13; that is,

$\alpha_1 = 90^0 - \theta_1$ and $\alpha_2 = 90^0 - \theta_2$

Since $E_{1n} = 3$ and $E_{1t} = \sqrt{25+4} = \sqrt{29}$,

Thus, $\tan\theta_1 = \frac{E_{1t}}{E_{1n}} = \frac{\sqrt{29}}{3} = 1.795 \rightarrow \theta_1 = 60.9^0$

Hence, $\alpha_1 = 29.1^0$

Alternatively, $\vec{E}_1.\hat{a}_n = |\vec{E}_1|.1.\cos\theta_1$, or $\cos\theta_1 = \frac{3}{\sqrt{38}} = 0.4867 \rightarrow \theta_1 = 60.9^0$

Similarly, $E_{2n} = 4$, $E_{2t} = E_{1t} = \sqrt{29}$

$\tan\theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{\sqrt{29}}{4} = 1.346 \rightarrow \theta_2 = 53.4^0$ , hence, $\alpha_2 = 36.6^0$

Note that $\frac{\tan\theta_1}{\tan\theta_1} = \frac{k_1}{k_2}$ is satisfied.

**Problem.4(H.W):** A homogenous dielectric ($k = 2.5$) fills region 1 ($x \le 0$) while region 2 ($x \ge 0$) is free space. a) If $\vec{D}_1 = 12\hat{a}_x - 10\hat{a}_y + 4\hat{a}_z$ nC/m$^2$ , find $\vec{D}_2$ and $\theta_2$. b) If $E_2$=12V/m and $\theta_2 = 60^0$, find $E_1$ and $\theta_1$, take $\theta_1$ and $\theta_2$ as defined in problem.3. Ans: a) $\vec{D}_2 = 12\hat{a}_x - 4\hat{a}_y + 1.6\hat{a}_z$ nC/m$^2$ , 19.75$^0$ , b) 10.67V/m, 77$^0$.

## 8. Boundary Value Problems Involving Dielectrics

The fundamental equation which has been developed in this chapter is

$$div\,\vec{D} = \rho \qquad (43)$$

Where $\rho$ is the free charge density. If the dielectrics are linear, isotropic, and homogeneous, then $\vec{D} = \varepsilon\vec{E}$, where ε is a constant characteristic of the material, and we may write

$$div\,\vec{E} = \frac{1}{\varepsilon}\rho \qquad (44)$$

Since, $\vec{E} = -\boldsymbol{grad}\,U$, so that

$$\nabla^2 U = -\frac{1}{\varepsilon}\rho \qquad (45)$$

Thus the potential in the dielectric satisfies Poisson's equation. The only difference between (45) and the corresponding equation for the potential in vacuum is that ε replaces $\varepsilon_0$.

In most cases of interest the dielectric contains no free charge distributed throughout its volume, that is, $\rho = 0$ inside the dielectric material. The free charge exists on the surfaces of conductors or is concentrated in the form of point charges to be embedded in the dielectric. *In these circumstances, the potential satisfies Laplace's equation throughout the body of the dielectric*

$$\nabla^2 U = 0 \qquad (46)$$

In some problems there may be a surface density of free charge, $\sigma$ , on the surface of a dielectric body or on the interface between two dielectric materials, but this does not alter the situation, and equation (46) still applies so long as $\rho = 0$.

## 9. Dielectric Sphere in a Uniform Electric Field.

We should like to determine how the lines of force are modified, when a dielectric sphere of radius a is placed in a region of space containing an initially uniform electric field, $\vec{E}_0$. Let us assume the dielectric to be linear, isotropic, and homogeneous, and to be characterized by the dielectric constant $k$. Furthermore, it bears no free charge. The origin of our coordinate system may be taken at the center of the sphere, and the direction of $E_o$ as the polar direction (z-direction); the potential may then be expressed as a sum of zonal harmonics.

The solution of Laplace's equation ( the zonal harmonics) is

$$U(r,\theta) = \sum_{n=0}^{\infty} P_n(\cos\theta)\left[A_n r^n + B_n r^{-(n+1)}\right]$$

$$= A_0 + \frac{B_0}{r} + A_1 r\cos\theta + \frac{B_1}{r^2}\cos\theta + \frac{A_2}{2}r^2(3\cos^2\theta - 1) + \frac{B_2}{2r^3}(3\cos^2\theta - 1) + \cdots \ldots \ldots$$

Since there is no charges on the sphere, thus, $B_0 = 0$, and because there is no constant potential on the sphere, thus, $A_0 = 0$, and as in Section 3.5, all boundary conditions can be satisfied by means of the two lowest-order harmonics, and we write

The solution in the vacuum region (1) outside the sphere given by

$$U_1(r,\theta) = A_1 r\ \cos\theta + A_2 r^{-2}\cos\theta \qquad r \geq a \qquad (47)$$

and that for the dielectric region is

$$U_2(r,\theta) = B_1 r\ \cos\theta + B_2 r^{-2}\cos\theta \qquad r \leq a \qquad (48)$$

for the dielectric region (2). The constants $A_1$, $A_2$, $B_1$, and $B_2$ are unknown and must be determined from the boundary conditions.

When $r \to 0$ then, $U_2 \to \infty$ , thus, $B_2$ must vanishes, i.e. $B_2 = 0$, thus the solutions become

$$U_1(r,\theta) = A_1 r\ \cos\theta + A_2 r^{-2}\cos\theta \qquad r \geq a$$

$$U_2(r,\theta) = B_1 r\ \cos\theta \qquad r \leq a$$

At distance far from the sphere, the electric field will retain its uniform character, and $U_1(r,\theta) = A_1 r\ \cos\theta = -E_0 r\ \cos\theta$, hence, $A_1 = -E_0$, then, the solution becomes

$$U_1(r,\theta) = -E_0 r\ \cos\theta\ + A_2 r^{-2}\cos\theta \qquad r \geq a$$

$$U_2(r,\theta) = B_1 r\ \cos\theta \qquad r \leq a$$

The remaining constants $A_2$ and $B_1$ may be obtained from the boundary conditions of section-7 as follows:

Continuity of the potential across the interface between the dielectric and vacuum required that $U_1 = U_2$ at $r = a$, i.e,

$-E_0 a\ \cos\theta\ + \frac{A_2}{a^2}\cos\theta = B_1 a\ \cos\theta$, or

$$-E_0 a + \frac{A_2}{a^2} = B_1 a \qquad (49)$$

Since there is no free charge on the surface of the dielectric, thus, the normal component of $\vec{D}$ must be continuous, thus, the normal component of $\vec{D}$ at the interface is $D_r = -\varepsilon\frac{\partial U}{\partial r}$ requires that $D_{1r} = D_{2r}$ at $r = a$, or

$$D_{1r} = -\varepsilon_0\frac{\partial U_1}{\partial r} = -\varepsilon_0\left[-E_0\cos\theta - 2\frac{A_2}{r^3}\cos\theta\right]$$

$D_{2r} = -\varepsilon\frac{\partial U_2}{\partial r} = -\varepsilon B_1\cos\theta$ and since $\varepsilon = k\varepsilon_0$, then, $D_{2r} = -k\varepsilon_0 B_1\cos\theta$

By applying the boundary condition, $D_{1r} = D_{2r}$ at $r = a$, we obtain

$-\varepsilon_0\left[-E_0\cos\theta\ - 2\frac{A_2}{a^3}\cos\theta\right] = -k\varepsilon_0 B_1\cos\theta$ , or $-E_0 - 2\frac{A_2}{a^3} = kB_1$, which can be written as

$$a^3 E_0 + 2A_2 = -ka^3 B_1 \qquad (50)$$

Continuity of $E_t$ at $r = a$ is equivalent to equation (49), solving equations (49) and (50), as follows. By multiply equation (49) by $a^2$, results that

$$-E_0 a^3 + A_2 = B_1 a^3 \qquad (51)$$
$$a^3 E_0 + 2A_2 = -ka^3 B_1 \qquad (52)$$

By multiply equation (51) by k, the system of equation becomes

$-E_0ka^3 + kA_2 = kB_1a^3$
$a^3E_0 + 2A_2 = -ka^3B_1$

By adding the latter two equations

$a^3E_0 - E_0ka^3 + kA_2 + 2A_2 = 0$

$a^3E_0(1-k) = -A_2(k+2)$ , thus, $A_2 = \frac{a^3E_0(k-1)}{k+2}$ and by substituting $A_2$ by any one of equations (51) or (52), we obtain $B_1 = -\frac{3E_0}{k+2}$ .

Thus, the potential functions which are the solution can be obtained as follows:

$$U_1(r,\theta) = -E_0 r\cos\theta + \frac{E_0(k-1)}{k+2}\frac{a^3}{r^2}\cos\theta \quad r \geq a \qquad (53)$$

$$U_2(r,\theta) = -\frac{3E_0}{k+2} r\cos\theta \qquad r \leq a \qquad (54)$$

The electric field components at any point $(r,\theta,\varphi)$ by differentiation as follows:

$$\vec{E}_1 = -\frac{\partial U_1}{\partial r}\hat{a}_r - \frac{1}{r}\frac{\partial U_1}{\partial \theta}\hat{a}_\theta = E_0\cos\theta\left[1 - \frac{2(k-1)}{k+2}\frac{a^3}{r^3}\right]\hat{a}_r - E_0\sin\theta\left[1 - \frac{(k-1)}{k+2}\frac{a^3}{r^3}\right]\hat{a}_\theta \qquad (55)$$

$$\vec{E}_2 = -\frac{\partial U_2}{\partial r}\hat{a}_r - \frac{1}{r}\frac{\partial U_2}{\partial \theta}\hat{a}_\theta = \frac{3E_0}{K+1}[\cos\theta\ \hat{a}_r - \sin\theta\ \hat{a}_\theta]$$

The lines of displacement and lines of force are shown in Fig.14.

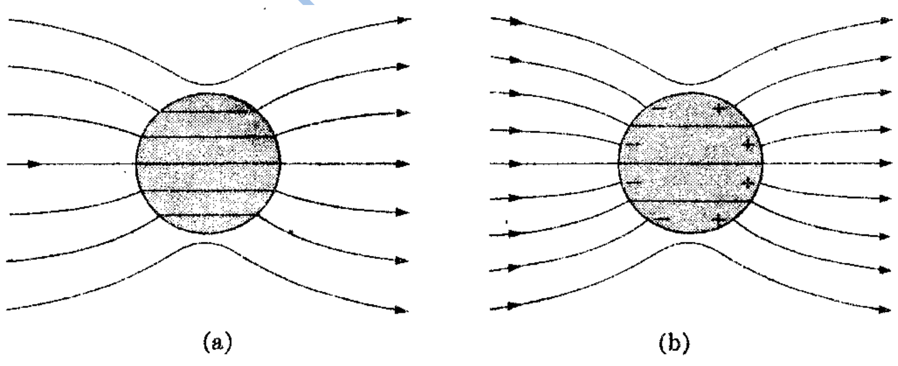

*Fig.(14): A uniform electric field is distorted by the presence of a dielectric sphere: (a) lines of electric displacement, (b) lines of the electric field.*

**_Depending References:_**

1- *Foundations of Electromagnetic Theory, Second Edition, by Reitz*
2- *Elements of Electromagnetics, Sadiku, 2000*