

7. Conditional AND Biconditional Statements

Many statements, in mathematics, are of the form "If p then q ." Such statements are called conditional statements and are denoted by: $p \rightarrow q$

The conditional $p \rightarrow q$ is frequently read "p implies q" or "p only if q."

Another statement is of the form "p if and only if q." Such statements are called biconditional statements and are denoted by $p \leftrightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(a) $p \rightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(b) $p \leftrightarrow q$

(a) The conditional $p \rightarrow q$ is false only when the first part p is true and the second part q is false. Accordingly, when p is false, the conditional $p \rightarrow q$ is true regardless of the truth value of q .

(b) The biconditional $p \leftrightarrow q$ is true whenever p and q have the same truth values and false otherwise.

Note that the truth table of $p \rightarrow q$ and $\neg p \vee q$ are identical, that is, they are both false only in the second case. Accordingly, $p \rightarrow q$ is logically equivalent to $\neg p \vee q$; that is,

$$p \rightarrow q \equiv \neg p \vee q$$

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(c) $\neg p \vee q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(a) $p \rightarrow q$

