

Example:

Show that $(p \wedge \neg q) \wedge (\neg p \vee q)$ is a contradiction.

Solution:

p	Q	$\neg q$	$p \wedge \neg q$	$\neg p$	$(\neg p \vee q)$	$(p \wedge \neg q) \wedge (\neg p \vee q)$
T	T	F	F	F	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

The last column shows that $(p \wedge \neg q) \wedge (\neg p \vee q)$ is always false, no matter what the truth values of p and q.

Hence $(p \wedge \neg q) \wedge (\neg p \vee q)$ is a contradiction.

6. Logical Equivalence

Two propositions $P(p, q, \dots)$ and $Q(p, q, \dots)$ are said to be logically equivalent, or equal, denoted by $P(p, q, \dots) \equiv Q(p, q, \dots)$ if they have identical truth tables.

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

(a) $\neg(p \wedge q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(b) $\neg p \vee \neg q$

for example, the truth tables of $\neg(p \wedge q)$ and $\neg p \vee \neg q$, both truth tables are the same, that is, both propositions are false in the first case and true in the other three cases. Accordingly, we can write:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

