

2) The second way by matrix:

$$MR = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{cccc} a & b & c & d \\ \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

$$MS = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{ccc} x & y & z \\ \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \end{array}$$

$$R \circ S = MR \cdot MS =$$

$$\begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{ccc} x & y & z \\ \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{array} \right] \end{array}$$

$$R \circ S = \{(2,z), (3,x), (3,z)\}$$

*Theorem* : Let  $A$ ,  $B$ ,  $C$  and  $D$  be sets. Suppose  $R$  is a relation from  $A$  to  $B$ ,  $S$  is a relation from  $B$  to  $C$ , and  $T$  is a relation from  $C$  to  $D$ . Then

$$(R \circ S) \circ T = R \circ (S \circ T)$$

