Relations

Relation:

We define a relation simply in terms of ordered pairs of objects.

Product sets:

Consider two arbitrary sets A and B. The set of all ordered pairs (a,b) where $a \in A$ and $b \in B$ is called the product, or cartesian product, of A and B.

$$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$$

Example: *Let $A = \{1,2\}$ and $B = \{a, b, c\}$ then*

$$A \times B = \{1,a), (1,b),(1,c),(2,a),(2,b),(2,c)\}$$

$$\text{Also, } A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

- The order in which the sets are considered is important, so

$A \times B \neq B \times A$.

Let A and B be sets. A binary relation, R, from A to B is a subset of $A \times B$. If $(x,y) \in R$,

we say that x is R-related to y and denote this by xRy

if $(x,y) \notin R$, we write $x \not R y$ and say that x is not R-related to y.

if R is a relation from A to A, i.e. R is a subset of $A \times A$, then we say that R is a relation on A.

*The **domain** of a relation R is the set of all first elements of the ordered pairs which belong to R, and the **range** of R is the set of second elements.*