2- If matrix B results from matrix A by interchanging two rows (columns) of A then
$|B| = -|A|$.

Ex// $\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 7$ and $\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -7$

3- If two rows (columns) of A are equal, then $|A| = 0$

Ex//

$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 7 \\ 1 & 2 & 3 \end{bmatrix}$ det (A) = 0

$B = \begin{bmatrix} 4 & 0 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ det (B) = 0

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 7 \\ 0 & 0 & 0 \end{bmatrix}$ det (C) = 0

$D = \begin{bmatrix} 3 & 0 & 3 \\ 4 & 0 & 4 \\ 1 & 0 & 0 \end{bmatrix}$ det (D) = 0

4- If a matrix $A = [a_{ij}]$ is upper (lower) triangular , then $|A| = a_{11}a_{22}a_{33}\ldots\ldots a_{nn}$

Ex//

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix}$ det (A) = (1)(2)(4) = 8

$B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 4 & 7 \end{bmatrix}$ det (B) = (1)(3)(7) = 21

Cofactor Expansion

Def: Let $A = [a_{ij}]$ be an nxn matrix . Let M_{ij} be the (n-1)x(n-1) sub matrix of A abtained by deleting the ith row and jth Colum of A. the determinant $|M_{ij}|$ is called the minor of a_{ij} . the cofactor of a_{ij} is defined as

$A_{ij} = (-1)^{i+j}\,|M_{ij}|$

Ex// Find the determinant by using cofactor

$A = \begin{bmatrix} 2 & 3 & 0 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

$\text{Det}(A) = 2\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$

$= 2(1+2) - 3(2-2) + 0 = 6$