

Chapter Ten – Collisions

10 – 1 The Concept of Collision

A collision is a short-duration interaction between two objects. The collision is the term that represent the event of two particles coming together for a short time and thereby producing impulse forces on each other. These forces are assumed to be much greater than any external forces present.

When two particles 1 and 2 of masses (m_1) and (m_2) collide as shown in figure (10-1), the impulsive forces may vary in time in complicated ways.

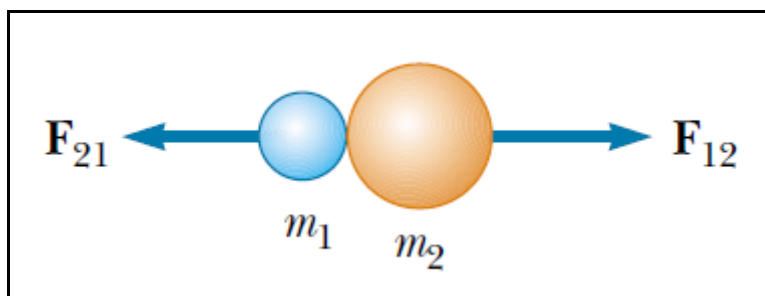


Figure (10-1), collide of two particles

If $(\vec{F}_{21})$ is the force exerted by particle 2 on particle 1 and if we assume that no external forces act on the particles, then the change in momentum of particle 1 due to the collision is given by:

$$\Delta \vec{p}_1 = \int_{t_1}^{t_2} \vec{F}_{21} dt$$

Likewise if $(\vec{F}_{12})$ is the force exerted by particle 1 on particle 2, then the change in momentum of particle 2 is:

$$\Delta \vec{p}_2 = \int_{t_1}^{t_2} \vec{F}_{12} dt$$

From Newton's third law, we conclude that:

$$\Delta \vec{p}_1 = -\Delta \vec{p}_2$$

$$\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0$$

Because the total momentum of the system is:

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 = \text{constant}$$

Because the impulsive forces are internal, they do not change the total momentum of the system (only external forces can do that). Therefore, we conclude that the total momentum of an isolated system just before a collision equals the total momentum of the system just after the collision.

Note: Momentum is constant in all collisions but kinetic energy is constant only in elastic collisions.

The collisions are divided into two fundamental types like explained in figure (10-2), which are:

- A – Elastic collision : One in which the total kinetic energy of the system (K) is the same before and after the collision
- B – Super elastic collision : One in which (K) after the collision is bigger than that before.
- C – Inelastic collision : One in which (K) after the collision is less than that before.
- D – Completely inelastic collision : One where the objects stick together after colliding.

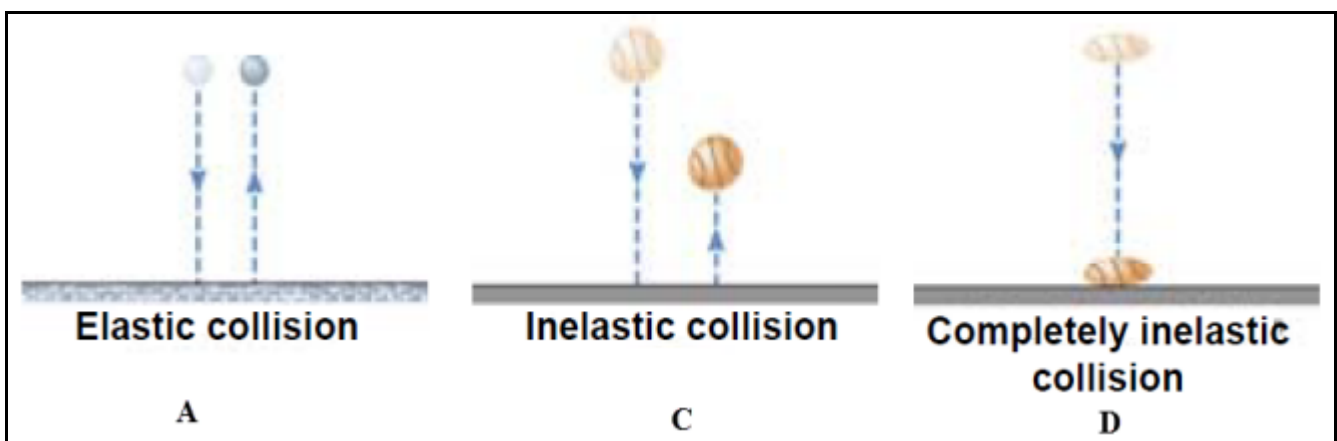


Figure (10-2), types of collision

10 – 2 Elastic and Inelastic Collision in One Dimension

A – Elastic Collision

A collision in which the total kinetic energy and momentum are unchanged before and after the collision. An elastic collision between two objects is one in which the total kinetic energy as well as the total momentum is the same before and after the collision. For example:

- 1 – Billiard-ball collisions.
- 2 – The collisions of air molecules with the walls of a container at ordinary temperature are approximately elastic.

Consider two particles that undergo an elastic head-on collision as shown in figure (10-3).

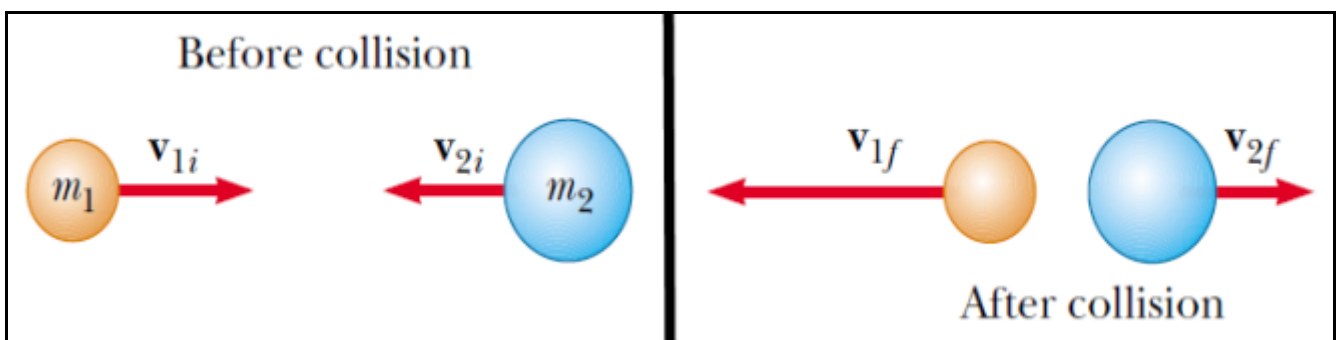


Figure (10-3), diagram representation of elastic collision of two particles, before and after

In this case, both momentum and kinetic energy are conserved, therefore, we have:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \dots \dots \dots (1)$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \dots \dots \dots (2)$$

After simplifying the eq. (2) and rearranging we get:

$$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2) \dots \dots \dots (3)$$

After factoring (analyzing the brackets), both side of eq. (3), we get:

$$m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i}) \dots \dots \dots (4)$$

From eq. (1), we get: $m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i}) \dots \dots \dots (5)$

After dividing eq. (4) by eq. (5), we get:

$$\begin{aligned} v_{1i} + v_{1f} &= v_{2f} + v_{2i} \\ v_{1i} - v_{2i} &= -(v_{1f} - v_{2f}) \dots \dots \dots (6) \end{aligned}$$

According to eq. (6), the relative speed of the two particles before the collision $(v_{1i} - v_{2i})$, is equal to the negative of their relative speed after the collision, $(-(v_{1f} - v_{2f}))$.

Suppose that the masses and initial velocities of both particles are known, eq. (1) and eq. (6) can be solved for the final speeds in terms of the initial speeds because there are two equations and two unknowns:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i} \dots \dots \dots (7)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i} \dots \dots \dots (8)$$

Note : The appropriate signs for (v_{1i}) and (v_{2i}) must be included in eq. (7) and eq. (8).

For example, if particle 2 is moving to the left initially, then (v_{2i}) is negative.

Let us consider some special cases:

(a) : If $(m_1 = m_2)$, then $(v_{1f} = v_{2i})$ and $(v_{2f} = v_{1i})$. That is the particles exchange speeds if they have equal masses.

(b) : If particle 2 is initially at rest, then $(v_{2i} = 0)$ and eq. (7) and eq. (8) becomes:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} \quad \dots \dots \dots (9)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} \quad \dots \dots \dots (10)$$

(c) : If (m_1) , is much greater then (m_2) and $(v_{2i} = 0)$, from eq. (9) and eq. (10), we see that:

$$v_{1f} \approx v_{1i} \quad \text{and} \quad v_{2f} \approx 2v_{1i}$$

That is, when a very heavy particle collide head-on with a very light one that is initially at rest, the heavy particle continues its motion unaltered after the collision, and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle.

(d) : If (m_2) , is much greater then (m_1) and particle 2 is initially at rest, then:

$$v_{1f} \approx -v_{1i} \quad \text{and} \quad v_{2f} \approx v_{2i} = 0$$

That is, when a very light particle collide head-on with a very heavy particle that is initially at rest, the light particle has its velocity reversed and the heavy one remains approximately at rest.

Example 10.1 : A block of mass $(m_1 = 1.6 \text{ kg})$ initially moving to the right with a speed of (4 m/s) on a frictionless horizontal track collides with a spring attached to a second block of mass $(m_2 = 2.1 \text{ kg})$ initially moving to the left with a speed of (2.5 m/s) , as shown in figure (10-4 a). The spring constant is (600 N/m) .

- (a) At the instant block (m_1) is moving to the right with a speed of (3 m/s) , determine the velocity of block (m_2) , as shown in figure (10-4 b).
 (b) Determine the distance the spring is compressed at that instant.

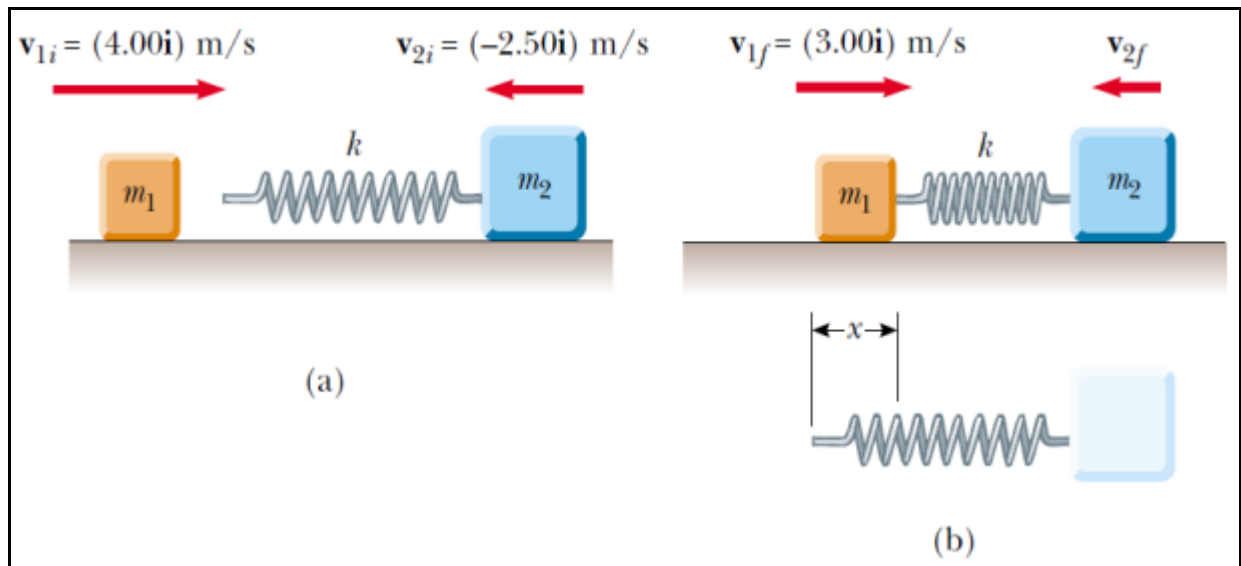


Figure (10-4, a and b), diagram representation of Example 10.1

Solution : (a) – By applying eq. (1), so: $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$$(1.60)(4.00) + (2.10)(-2.50) = (1.60)(3.00) + (2.10)v_{2f}$$

$$v_{2f} = \frac{6.40 - 5.25 - 4.80}{(2.10)} = \frac{-3.65}{2.10} = -1.7381 \text{ m/s}$$

(b) – By applying eq. (2) and adding kinetics by spring, so:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 + \frac{1}{2} kx^2$$

The left side of the equation is: $\frac{1}{2} (1.60)(4.00)^2 + \frac{1}{2} (2.10)(-2.50)^2$

$$= 12.8000 + 6.5625 = 19.3625$$

The right side of the equation is: $\frac{1}{2} (1.60)(3.00)^2 + \frac{1}{2} (2.10)(-1.7381)^2 + \frac{1}{2} (600)x^2$

$$= 11.7000 + 3.1720 + 300 x^2 = 14.8720 + 300 x^2$$

$$x^2 = \frac{19.3625 - 14.8720}{300} = \frac{4.4905}{300}$$

$$x = \sqrt{\frac{4.4905}{300}} = \sqrt{0.0150} = 0.1223 \text{ m} = 12.23 \text{ cm}$$

B – Inelastic Collisions

A collision in which the total kinetic energy is not the same before and after the collision, but momentum is the same. An inelastic collision is one in which the total kinetic energy is not the same before and after the collision (even though momentum is constant).

Inelastic collisions are of two types, as explained in figure (10-2), the types are:

- (a) When the colliding objects stick together after the collision, the collision is called perfectly or completely inelastic.
- (b) When the colliding objects do not stick together, but some kinetic energy is lost, the collision is called inelastic.

In most collisions, kinetic energy is not the same before and after the collision because some of it is converted to internal energy, to elastic potential energy when the objects are deformed, and to rotational energy.

The important distinction between elastic and inelastic collisions is that, the momentum is constant in all collisions, but kinetic energy is constant only in elastic collision.

In the case of perfectly inelastic collision as explained in figure (10-5), consider two particles of masses (m_1) and (m_2) moving with initial velocity $(\vec{v}_{1i})$ and $(\vec{v}_{2i})$ along a straight line as shown in figure (10-5) before. The two particles collide head-on, stick together, and then move with same common velocity $(\vec{v}_f)$ after the collision. Because momentum is conserved in any collision, we can say that the total momentum before the collision equals the total momentum of the composite system after the collision:

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f \quad \dots \dots \dots (11)$$

$$\vec{v}_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{(m_1 + m_2)} \quad \dots \dots \dots (12)$$

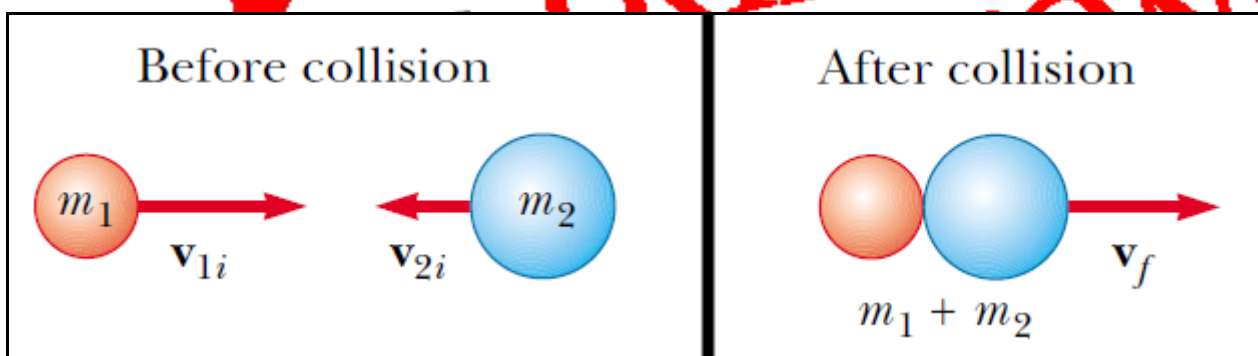


Figure (10-5), diagram representation of perfectly inelastic collision of two particles

Example 10.2 : A car of mass (1800 kg) stopped at a traffic light is struck from the rear by a (900 kg) car, and the two become entangled. If the smaller car was moving at (20.0 m/s) before the collision, what is the velocity of the entangled cars after the collision?

Solution : *momentum before collision = momentum after collision*, so:

$$p_i = p_f$$

$$p_i = m_1 v_{1i} = (900)(20.0) = 18000 \text{ kg.m/s}$$

$$p_f = (m_1 + m_2)v_f = (900 + 1800)v_f = 2700 v_f$$

$$v_f = \frac{18000}{2700} = 6.6667 \text{ m/s}$$

Example 10.3 : A ball of mass ($m_1 = 0.25 \text{ kg}$) and velocity ($v_{1i} = 5 \text{ m/s}$) collides head-on with a ball of mass ($m_2 = 0.8 \text{ kg}$) that is initially at rest ($v_{2i} = 0 \text{ m/s}$). No external forces act on the balls. If the collision is elastic, what are the velocities of the balls after the collision?

Solution : *momentum before collision = momentum after collision*, so

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$$

$$(0.25)(5) + (0.80)(0) = (0.25 + 0.80)v_f$$

$$v_f = \frac{1.25}{1.05} \text{ m/s}$$

Example 10.4 : The ballistic pendulum is a system used to measure the speed of a fast-moving projectile which explained in figure (10-5), such as a bullet. The bullet is fired into a large block of wood suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height (h). The collision is perfectly inelastic. Find the initial speed of the bullet.

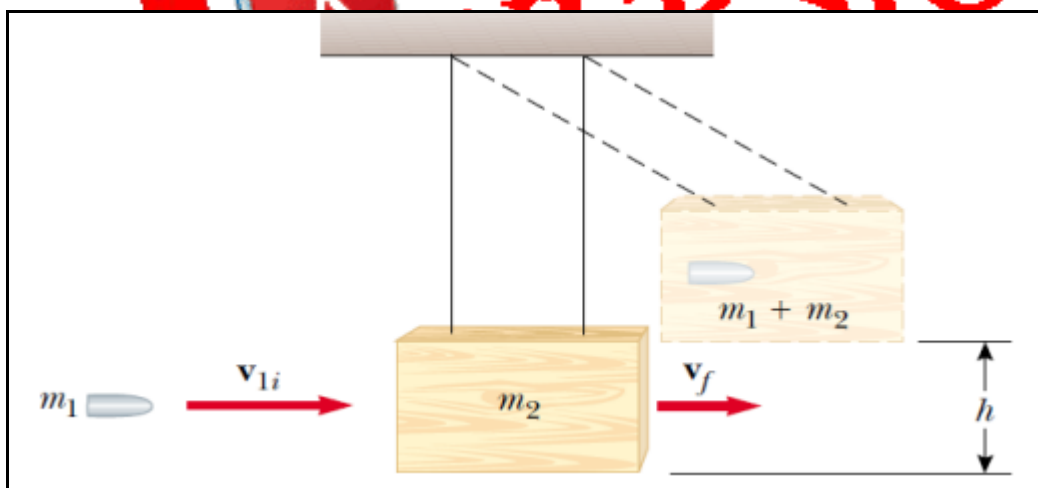


Figure (10-5), diagram representation of ballistic pendulum, example 10.5

Solution : from figure (10-5), (m_1) is the mass of bullet, and (m_2) is the mass of the large block of wood, as used in ballistic pendulum. Applying eq. (11), and (12), where (v_{1i}) is

the velocity of bullet, and (v_{2i}) is the velocity of the block, before the collision
 $(v_{2i} = 0)$, the collision is inelastic, so:

$$K_f = \frac{1}{2}(m_1 + m_2)v_f^2 \quad \text{and} \quad v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{(m_1 + m_2)} \quad \text{where } v_{2i} = 0, \text{ so:}$$

$$K_f = \frac{1}{2}(m_1 + m_2) \left(\frac{m_1 v_{1i} + m_2 v_{2i}}{(m_1 + m_2)} \right)^2 = \frac{m_{1i}^2 v_{1i}^2}{2(m_1 + m_2)}$$

The kinetic energy immediately after the collision is less than the initial kinetic energy of the bullet, and there is an loss in mechanical energy due to the collision, therefore, the kinetic energy of the block and bullet at the bottom is transformed to potential energy at the height (h) , so:

$$\frac{m_{1i}^2 v_{1i}^2}{2(m_1 + m_2)} = (m_1 + m_2)gh$$

$$m_{1i}^2 v_{1i}^2 = (m_1 + m_2)^2 2gh \Rightarrow v_{1i}^2 = \frac{(m_1 + m_2)^2}{m_{1i}^2} 2gh$$

$$v_{1i} = \left(\frac{m_1 + m_2}{m_1} \right) \sqrt{2gh}$$

Example 10.5 : In a ballistic pendulum experiment, suppose that $(h = 5 \text{ cm})$, $(m_1 = 5 \text{ gm})$, and $(m_2 = 1.0 \text{ kg})$. Find:

(a) : the initial speed of the bullet.

(b) : the loss in mechanical energy due to the collision.

Solution : (a) : the initial speed of the bullet is: $v_{1i} = \left(\frac{m_1 + m_2}{m_1} \right) \sqrt{2gh}$

$$v_{1i} = \left(\frac{0.005 + 1.0}{0.005} \right) \sqrt{2 \times 9.8 \times 0.05} = 201 \times 0.9899 = 198.9699 \text{ m/s}$$

(b) : the loss in mechanical energy due to the collision.

$$K_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} (0.005)(198.9699)^2 + \frac{1}{2} (1.0)(0)^2$$

$$K_i = 98.9726 \text{ J}$$

$$K_f = \frac{1}{2} (m_1 + m_2) \left(\frac{m_1 v_{1i} + m_2 v_{2i}}{(m_1 + m_2)} \right)^2 = \frac{m_{1i}^2 v_{1i}^2}{2(m_1 + m_2)}$$

$$K_f = \frac{(0.005)^2 (198.9699)^2}{2(0.005 + 1.0)} = \frac{0.9897}{2.0100} = 0.4924 \text{ J}$$

$$K_{loss} = K_i - K_f = 98.9726 - 0.4924 = 98.4802 \text{ J}$$

Example 10.6 : Two bodies of masses (8 kg) and (4 kg) move along the (x) axis in opposite direction with velocities of (11 m/s) and (-7 m/s) , respectively. They collide and stick together. Find their velocity just after collision.

Solution : *momentum before collision = momentum after collision*, so

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$(8)(11) + (4)(-7) = (8 + 4) v_f \Rightarrow 88 - 28 = 12 v_f$$

$$v_f = \frac{60}{12} = 5\text{ m/s}$$

Example 10.6 : Two clay balls collide head-on in a perfectly inelastic collision. The first ball has a mass of (0.5 kg) and an initial velocity of (4 m/s) to the right. The second ball has a mass of (0.25 kg) and an initial velocity of (3 m/s) to the left. What is the decrease in kinetic energy during the collision?

Solution : *momentum before collision = momentum after collision*, so:

$$p_i = p_f$$

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (0.5)(4) + (0.25)(-3) = 2 - 0.75 = 1.25\text{ kg.m/s}$$

$$p_f = (m_1 + m_2) v_f = (0.5 + 0.25) v_f = 0.75 v_f$$

$$v_f = \frac{p_i}{(m_1 + m_2)} = \frac{1.25}{0.75} = 1.6667\text{ m/s}$$

$$K_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} (0.5)(4)^2 + \frac{1}{2} (0.25)(-3)^2 = 4 + 1.1250$$

$$K_i = 5.125\text{ J}$$

$$K_f = \frac{1}{2} (m_1 + m_2) v_f^2 \Rightarrow K_f = \frac{1}{2} (0.5 + 0.25) (1.6667)^2$$

$$K_f = \frac{1}{2} (0.75) (2.7779) = 1.0417\text{ J}$$

The decrease in kinetic energy during the collision is:

$$\Delta K = K_i - K_f = 5.1250 - 1.0417 = 4.0833\text{ J}$$

Table (10-1), summarizes the properties of types of the collisions in physics.

Table (10-1), descriptions type of collisions

Type	Description	Happens	Conserved quantity
Elastic	Total kinetic energy as well as total momentum, is the same before and after the collision.	The two objects bounce after the collision so that they move separately	Momentum and kinetic energy
Inelastic	Total kinetic energy is not the same before and after the collision, even though momentum is constant. When the colliding objects do not stick together, but some kinetic energy is lost, as in the case of a rubber ball colliding with a hard surface, the collision is called inelastic with no modifying adverb	The two objects deform during the collision so that the total kinetic energy decreases, but the objects move separately after the collision	Only Momentum
Perfectly inelastic	Total kinetic energy is not the same before and after the collision, even though momentum is constant. When the colliding objects stick together after the collision, as happens when a meteorite collides with the Earth, the collision is called perfectly inelastic.	The two objects stick together after the collision so that their final velocities are the same	Only Momentum

10 – 3 Two Dimensional Collisions

In a two or three dimensional collisions, the components of the momentum in each of the three directions (x, y, z) are conserved independently.

For two dimensional collision, we obtain two component equation for conservation of momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx} \dots \dots \dots (13)$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy} \dots \dots \dots (14)$$

Consider that particle 1 of mass (m_1) collides with particle 2 of mass (m_2) , where particle 2 is initially at rest as in figure (10-6).

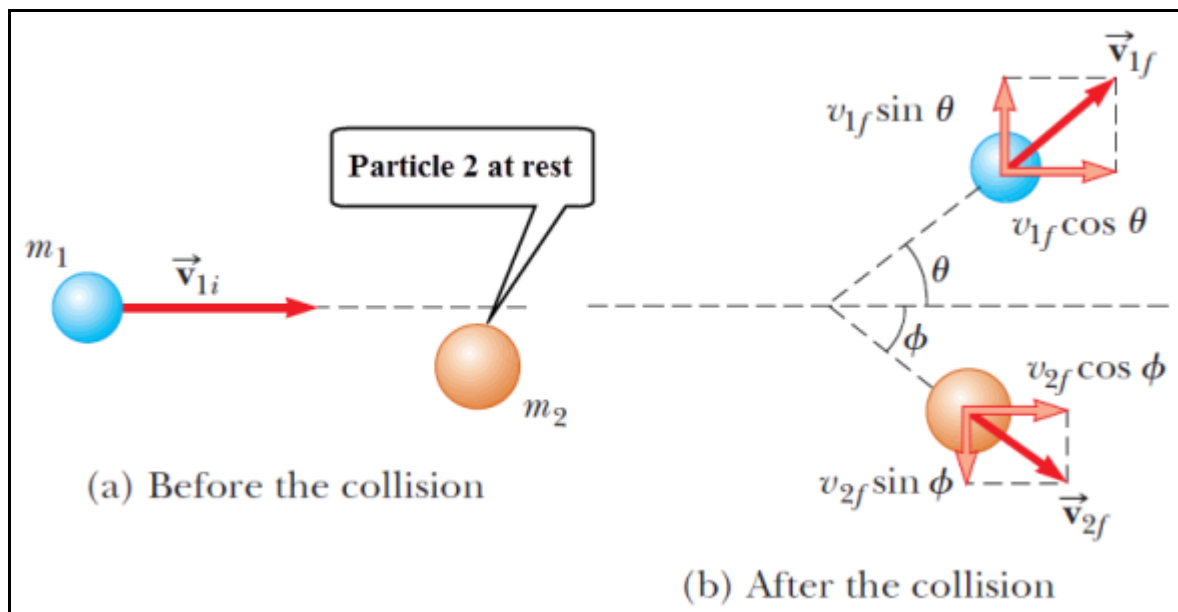


Figure (10-6), diagram representation of collision in two dimensions, of two particles

After the collision particle 1 moves at an angle (θ) with respect to the horizontal and particle 2 moves at an angle (ϕ) with respect to the horizontal. This is called a glancing collision. Applying the law of conservation of momentum in component form, and noting that the initial (y) component of velocity that is downward.

If the collision is elastic, we can use the eq. (2) of:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \dots\dots\dots (2)$$

The conservation of kinetic energy, with $(v_{2i} = 0)$ to give;

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \dots\dots\dots (15)$$

If the collision is inelastic, kinetic is not conserved and eq. (15) does not apply.

Example 10.7 : In a game of billiards, a player wishes to sink a target ball 2 (violet ball) in the corner pocket, as shown in Figure (10-7). If the angle to the corner pocket is (35°) , at what angle is the cue ball 1 (white ball) deflected? Assume that friction and rotational motion are unimportant and that the collision is elastic.

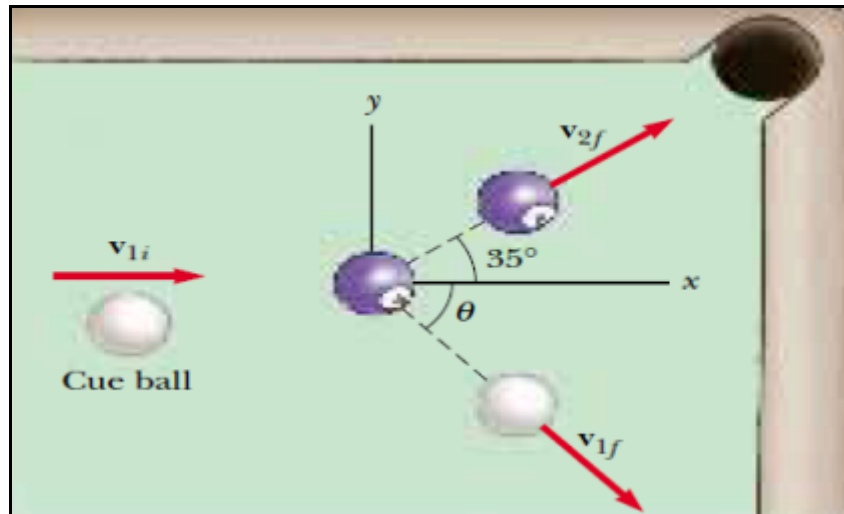


Figure (10-7), diagram representation of Example 10.7

Solution : In the figure there are two balls, cue ball and target ball, target ball is initially at rest, and $(m_1 = m_2)$, so that:

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \Rightarrow v_{1i}^2 = v_{1f}^2 + v_{2f}^2 \dots\dots (1)$$

Applying conservation of momentum to the two-dimensional collisions, which gives:

$$m_1v_{1i} + m_2v_2 = m_1v_{1f} + m_2v_{2f} \Rightarrow m_1v_{1i} + m_2(0) = m_1v_{1f} + m_2v_{2f}$$

$$v_{1i} = v_{1f} + v_{2f} \dots\dots (2)$$

If we square both side of eq. (2), and use the definition of dot product of two vectors, we get:

$$v_{1i}^2 = (v_{1f} + v_{2f}) \cdot (v_{1f} + v_{2f}) \Rightarrow v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2v_{1f} \cdot v_{2f}$$

From figure the angle between (v_{1f}) and (v_{2f}) is $(\theta + 35^\circ)$, so:

$$v_{1f} \cdot v_{2f} = v_{1f}v_{2f} \cos(\theta + 35^\circ), \text{ so:}$$

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f} \cos(\theta + 35^\circ) \dots\dots (3)$$

Rearranging eq. (3) to simplify the equation, which gives:

$$v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f} \cos(\theta + 35^\circ) - v_{1i}^2 = 0 \dots\dots (4)$$

Substituting on (v_{1i}^2) from eq. (1) into eq. (4), we get:

$$v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f} \cos(\theta + 35^\circ) - (v_{1f}^2 + v_{2f}^2) = 0 \dots\dots (4)$$

$$2v_{1f}v_{2f} \cos(\theta + 35^\circ) = 0 \Rightarrow v_{1f}v_{2f} \cos(\theta + 35^\circ) = 0 \dots\dots (5)$$

$$\cos(\theta + 35^\circ) = \frac{0}{v_{1f}v_{2f}} = 0 \Rightarrow \cos(\theta + 35^\circ) = 0 \Rightarrow \beta = \cos^{-1}(0)$$

$$\beta = 90^\circ \Rightarrow 90^\circ = \theta + 35^\circ \Rightarrow \theta = 90^\circ - 35^\circ = 55^\circ$$

The angle that the cue ball 1 (white ball) is deflect.

Example 10.8 : Proton 1 collides elastically with proton 2 that is initially at rest. Proton 1 has an initial speed of $(3.5 \times 10^5 \text{ m/s})$ and makes a glancing collision with proton 2, as was shown in figure (10-8). After the collision, proton 1 moves at an angle of $(\theta = 37^\circ)$ to the horizontal axis, and proton 2 deflects at an angle (ϕ) to the

same axis. Find the final speeds of the two protons and the angle (ϕ).

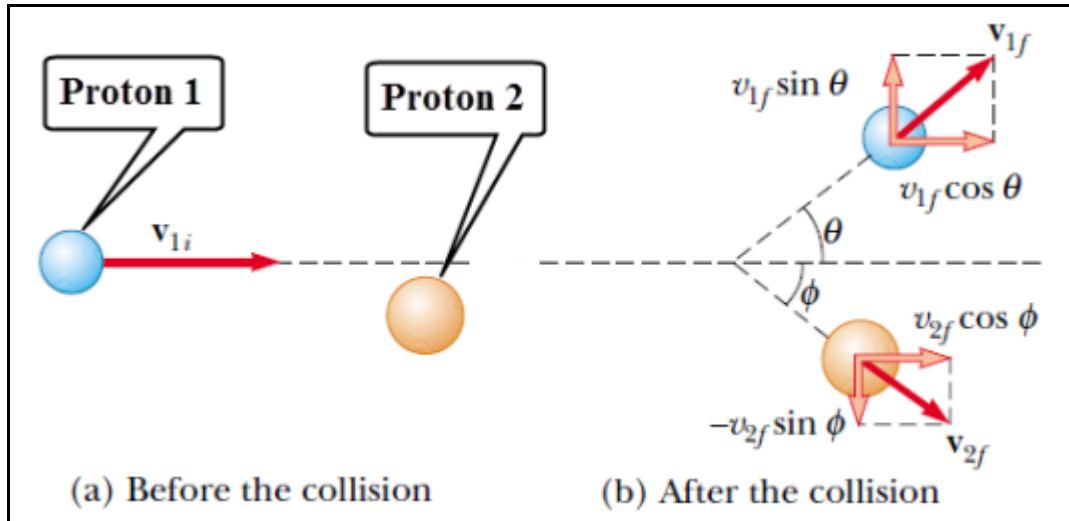


Figure (10-8), diagram representation of Example 10.8

Solution : Because both particles are protons, we know that ($m_1 = m_2$), and from figure, so:
Applying conservation of momentum to the two-dimensional collisions, which gives:

$$m_1 v_{1i} + m_2 v_2 = m_1 v_{1f} + m_2 v_{2f} \Rightarrow m_1 v_{1i} + m_2 (0) = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1i} = v_{1f} + v_{2f}$$

The resultant speed in (x - direction) is:

$$v_{1i} = v_{1f} \cos(\theta) + v_{2f} \cos(\phi) \Rightarrow 3.5 \times 10^5 = v_{1f} \cos(37^\circ) + v_{2f} \cos(\phi)$$

$$v_{2f} \cos(\phi) = 3.5 \times 10^5 - v_{1f} \cos(37^\circ) \dots \dots (1)$$

The resultant speed in (y - direction) is:

$$v_{1i} = v_{1f} \sin(\theta) - v_{2f} \sin(\phi) \Rightarrow 0 = v_{1f} \sin(37^\circ) - v_{2f} \sin(\phi)$$

$$v_{1f} \sin(37^\circ) = v_{2f} \sin(\phi) \dots \dots (2)$$

The resultant speed in (x and y - directions) is:

$$v_{1f}^2 + v_{2f}^2 = v_{1i}^2 \Rightarrow v_{1f}^2 + v_{2f}^2 = (3.5 \times 10^5 \text{ m/s})^2$$

$$v_{1f}^2 + v_{2f}^2 = 1.2250 \times 10^{11} \dots \dots (3)$$

By squaring both two sides of equations (1), and (2), so:

$$(v_{2f} \cos(\phi))^2 = (3.5 \times 10^5 - v_{1f} \cos(37^\circ))^2 \dots \dots (4)$$

$$v_{2f}^2 \cos^2(\phi) = 1.2250 \times 10^{11} - 7 \times 10^5 v_{1f} \cos(37^\circ) + v_{1f}^2 \cos^2(37^\circ) \dots \dots (5)$$

$$(v_{1f} \sin(37^\circ))^2 = (v_{2f} \sin(\phi))^2$$

$$v_{1f}^2 \sin^2(37^\circ) = v_{2f}^2 \sin^2(\phi) \dots \dots (6)$$

$$v_{1f}^2 + v_{2f}^2 = 1.2250 \times 10^{11} \dots \dots (7)$$

$$v_{1f}^2 = 1.2250 \times 10^{11} - v_{2f}^2 \dots \dots (8)$$

$$v_{1f} = \sqrt{1.2250 \times 10^{11} - v_{2f}^2} \dots \dots (9)$$

By adding eq. (6) to eq. (5) rearrange of their terms and simplify, we get that:

The left side of eq. (10), is:

$$A = v_{2f}^2 \cos^2(\phi) + v_{2f}^2 \sin^2(\phi) \Rightarrow A = v_{2f}^2 \{ \cos^2(\phi) + \sin^2(\phi) \}, \text{ so:}$$

$$A = v_{2f}^2 \dots \dots (10 a)$$

The right side of eq. (10), is:

$$B = 1.2250 \times 10^{11} - 7 \times 10^5 v_{1f} \cos(37^\circ) + v_{1f}^2 \cos^2(37^\circ) + v_{1f}^2 \sin^2(37^\circ)$$

$$B = 1.2250 \times 10^{11} - 7 \times 10^5 v_{1f} \times 0.7986 + v_{1f}^2 \{ \cos^2(37^\circ) + \sin^2(37^\circ) \}$$

$$B = 1.2250 \times 10^{11} - 5.5904 \times 10^5 v_{1f} + v_{1f}^2 \dots \dots (10 b)$$

So eq. (10) is:

$$v_{2f}^2 = 1.2250 \times 10^{11} - 5.5904 \times 10^5 v_{1f} + v_{1f}^2 \dots \dots (10)$$

Now substituting eq. (10) in eq. (3), so we get:

$$v_{1f}^2 + 1.2250 \times 10^{11} - 5.5904 \times 10^5 v_{1f} + v_{1f}^2 = 1.2250 \times 10^{11}$$

$$2v_{1f}^2 - 5.5904 \times 10^5 v_{1f} = 0 \dots \dots (11)$$

Dividing the two sides of eq. (11) by $(2v_{1f})$, so we get:

$$v_{1f} - 2.7952 \times 10^5 = 0 \Rightarrow v_{1f} = 2.7952 \times 10^5 \text{ m/s} \dots \dots (12)$$

Now substituting value of (v_{1f}) from eq. (12) into eq. (3), so we get:

$$(2.7952 \times 10^5)^2 + v_{2f}^2 = 1.2250 \times 10^{11} \dots \dots (3)$$

$$7.8131 \times 10^{10} + v_{2f}^2 = 1.2250 \times 10^{11}$$

$$v_{2f}^2 = 1.2250 \times 10^{11} - 0.78131 \times 10^{11} \Rightarrow v_{2f}^2 = 0.4437 \times 10^{11}$$

$$v_{2f} = \sqrt{(0.4437 \times 10^{11})} = 2.1064 \times 10^5 \text{ m/s}$$

To find (ϕ) we substituting values of (v_{1f}) and (v_{2f}) into eq. (1), so:

$$v_{2f} \cos(\phi) = 3.5 \times 10^5 - v_{1f} \cos(37^\circ) \dots \dots (1)$$

$$\cos(\phi) = \frac{3.5 \times 10^5 - v_{1f} \cos(37^\circ)}{v_{2f}}$$

$$\cos(\phi) = \frac{3.5 \times 10^5 - 2.7952 \times 10^5 \times 0.7986}{2.1064 \times 10^5} = \frac{(3.5 - 2.2322) \times 10^5}{2.1064 \times 10^5}$$

$$\cos(\phi) = \frac{1.2678 \times 10^5}{2.1064 \times 10^5} = 0.6019 \Rightarrow \phi = \cos^{-1}(0.6019)$$

$$\phi = 52.9939^\circ$$

Example 10.9 : Two players are collides in the point on the football stadium, the first moving from West to East with momentum $(p_1 = 500 \text{ kg.m/s})$, while the second moving from South to North with momentum $(p_2 = 300 \text{ kg.m/s})$, find the magnitude of total momentum and their direction.

Solution :

$p_1 = 500 \text{ kg.m/s}$ from West to East.

$p_2 = 300 \text{ kg.m/s}$ from South to North.

$$p_{total} = \sqrt{(500 \text{ kg.m/s})^2 + (300 \text{ kg.m/s})^2} = 583.0952 \text{ kg.m/s}$$

$$\theta_{p_{total}} = \tan^{-1} \left(\frac{p_2}{p_1} \right) \Rightarrow \theta_{p_{total}} = \tan^{-1} \left(\frac{300}{500} \right) \Rightarrow \theta_{p_{total}} = 30.9638^\circ$$

Example 10.10 : A (1500 kg) car 1 traveling from West to East with a speed of (25.0 i m/s) collides at an intersection with a (2500 kg) large car 2 traveling from south to North at a speed of (20.0 j m/s) , as shown in figure (10-9). Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).

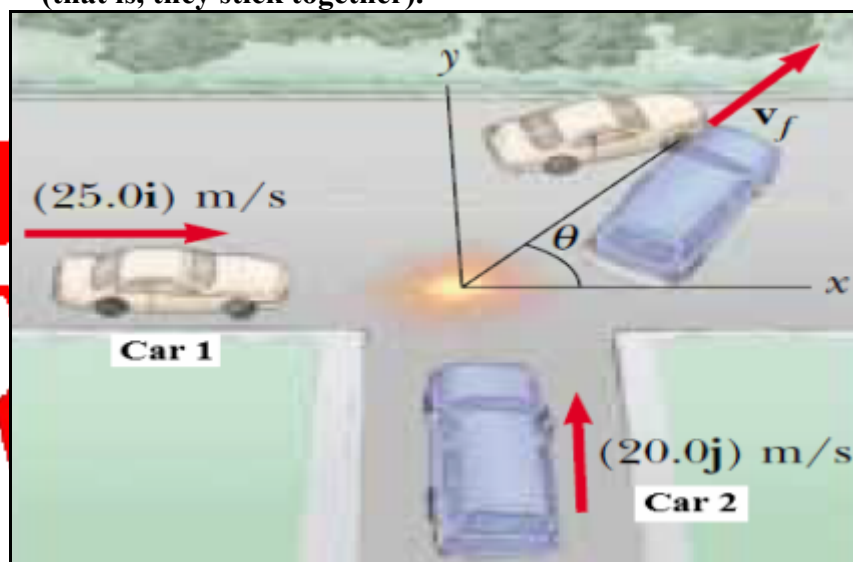


Figure (10-9), diagram representation of Example 10.10

Solution :

From the figure we see that the car 1 continues to moving along positive $(x - direction)$, and car 2 continues to moving along positive $(y - direction)$, before and after the collision.

The initial momentum of the system in the $(x - direction)$, and the total final momentum in the same direction, so:

$$\sum p_{x-before} = \sum p_{x-after}$$

$$\sum p_{xi} = m_1 v_1 = (1500 \text{ kg})(25.0 \text{ m/s}) = 37500 \text{ kg.m/s}$$

$$\sum p_{xi} = 3.75 \times 10^4 \text{ kg.m/s}$$

From the figure we see that the wreckage cars after the collision, moving in (θ°) direction, so:

$$\sum p_{xf} = (m_1 + m_2) v_f \cos(\theta) = ((1500 + 2500) \text{ kg}) ((v_f \cos(\theta)) \text{ m/s})$$

$$\sum p_{xf} = 4000 v_f \cos(\theta) \text{ kg.m/s}$$

Because the total momentum in the $(x - \text{direction})$ is constant, we can equate these two equations to obtain the eq. (1), so:

$$3.75 \times 10^4 \text{ kg.m/s} = 4000 \text{ kg} \times v_f \cos(\theta)$$

$$v_f \cos(\theta) = \frac{3.75 \times 10^4 \text{ kg.m/s}}{4000 \text{ kg}} = 9.3750 \text{ m/s} \dots\dots (1)$$

Similarly, the initial momentum of the system in the $(y - \text{direction})$, and the total final momentum in the same direction, so:

$$\sum p_{y\text{-before}} = \sum p_{y\text{-after}}$$

$$\sum p_{yi} = m_2 v_2 = (2500 \text{ kg})(20.0 \text{ m/s}) = 50000 \text{ kg.m/s}$$

$$\sum p_{yi} = 5.0 \times 10^4 \text{ kg.m/s}$$

$$\sum p_{yf} = (m_1 + m_2) v_f \sin(\theta) = ((1500 + 2500) \text{ kg}) ((v_f \sin(\theta)) \text{ m/s})$$

$$\sum p_{yf} = 4000 v_f \sin(\theta) \text{ kg.m/s}$$

$$5.0 \times 10^4 \text{ kg.m/s} = 4000 v_f \sin(\theta) \text{ kg.m/s}$$

$$v_f \sin(\theta) = \frac{5.0 \times 10^4 \text{ kg.m/s}}{4000 \text{ kg}} = 12.5 \text{ m/s} \dots\dots (2)$$

If we divide eq. (2) by eq. (1), we get:

$$\frac{v_f \sin(\theta)}{v_f \cos(\theta)} = \frac{12.5 \text{ m/s}}{9.3750 \text{ m/s}} \Rightarrow \tan(\theta) = 1.3333$$

$$\theta = \tan^{-1}(1.3333) \Rightarrow \theta = 53.1301^\circ$$

When substituted (θ) into eq. (2), or eq. (1), the (v_f) is:

$$v_f \sin(\theta) = 12.5 \text{ m/s} \Rightarrow v_f = \frac{12.5 \text{ m/s}}{\sin(53.1301^\circ)} = 15.6250 \text{ m/s}$$

$$v_f \cos(\theta) = 9.3750 \text{ m/s} \Rightarrow v_f = \frac{9.3750 \text{ m/s}}{\cos(53.1301^\circ)} = 15.6250 \text{ m/s}$$

Another method of solution :

$$\sum p_{\text{before collision}} = \sum p_{\text{after collision}}$$

Using vector method for car 1 the momentum before the collision is:

$$p_{1\text{-before}} = m_1 \times v_1 = 1500 \text{ kg} \times 25.0 \text{ m/s} = 3.75 \times 10^4 \text{ kg.m/s}$$

Using vector method for car 2 the momentum before the collision is:

$$p_{2\text{-before}} = m_2 \times v_2 = 2500 \text{ kg} \times 20.0 \text{ m/s} = 5.00 \times 10^4 \text{ kg.m/s}$$

Because (p_2) perpendicular to (p_1) , using Phitagors theory to find the resultant momentum, so:

$$p_{total-after} = \sqrt{(p_{1-before})^2 + (p_{2-before})^2} = \sqrt{(3.75 \times 10^4)^2 + (5.00 \times 10^4)^2}$$

$$p_{total-after} = 62500 = 6.25 \times 10^4 \text{ kg.m/s}$$

To find the direction of $(p_{total-after})$, using the inverse of tangent angle, so:

$$\theta = \tan^{-1} \left(\frac{p_{2-before}}{p_{1-before}} \right) \Rightarrow \theta = \tan^{-1} \left(\frac{5.00 \times 10^4 \text{ kg.m/s}}{3.75 \times 10^4 \text{ kg.m/s}} \right) = 53.1301^\circ \text{ with } x\text{-axis}$$

To find the magnitude of (v_{final}) , using the inverse of tangent angle, so:

$$p_{total-after} = m_{total} \times v_{final} \Rightarrow p_{total-after} = (m_1 + m_2) \times v_{final}$$

$$6.25 \times 10^4 \text{ kg.m/s} = ((1500 + 2500) \text{ kg}) \times v_{final}$$

$$v_{final} = \frac{6.25 \times 10^4 \text{ kg.m/s}}{4000 \text{ kg}} = 15.6250 \text{ m/s}$$

Example 10.11 : Two cars of equal mass Collide at right angles to one another in an intersection. Their direction of motion after the collision is as shown in figure (10-10). Choose the correct answer from the following, and which car had the greater velocity before the collision?

- (a) : Car A. (b) : Car B. (c) : Their velocities were equal in magnitude.
(d) : It is impossible to tell from this graph.

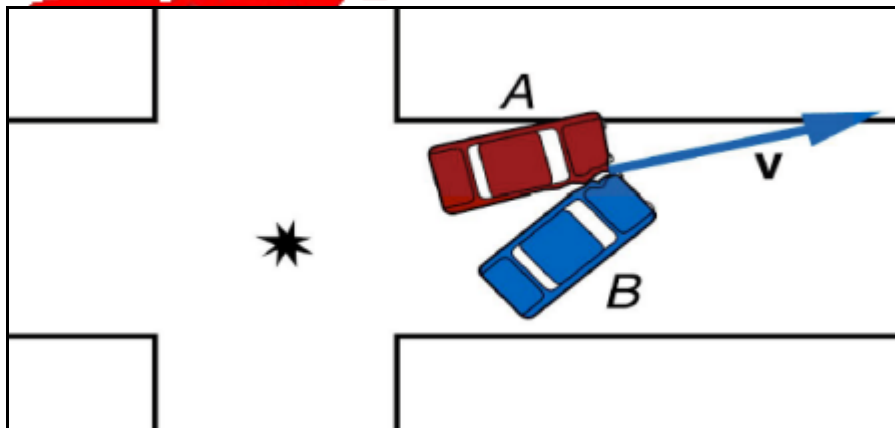


Figure (10-10), diagram representation of Example 10.11

Solution :

The correct answer is (a). And since the angle with respect to the original direction of Car A is smaller than (45°) , car A must have had a larger momentum and thus was traveling faster.

Example 10.12 : Four railroad cars, all with the same mass of $(20,000 \text{ kg})$, sit on a track. A fifth car of identical mass approaches them with a velocity of (15 m/s) , as explained in figure (10-?). This car collides and couples with the other four cars.

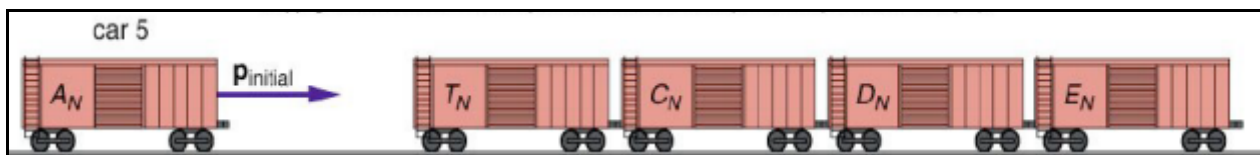


Figure (10-11), diagram representation of Example 10.12

What is the initial momentum of the system?

A : Select the correct answer from the following values by proof:

- (a) : $200,000 \text{ kg.m/s}$. (b) : $300,000 \text{ kg.m/s}$.
 (c) : $600,000 \text{ kg.m/s}$. (d) : $1,200,000 \text{ kg.m/s}$.

Solution : (b) : $300,000 \text{ kg.m/s}$. The proof is:

$$p_{\text{initial}} = m_5 \times v_5 \Rightarrow p_{\text{initial}} = 20000 \text{ kg} \times 15 \text{ m/s}$$

$$p_{\text{initial}} = 300000 \text{ kg.m/s}$$

What is the velocity of the five coupled cars after the collision?

B : Select the correct answer from the following values by proof:

- (a) : 1 m/s . (b) : 3 m/s . (c) : 5 m/s . (d) : 10 m/s .

Solution : (b) : 3 m/s . The proof is:

$$m_{\text{total}} = m_1 + m_2 + m_3 + m_4 + m_5$$

$$m_{\text{total}} = 20000 + 20000 + 20000 + 20000 + 20000 = 100000 \text{ kg}$$

$$p_{\text{initial}} = p_{\text{final}} \Rightarrow p_{\text{final}} = m_{\text{total}} \times v_{\text{final}} \Rightarrow v_{\text{final}} = p_{\text{final}} / m_{\text{total}}$$

$$v_{\text{final}} = \frac{300000 \text{ kg.m/s}}{100000 \text{ kg}} = 3 \text{ m/s}$$

Is the kinetic energy after the railroad cars collide equal to the original kinetic energy of car 5?

C : Select the correct answer from the following values by proof:

- (a) : Yes. (b) : No. (c) : It depends.

Solution : (b) : No. The proof is:

$$E_{\text{initial}} = \frac{1}{2} m_5 v_5^2 \Rightarrow E_{\text{initial}} = \frac{1}{2} \times 20000 \text{ kg} \times (15 \text{ m/s})^2$$

$$E_{\text{initial}} = 2250000 \text{ J} = 2250 \text{ kJ}$$

$$E_{\text{final}} = \frac{1}{2} m_{\text{total}} v_{\text{final}}^2 \Rightarrow E_{\text{final}} = \frac{1}{2} \times 100000 \text{ kg} \times (3 \text{ m/s})^2$$

$$E_{\text{final}} = 450000 \text{ J} = 450 \text{ kJ}$$

$$E_{\text{final}} \neq E_{\text{initial}}$$

Example 10.13 : Which of the following equations can be used to directly calculate an object's momentum, (p) ?

- (a) : $p = mv$. (b) : $p = F\Delta t$. (c) : $\Delta p = F\Delta t$.

Solution : The equation is: (a) : $p = mv$

Example 10.14 : Two objects move separately after colliding, and both the total momentum and total kinetic energy remain constant. Identify the type of collision from the following answers:

(a) : elastic. (b) : nearly elastic. (c) : inelastic. (d) : perfectly inelastic.

Solution : The answer is: (a) : elastic.

Example 10.15 : If a force is exerted on an object, which statement is true?

(a) : A large force always produces a large change in the object's momentum.

(b) : A small force always produces a large change in the object's momentum.

(c) : A small force applied over a long time interval can produce a large change in the object's momentum.

(d) : A large force produces a large change in the object's momentum only if the force is applied over a very short time interval.

Solution : Homework to student.

Example 10.16: When comparing the momentum of two moving objects, which of the following is correct?

(a) : The object with the higher velocity will have less momentum if the masses are equal.

(b) : The more massive object will have less momentum if its velocity is greater.

(c) : The less massive object will have less momentum if the velocities are the same.

(d) : The more massive object will have less momentum if the velocities are the same.

Solution : Homework to student.

Homework Chapter Ten

Q1 – On a touchdown attempt, a (95 kg) running back runs toward the end zone at (3.75 m/s) . A (111 kg) line backer moving at (4.1 m/s) meets the runner in a head-on collision and locks his arms around the runner.

(a) Find their velocity immediately after the collisions.

(b) Find the initial and final kinetic energies and the energy (ΔK) lost in the collision.

Q2 – A $(m_1 = 1500\text{ kg})$ car traveling East with a speed of $(v_1 = 25\text{ m/s})$ collides at an intersection with a $(m_2 = 2500\text{ kg})$ van traveling North at speed of $(v_2 = 20\text{ m/s})$. If the vehicles stick together afterwards, calculate the magnitude and velocity of the cars after the collision.

Q3 – Proton 1 collides elastically with proton 2 that is initially at rest. Proton 1 has an initial speed of $(3.5 \times 10^5\text{ m/s})$. After the collision, proton 1 moves at an angle of (37°) to the horizontal axis, and proton 2 deflects at an angle (φ) to the same axis. Find the final speeds of the two protons and the angle (φ) .

Q4 – A (0.015 kg) marble moving to the right at (0.225 m/s) makes an elastic head-on collision with a (0.03 kg) shooter marble moving to the left at (0.18 m/s) . After the collision, the

smaller marble moves to the left at (0.315 m/s) . Assume that either marble rotates before or after the collision and that both marbles are moving on a frictionless surface. What is the velocity of the (0.03 kg) marble after the collision?

Q5 – In ballistic pendulum the mass of the block of wood is (2.5 kg) and the mass of the bullet is (0.01 kg) . The block swings to a maximum height of (0.650 m) above the initial position. Find the initial speed of the bullet.

Q6 – A (0.025 kg) bullet is accelerated from rest to a speed of (550 m/s) in a (3.000 kg) rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder.

(a) : Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder.

(b) : How much kinetic energy does the rifle gain?

(c) : What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass (28.0 kg) ?

(d) : How much kinetic energy is transferred to the rifle-shoulder combination?

Q7 – An (80 kg) roller skating grandma collides with and picks up a (40 kg) kid. What is their velocity after the collision?

