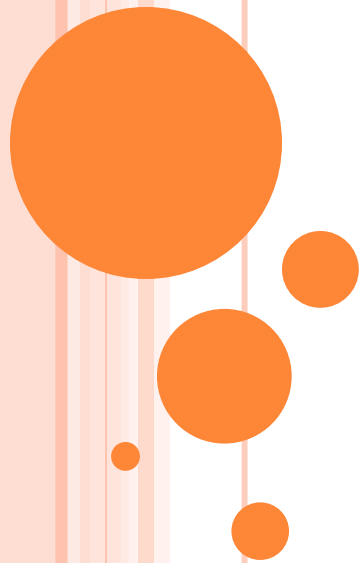


IMAGE PROCESSING

Histogram

Ch3



Histogram

In an image processing context, the histogram of an image normally refers to a histogram of the pixel intensity values. This histogram is a graph showing the number of pixels in an image at each different intensity value found in that image. For an 8-bit grayscale image there are 256 different possible intensities, and so the histogram will graphically display 256 numbers showing the distribution of pixels amongst those grayscale values. Histograms can also be taken of color images either individual histogram of red, green and blue channels can be taken, or a 3-D histogram can be produced, with the three axes representing the red, blue and green channels, and brightness at each point representing the pixel count. The exact output from the operation depends upon the implementation it may simply be a picture of the required histogram in a suitable image format, or it may be a data file of some sort representing the histogram statistics.

Histogram

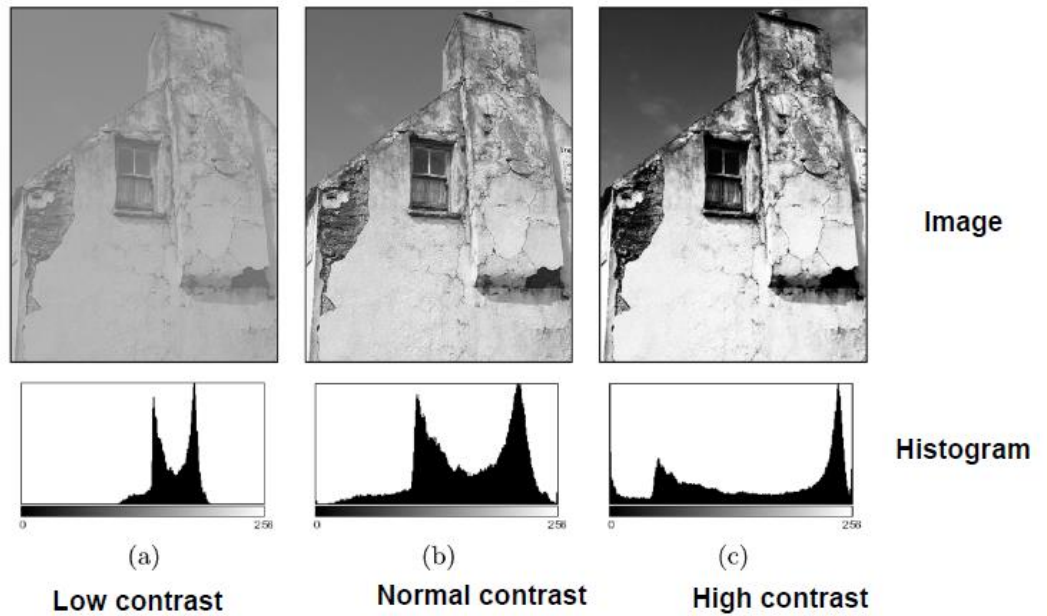
Using this graph we are able to see immediately:

1. Whether an image is dark or light, and high or low contrast.
2. Give us our first clues about what contrast enhancement would be appropriately applied to make the image more subjectively pleasing to an observer, or easier to interpret by succeeding image analysis operations.
3. The shape of histogram provides us with information about the number of distinct pixels in an image, called its Dynamic Range. High dynamic range means very bright and very dark parts in a single image (many distinct values).

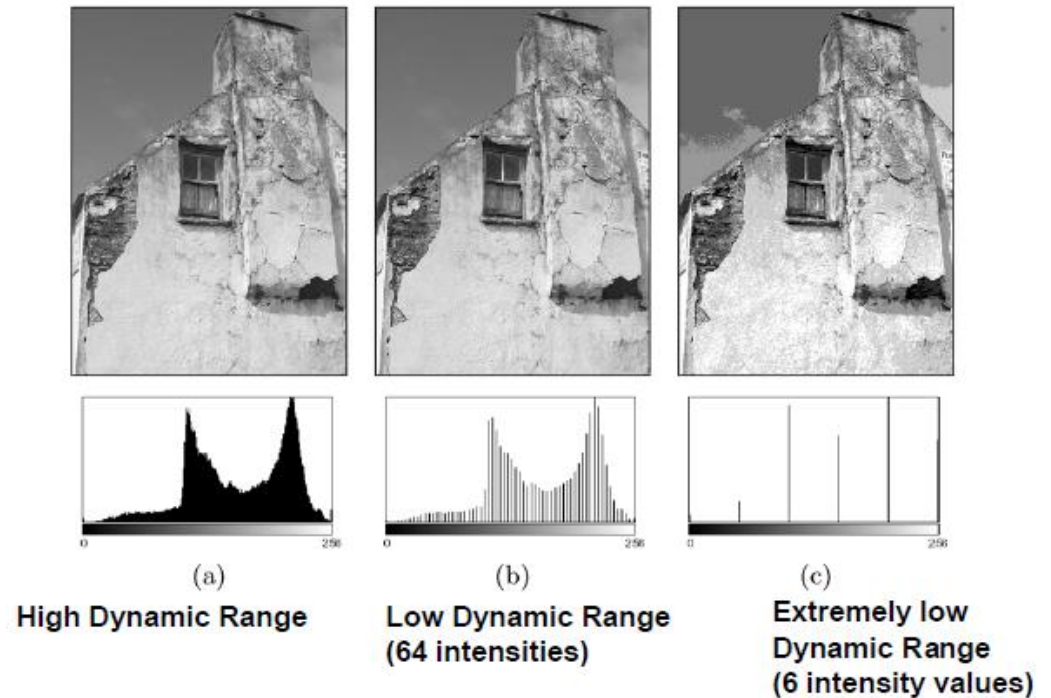


Histogram

Histograms and Contrast



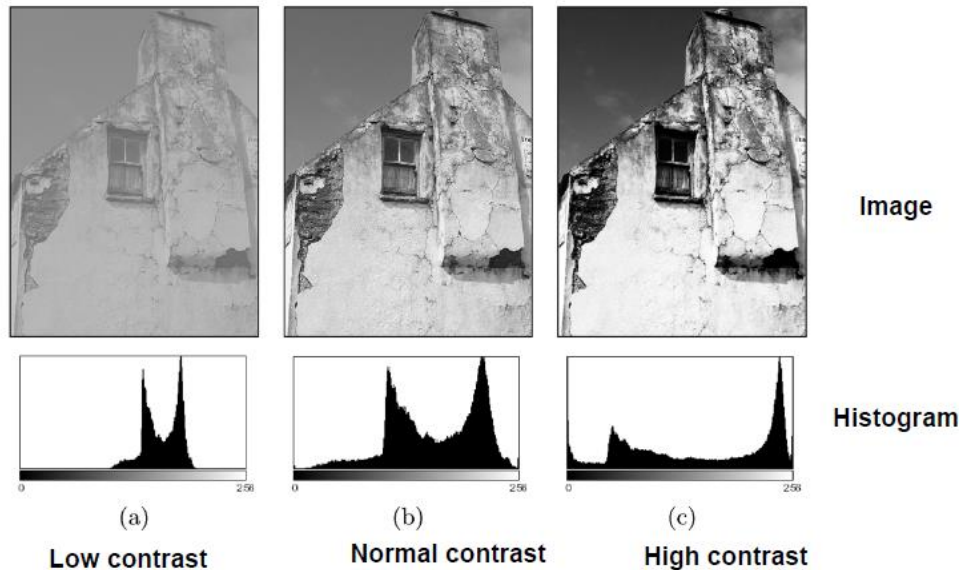
Histograms and Dynamic Range.



Histogram

So the shape of histogram provide us with information about nature of the image or sub image if we considering an object within the image. For example:

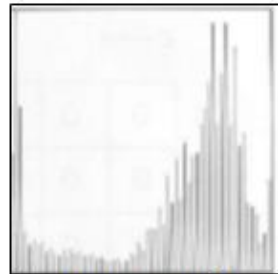
1. Very narrow histogram implies a low-contrast image.
2. Histogram skewed towards the high end implies a bright image.
3. Histogram with two major peaks, called bimodal, implies an object that is in contrast with the background.



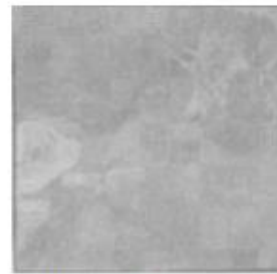
Histogram



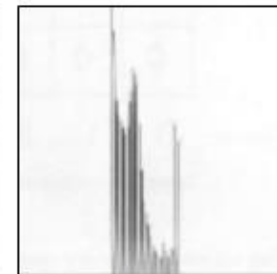
a. Object in Contrast with Background



Histogram of bimodal Shape



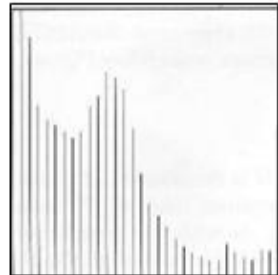
b. Low Contrast Image



Histogram Clustered



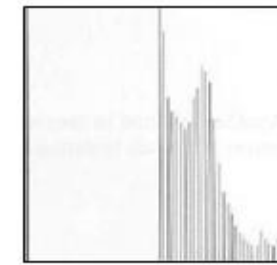
c. High Contrast Image



Histogram Spread-out



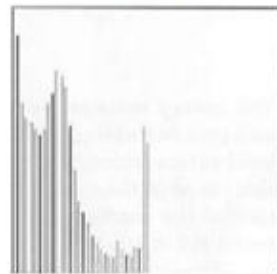
d. Bright Image



Histogram shifted to the Right



e. Dark Image



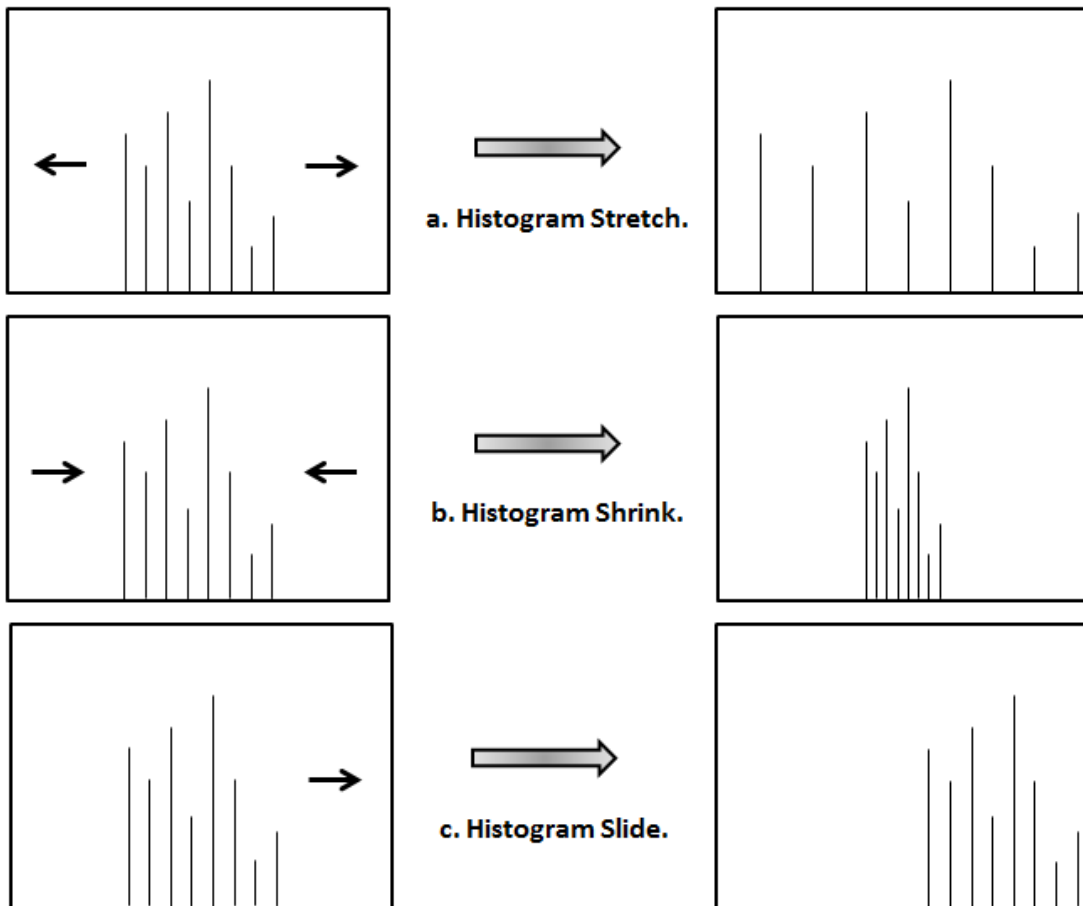
Histogram shifted to the Left

Examples of the different types of histograms



Histogram Modifications

The gray level histogram of an image is the distribution of the gray level in an image. The histogram can be modified by mapping functions, which will stretch, shrink (compress), or slide the histogram.



Histogram Modifications

The mapping function for **histogram stretch** can be found by the following equation:

$$\text{Stretch}(I(r, c)) = \left[\frac{I(r, c) - I(r, c)_{\min}}{I(r, c)_{\max} - I(r, c)_{\min}} \right] (MAX - MIN) + MIN$$

Where, $I(r, c)_{\max}$ is the largest gray- level in the image $I(r, c)$.

$I(r, c)_{\min}$ is the smallest gray- level in the image $I(r, c)$.

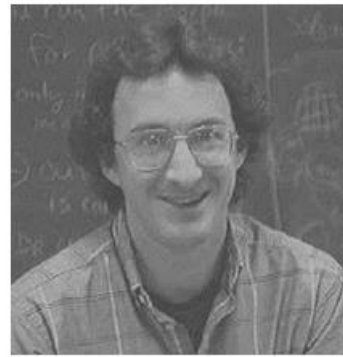
MAX and MIN correspond to the maximum and minimum gray – level values possible (for an 8-bit image these are 255 and 0).

This equation will take an image and stretch the histogram across the entire gray-level range which has the effect of increasing the contrast of a low contrast image.

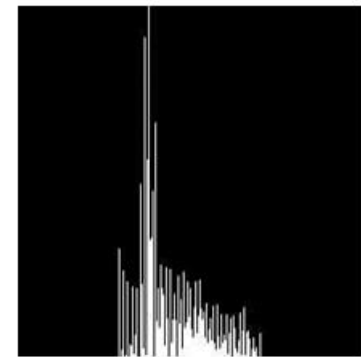


Histogram Modifications

histogram stretch



Low-contrast image



Histogram of low-contrast image

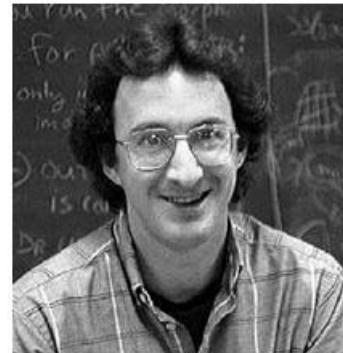
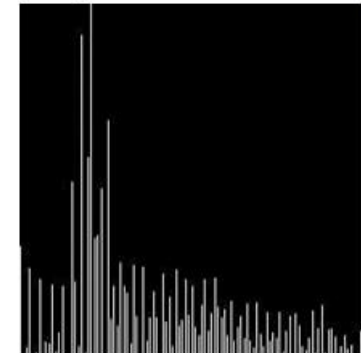


Image after histogram stretching



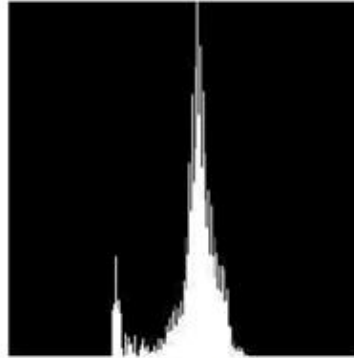
Histogram of image after stretching

In most of the pixel values in an image fall within small range, but a few outlines force the histogram to span the entire range, a pure histogram stretch will not improve the image. In this case it is useful to allow a small proceeding of the pixel values to be clipped at the low and high end of the range (for an 8-bit image this means truncating at 0 and 255).

Histogram Modifications



Original Image



Histogram of the original image

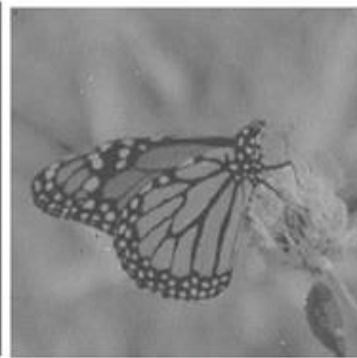
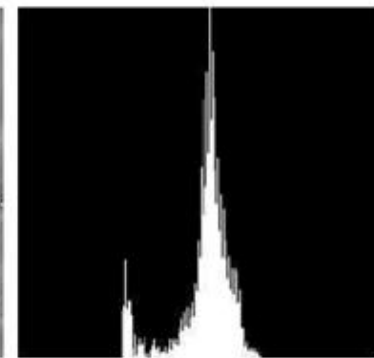


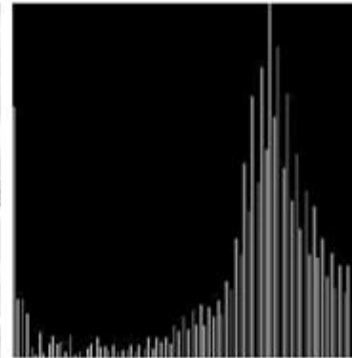
Image after histogram stretching without clipping



Histogram of the image



Image after histogram stretching with clipping 3% low and high value



Histogram of the image

Histogram Stretching (Clipping).



Histogram Modifications

Q/ Apply histogram stretching for the following sub image ?

$$\begin{pmatrix} 7 & 12 & 8 \\ 20 & 9 & 6 \\ 10 & 15 & 1 \end{pmatrix}$$

Q/ Apply histogram stretching for the following sub image ?

$$\text{Stretch}(I(r,c)) = \left[\frac{I(r,c) - I(r,c)_{\min}}{I(r,c)_{\max} - I(r,c)_{\min}} \right] (MAX - MIN) + MIN$$

$$I(r,c)_{\min} = 1 ; I(r,c)_{\max} = 20 ; Max = 255 ; Min = 0$$

$$I(0,0) = [7-1 / 20-1] * [255 - 0] + 0 = 80.5$$

$$I(0,1) = [12-1 / 20-1] * [255 - 0] + 0 = 147.6$$

$$I(0,2) = [8-1 / 20-1] * [255 - 0] + 0 = 93.9$$

$$I(1,0) = [20-1 / 20-1] * [255 - 0] + 0 = 255$$

$$I(1,1) = [9-1 / 20-1] * [255 - 0] = 107.3$$

$$I(1,2) = [6-1 / 20-1] * [255 - 0] + 0 = 67.1$$

$$I(2,0) = [10-1 / 20-1] * [255 - 0] + 0 = 120.7$$

$$I(2,1) = [15-1 / 20-1] * [255 - 0] + 0 = 187.8$$

$$I(2,2) = [1-1 / 20-1] * [255 - 0] + 0 = 0$$

$$\text{Istretch} = \begin{pmatrix} 80.5 & 147.6 & 93.9 \\ 255 & 107.3 & 67.1 \\ 120.7 & 187.8 & 0 \end{pmatrix}$$

Histogram Modifications

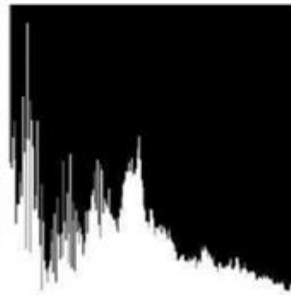
The opposite of a histogram stretch is a **histogram shrink**, which will decrease image contrast by compressing the gray levels. The mapping function for a histogram shrinking can be found by the following equation:

$$\text{Shrink}(I(r, c)) = \left[\frac{\text{shrink}_{\max} - \text{shrink}_{\min}}{I(r, c)_{\max} - I(r, c)_{\min}} \right] [I(r, c) - I(r, c)_{\min}] + \text{shrink}_{\min}$$

Shrink_{\max} and shrink_{\min} correspond to the maximum and minimum desired in the compressed histogram. In general, this process produces an image of reduced contrast and may **not seem to be useful an image enhancement**.



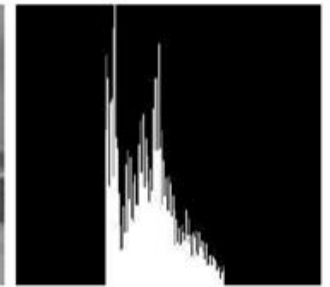
Original Image



Histogram of original image



Image after histogram shrink to the range [75, 175]



Histogram of the image

Histogram Modifications

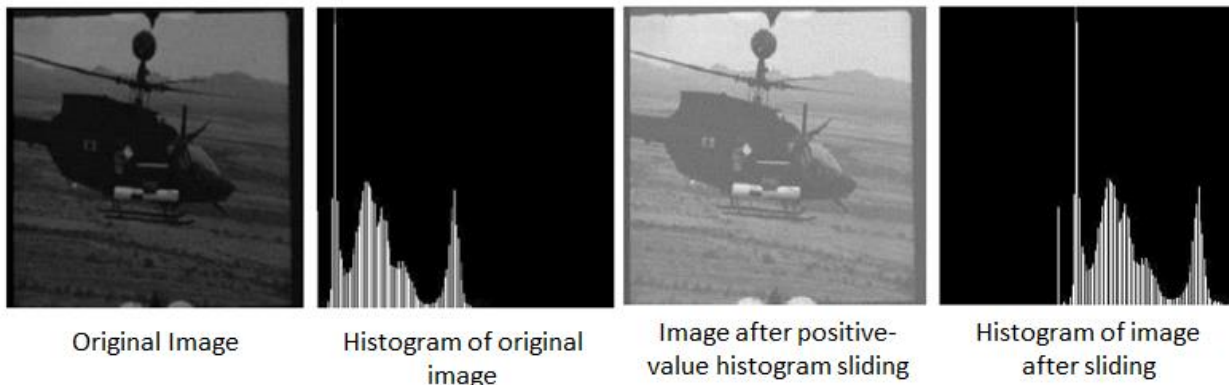
The **histogram slide** techniques can be used to make an image either darker or lighter but retain the relationship between gray-level values.

This can be accomplished by simply adding or subtracting a fixed number for all the gray-level values, as follows:

$$\text{Slide } (I(r,c)) = I(r,c) + \text{OFFSET.}$$

Where OFFSET values is the amount to slide the histogram.

In this equation, a positive OFFSET value will increase the overall brightness; whereas a negative OFFSET will create a darker image.




Histogram Equalization

Is a popular technique used for improving the appearance of a poor image. It's a function is similar to that of a histogram stretch but often provides more visually pleasing results across a wide range of images.

Histogram equalization is a technique where the histogram of the resultant image is as flat as possible (with histogram stretching the overall shape of the histogram remains the same).

The results in a histogram with a mountain grouped closely together to "spreading or flattening histogram makes the dark pixels appear darker and the light pixels appear lighter" (the key word is "appear" the dark pixels in a photograph cannot be any darker. If, however, the pixels that are only slightly lighter become much lighter, then the dark pixels will appear darker).



Histogram Equalization

The histogram equalization process for digital images consists of four steps:

1. Find the running sum of the histogram values
2. Normalize the values from step1 by dividing by total number of pixels.
3. Multiply the values from step2 by the maximum gray level value and round.
4. Map the gray-level values to the results from step 3, using a one-to-one correspondence.

The following example will help to clarify this process.



Histogram Equalization

Example: We have an image with 3 bit /pixel, so the possible range of values is 0 to 7. We have an image with the following histogram:

Gray-level value	0	1	2	3	4	5	6	7
No of Pixel Histogram value	10	8	9	2	14	1	5	2

Step 1: Great a running sum of histogram values. This means that the first values is 10, the second is $10+8=18$, next is $10+8+9=27$, and soon. Here we get 10,18,29,43,44,49,51.

Step 2: Normalize by dividing by total number of pixels. The total number of pixels is $10+8+9+2+14+1+5+0=51$.

Step 3 : Multiply these values by the maximum gray – level values in this case 7 , and then round the result to the closet integer. After this is done we obtain 1,2,4,4,6,6,7,7.

Step 4: Map the original values to the results from step3 by a one-to-one correspondence.



Histogram Equalization

Show the previous steps by table:

Gray-level value	0	1	2	3	4	5	6	7
No of Pixel	10	8	9	2	14	1	5	2
Run Sum	10	18	27	29	43	44	49	51
Normalized	10/51	18/51	27/51	29/51	43/51	44/51	49/51	51/51
Multiply by 7	1	2	4	4	6	6	7	7
New Value	1	2	4	4	6	6	7	7

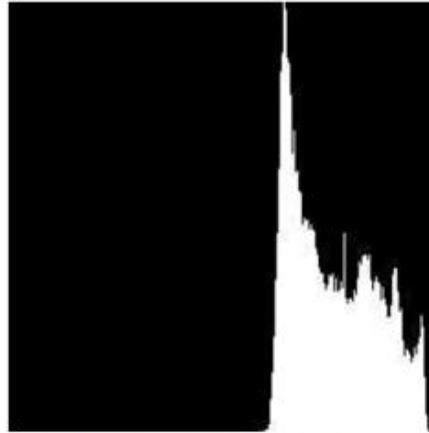
All pixel in the original image with gray level 0 are set to 1, values of 1 are set to 2, 2 set to 4, 3 set to 4, and so on (see figure (3-9)) histogram equalization, you can see the original histogram and the resulting histogram equalized histogram. Although the result is not flat, it is closer to being flat than the original.



Histogram Equalization



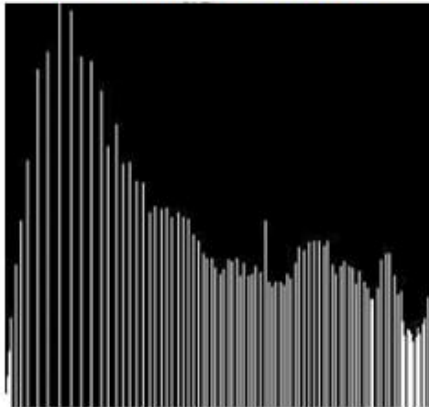
Original Image



Histogram of original image



Image after histogram equalization



Histogram after equalization

Histogram Equalization



Histogram Equalization

Q/ Apply histogram equalization technique for the following (3-bit) sub image?

H.W.

0	3	1	4	0	4	1	0	5
2	0	4	1	1	2	4	4	7
0	4	0	4	4	1	2	6	7
5	2	4	0	4	7	6	6	5
2	0	2	4	0	4	2	2	6
0	1	1	0	2	0	3	1	6




Histogram Features

The histogram features that we are considered are statically based features where the histogram is used as a model of the probability distribution of the gray levels.

These statistical features provide us with information about the characteristic of the gray – level distribution for the image or sub image. We define the **first – order histogram probability $P(a)$** as :

$$P(g) = \frac{N(g)}{M}$$

M is the number of pixels in the image or sub image (if the entire image is under consideration, then $M= N^2$ for $N \times N$), and $N(g)$ is the number of pixels at gray level g . as with any probability distribution, **all values for $P(g)$ are less than or equal to 1**, histogram probability are mean, standard deviation, skew, energy and entropy.



Histogram Features

1.Mean: the mean is the average value, so it tells us something about the general brightness of the image. A bright image will have a high mean, and a dark image will have a low mean. We will use L as the total number of gray levels available, so the gray levels range from 0 to L-1. For example, for typical 8-bit image data, L is 256 and ranges from 0 to 255. We can define the mean as follows:

$$\bar{g} = \sum_{g=0}^{L-1} gP(g) = \sum \sum \frac{I(r, c)}{M}$$

If we use the **second form of the equation**, we sum over the rows and columns corresponding to the pixels in the image or sub image under consideration.



Histogram Features

2. Standard deviation: Which is also known as the square root of the variance, tell us something about the contrast. It describes the spread in the data, so a **high contrast** image will have a high variance, and a **low – contrast** image will have a low variance. It is defined as follows:

$$\sigma = \sqrt{\sum_{g=0}^{L-1} (g - \bar{g})^2 P(g)}$$




Histogram Features

3.Energy :The energy measure tell us something about how the gray level are distributed

$$Energy = \sum_{g=0}^{L-1} [P(g)]^2$$

The energy measure has a maximum value of 1 for an image with a constant value and gets increasingly smaller as the pixel values are distributed a cross more gray level values(remember that all the $P(g)$ values are less than or equal to 1). The larger this value is, the easier it is to compress the image data. If the energy is high, it tells us that the number of gray levels in the image is few, that is, the distribution is concentrated in only a small number of different gray levels.



Histogram Features

4.Entropy: the entropy is a measure that tells us how many bits we need to code the image data and given by :

$$\text{Entropy} = \sum_{g=0}^{L-1} P(g) \log_2 [P(g)]$$

As the pixel values in the image are distributed among more gray levels, the entropy increases. This measure tends to vary inversely with the energy.

Note: ($\log_2 = \log_{10} * 3.33$).

